



# ANTENNAS , WAVE PROPAGATION &TV ENGG

# Topics to be covered

- Directivity
- Gain

# Directivity

## Definition 1:

*The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.*

$$D = U_{\text{given direc}} / U_{\text{av}}$$

## Definition 2:

The avg U is obtained by power radiated by  $4\pi$ , (rad int of isotropic antenna)

*The ratio of its radiation intensity (U) in a given direction over that of an isotropic source.*

$$D = U/U_0$$

# Antenna Directivity

Total power radiated  $P_{rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \varphi) \sin \vartheta d\vartheta d\varphi$

Average radiation intensity  $U_{avg} = \frac{P_{rad}}{4\pi}$

$$D = \frac{U}{U_{avg}} = \frac{U}{P_{rad}/4\pi}$$

- **NOTE:** **D Has no units**

# Antenna Directivity

- **Definition3:**

If the direction is not specified then we calculate directivity in the direction of maxima

*The ratio of radiation intensity in max. direction to the radiation intensity of isotropic source. ( $D_0$ )*

$$D_{max} = D_0 = U_{max}/U_0 = 4\pi U_{max}/P_{rad}$$

D=directivity

$D_0$ =

U=radiation intensity

$U_{max}$ =

$U_0$ =

$P_{rad}$ =

## Properties of D

- D=1 for isotropic source
- D>1 for non isotropic

(max. directivity is greater than 1,  $U_{max} > U_0$ )

- $D = 4\pi / \Omega_A$

smaller the beam area.....

# Antenna Radiation Efficiency

**Conduction and dielectric losses of an antenna ( $I^2R$  losses) =  $e_{cd}$  efficiency.**

Let  $R_{cd}$  represent the actual losses due to conduction and dielectric heating. Then the efficiency is given as

$$K / e_{cd} = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{cd} + R_{rad}}$$

Practically losses are there, so  $R_{cd} + R_{rad} > R_{cd}$

**Practical antenna:  $K < 1$**

**Ideal antenna:  $K = 1$**

# Overall Antenna Efficiency

The overall antenna efficiency is a coefficient that accounts for all the different losses present in an antenna system.

$$e = \overbrace{e_p e_r e_c e_d}^{e_t} = e_p \cdot e_r e_{cd}$$

$e_p$  = polarization mismatches

$e_r$  = reflection efficiency (impedance mismatch)

$e_c$  = conduction losses

$e_d$  = dielectric losses

$e_{cd}$  = conductor & dielectric losses

If antenna is perfectly matched,  $e_r=0$

$e_t = e_{cd}$ .



# Reflection Efficiency

The reflection efficiency through a reflection coefficient ( $\Gamma$ ) at the input (or feed) to the antenna.

$$e_r = 1 - |\Gamma|^2$$

$$\Gamma = \frac{R_{input} - R_{generator}}{R_{input} + R_{generator}}$$

$$R_{input} = \text{antenna input impedance}(\Omega)$$

$$R_{output} = \text{generator output impedance}(\Omega)$$

# Antenna Gain

- Directivity=  $D = \frac{4\pi U}{P_{rad}}$
- Gain=  $D = \frac{4\pi U}{P_{in}}$
- We know  $K = P_{rad}/P_{in}$

- $G = \frac{K 4\pi U}{P_{rad}}$

- So

$$G = KD$$

# Gain

- It can be measured by **comparing radiation intensity of the antenna under test (AUT) with a reference antenna.**
- Ref antenna =dipole, horn (whose gain can be calculated)

- $G = U(\text{AUT})/U(\text{ref})$

If ref antenna is isotropic

$$G = U(\text{AUT})/P_{\text{in}}/4\pi$$

$$\mathbf{G = 4\pi U (AUT)/P_{\text{in}}}$$

# Antenna Gain

- The directivity and gain are measures of the ability of an antenna to concentrate power in a particular direction.
- Directivity – power **radiated** by antenna ( $P_0$ )
- Gain – power **delivered** to antenna ( $P_T$ )

$$G = KD$$

$$K = \frac{P_{rad}}{P_{in}}$$

- K: radiation efficiency (50% - 75%)
- G has no units
  - Usually relates to the peak directivity of the main radiation lobe
  - Often expressed in dB
  - Known as “Absolute Gain” or “Isotropic Gain”

# Antennas – Gain

## Gain

The power gain,  $G$ , of an antenna is very much like its directive gain, but also takes into account efficiency

$$G(\theta, \phi) = eD(\theta, \phi)$$

The maximum power gain

$$G_{\max} = eD_{\max}$$

The maximum power gain is often expressed in dB.

$$G_{\max} (dB) = 10 \log_{10} (G_{\max})$$

### Example

D8.3: Suppose an antenna has  $D = 4$ ,  $R_{rad} = 40 \Omega$  and  $R_{diss} = 10 \Omega$ . Find antenna efficiency and maximum power gain. (Ans:  $e = 0.80$ ,  $G_{max} = 3.2$ ).

Antenna efficiency

$$e = \frac{R_{rad}}{R_{rad} + R_{diss}} = \frac{40}{10 + 40} = 0.8 \text{ (or) } 80\%$$

Maximum power gain

$$G_{max} = eD_{max} = (4)(0.8) = 3.2$$

Maximum power gain in dB

$$G_{max} (dB) = 10 \log_{10} (G_{max}) = 10 \log_{10} (3.2) = 5.05$$

# PFD vs. Antenna Gain

$$\begin{aligned} S(\vartheta, \varphi) &= \frac{\Phi(\vartheta, \varphi) \Delta \vartheta \Delta \varphi}{(r \Delta \vartheta)(r \Delta \varphi)} = \frac{\Phi(\vartheta, \varphi)}{r^2} \\ &= G(\vartheta, \varphi) \frac{P_0}{4\pi r^2} \\ &= G(\vartheta, \varphi) S_0 \end{aligned}$$

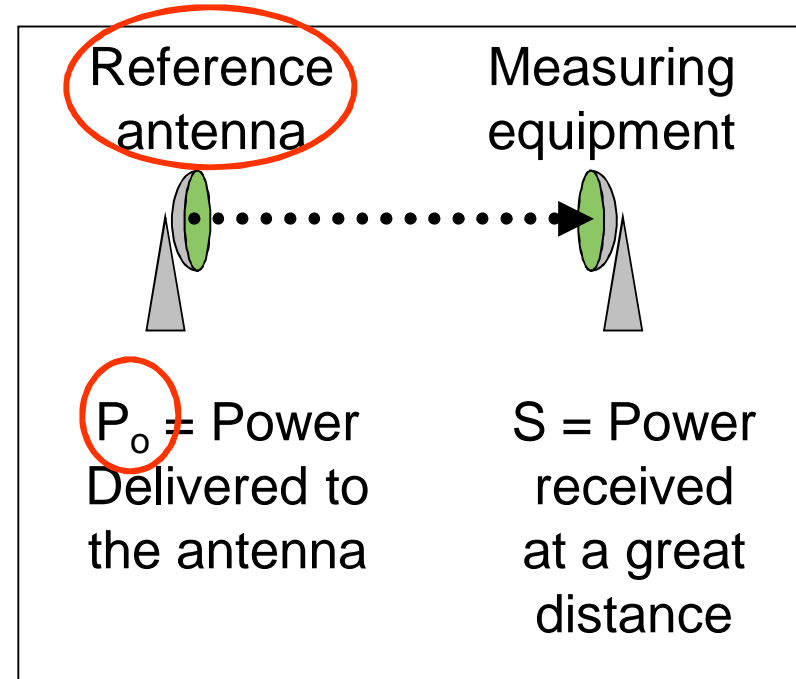
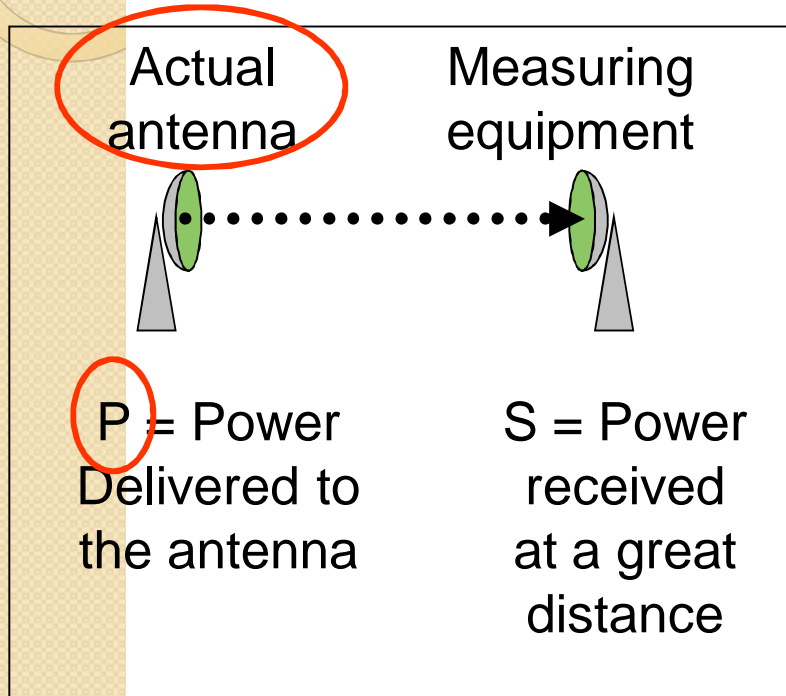
$S_0$  = PFD produced by a loss-less isotropic radiator

# Other Definitions of Gain

- For practical purposes, the antenna gain is defined as the ratio (usually in dB), of the power required at the input of a loss-free **reference antenna** to the power supplied to the input of the given antenna to produce, in a given direction, the same field strength or the same power flux-density at the same distance.
- When not specified otherwise, the gain refers to the direction of maximum radiation.
- The gain may be considered for a specified polarization. [RR 154]



# Antenna Gain



$$\text{Antenna Gain (in the specific direction)} = P / P_o$$

# Reference Antennas

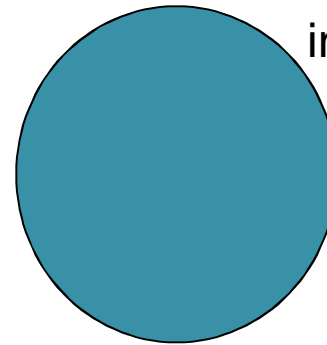
- Isotropic radiator
  - isolated in space ( $G_i$ , absolute gain, or isotropic gain)
- Half-wave dipole
  - isolated in space, whose equatorial plane of symmetry contains the given direction ( $G_d$ )
- Short vertical antenna
  - (much shorter than  $\lambda/4$ ), close to, and normal to a perfectly conducting plane which contains the given direction ( $G_v$ )

# Reference Antennas (1)

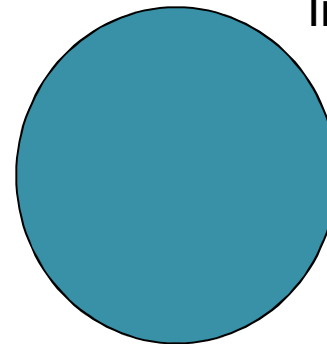
## Isotropic antenna

- Sends (receives) energy equally in (from) all directions
- Gain = 1 (= 0 dB)
- When supplied by  $P$ , produces at distance  $r$  power flux density =  $P / (4\pi r^2)$
- Theoretical concept, cannot be physically realized

Radiation pattern  
in vertical plane



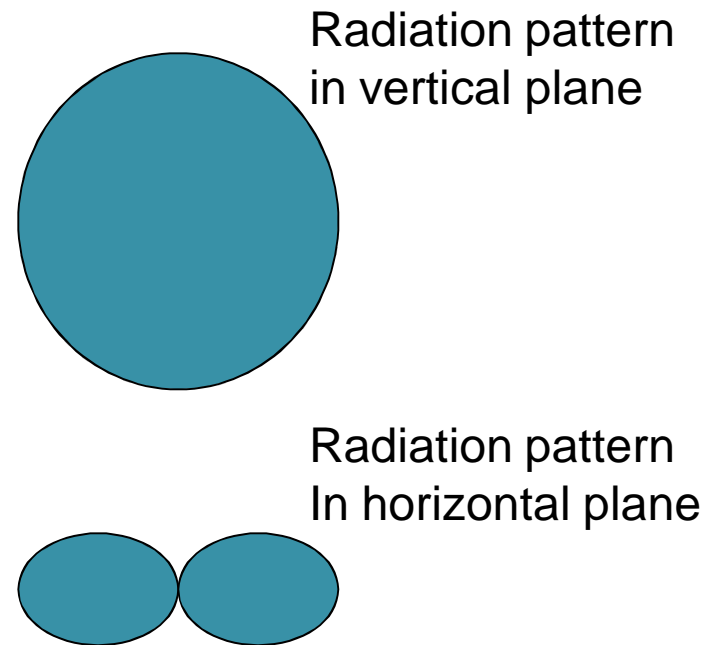
Radiation pattern  
In horizontal plane



# Reference Antennas (2)

## Half-Wave Dipole

- Linear antenna, realizable
- Gain = 1.64 (= 2,15 dB) in the direction of maximum radiation
- Figure-eight-shaped radiation pattern in the dipole plane, omnidirectional (circular) in the orthogonal plan



## Effective aperture and aperture efficiency

Receiving antenna extracts power from incident wave

$$P_{rec} = S_{in} \cdot A_e$$

Aperture and beam area are linked:  $A_e = \frac{\lambda^2}{\Omega_A}$

For some antennas, there is a clear physical aperture and an aperture efficiency can be defined

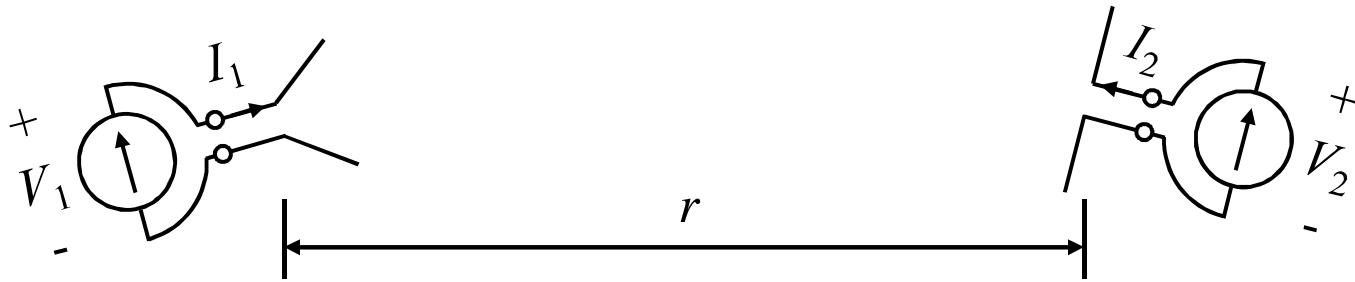
$$\epsilon_{ap} = \frac{A_e}{A_p}$$



## Reciprocity

- Transmission and reception antennas can be used interchangeably
- Medium must be linear, passive and isotropic
- Caveat: Antennas are usually optimised for reception or transmission not both !

# Receiving Antennas and Reciprocity



For a linear two-port

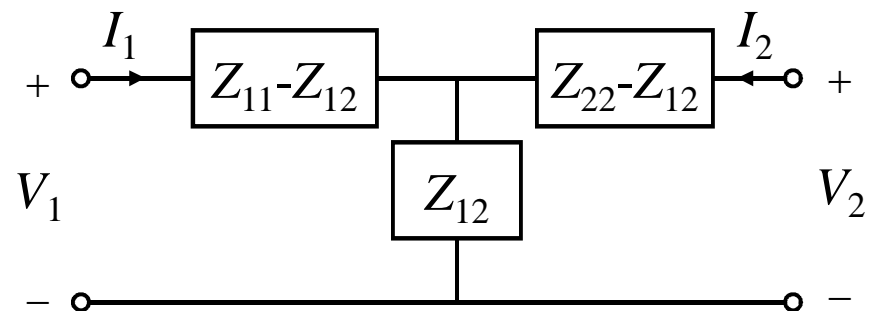
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Reciprocity

$$Z_{12} = Z_{21}$$

Equivalent Circuit

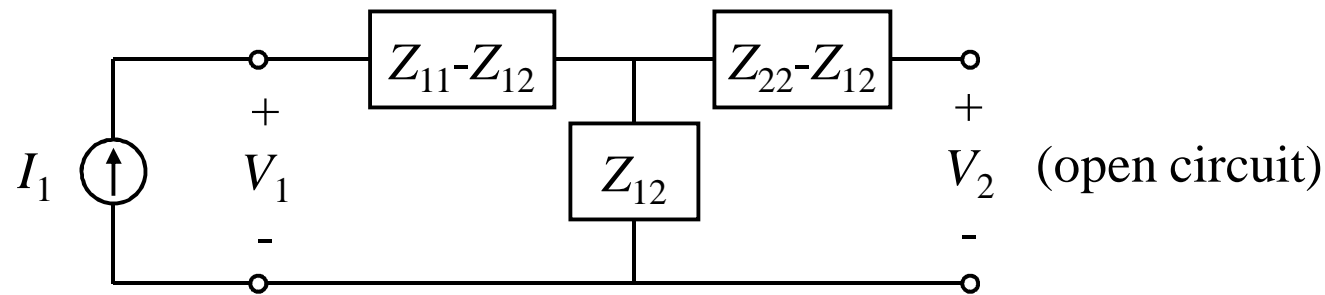


If  $I_2 = 0$ ,  $V_2 = Z_{12}I_1 \sim 1/r$

For  $r$  large,

$$|Z_{12}| \ll |Z_{11}|, |Z_{22}|$$

# Circuit Relation for Radiation into Free Space



$$V_1 = Z_{11}I_1$$

$$V_2 = V_{OC} = Z_{12}I_1$$

Transmitted power

$$P_T = (1/2)\text{Re}(V_1 I_1^*) = (1/2)\text{Re}(Z_{11}|I_1|^2) = (1/2)R_{r1}|I_1|^2$$

where  $R_{r1}$  = radiation resistance of antenna 1

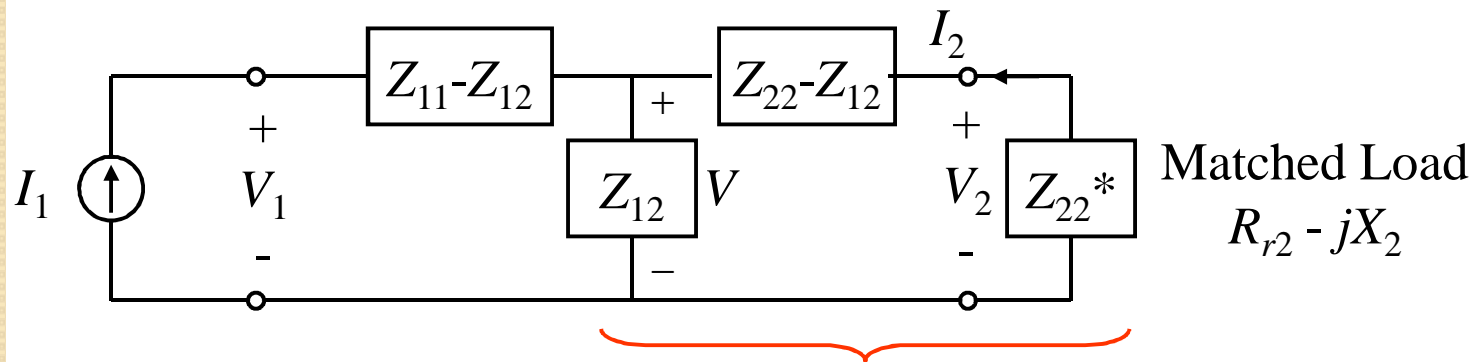
Therefore :  $Z_{11} = R_{r1} + jX_1$

Similarly :  $Z_{22} = R_{r2} + jX_2$

where  $R_{r2}$  = radiation resistance of antenna 2



# Received Power and Path Loss Ratio



Current  $I_1$  divides between branches 
$$I_2 = -I_1 \frac{Z_{12}}{Z_{12} + (Z_{22} - Z_{12} + Z_{22}^*)} = -I_1 \frac{Z_{12}}{2R_{r2}}$$

Received Power for Matched Load 
$$P_R = \frac{1}{2} |I_2|^2 R_{r2} = \frac{1}{2} \left| \frac{I_1 Z_{12}}{2R_{r2}} \right|^2 = |I_1|^2 \frac{|Z_{12}|^2}{8R_{r2}}$$

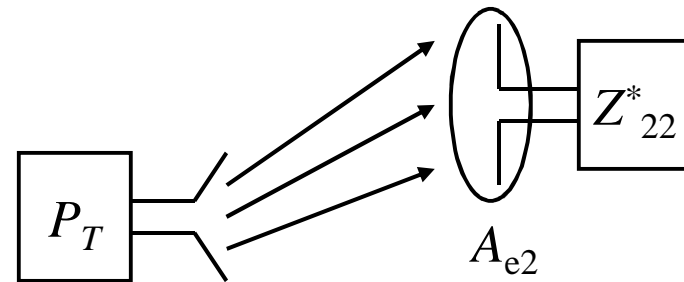
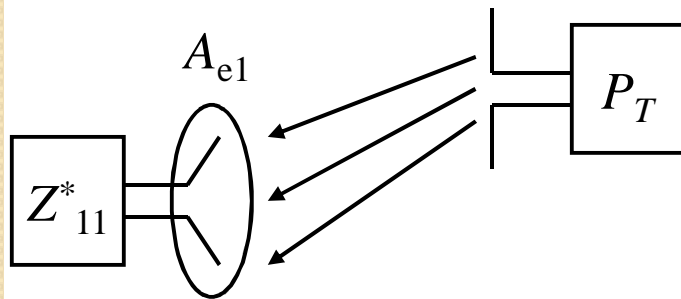
Path Gain 
$$PG \equiv \frac{P_R}{P_T} = \frac{|I_1|^2 |Z_{12}|^2 / 8R_{r2}}{|I_1|^2 R_{r1} / 2} = \frac{|Z_{12}|^2}{4R_{r1} R_{r2}}$$

Final expression for  $PG$  is the same if antenna 2 radiates and antenna 1 receives.

# Effective Area of Receiving Antenna

Effective Area =  $A_e$

$$P_R = \underline{P} \cdot \underline{a}_r A_e = P_T \frac{g(\theta, \phi)}{4\pi r^2} A_e$$



$$PG = \frac{P_R}{P_T} = \frac{g_2 A_{e1}}{4\pi r^2}$$

and by reciprocity

$$PG = \frac{P_R}{P_T} = \frac{g_1 A_{e2}}{4\pi r^2}$$

Therefore  $g_2 A_{e1} = g_1 A_{e2}$  or  $\frac{A_{e1}}{g_1} = \frac{A_{e2}}{g_2} =$  same for all antennas