COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- Decidability
- Decidable/Undecidable problems

ACCEPTING: DEFINITION

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be a TM.
- T accepts a string w in Σ^* if $(s,\underline{\Delta}w) \vdash_T^* (h, \underline{\Delta}1)$.
- T accepts a language $L \subseteq \Sigma^*$ if, for any string w in L, T accepts w.

CHARACTERISTIC FUNCTION

- For any language $L \subseteq \Sigma^*$, the characteristic function of L is the function $\chi_L(x)$ such that
  - $\chi_L(x) = 1$ if $x \in L$
  - $\chi_L(x) = 0$ otherwise
- Example

  Let $L = \{\omega \in \{0,1\}^* \mid n_1(\omega) < n_0(\omega) < 2n_1(\omega) \}$, where $n_x(\omega)$ is the number of x's in $\omega\}$.

  - $\chi_L(\omega) = 1$ if $n_1(\omega) < n_0(\omega) < 2n_1(\omega)$
  - $\chi_L(\omega) = 0$ otherwise

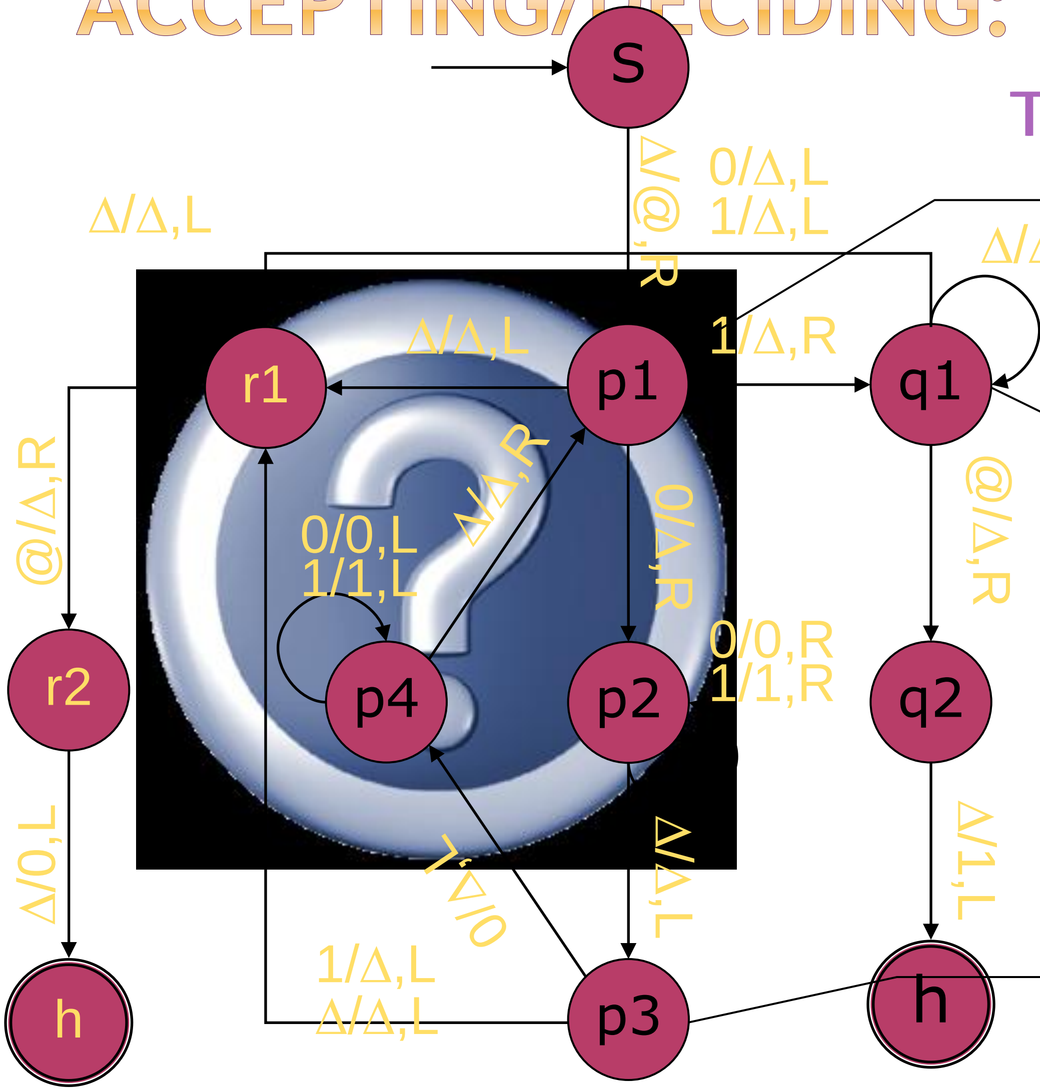
DECIDING: DEFINITION

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be a TM.
- T decides a language $L \subseteq \Sigma^*$ if T computes the characteristic function of L.
- T decides a language $L \subseteq \Sigma^*$ if
  - for any string w in L, T halts on w with output 1,
  - for any string w in $\overline{L}$, T halts on w with output 0.

ACCEPTING/DECIDING: EXAMPLE

TM deciding L = {0^n 1 0^m | n≥2}

TM accepting L = {0^n 1 0^n | n≥0}



Hang when input = 0^{2n}

Hang when input = 0^n 1 … 0^{n+m}

If the input x is in L,

T halts with output 1.

If the input x is not in L,

T hangs.

Hang when input = 0^{n+m} …0^n

RECURSIVELY ENUMERABLE LANGUAGES

- A language L is recursively enumerable if there is a Turing machine T accepting L.
- A language L is Turing-acceptable if there is a Turing machine T accepting L.
- Example:

  $\{0^n10^n | n \geq 0\}$ is a recursively-enumerable language.

RECURSIVE LANGUAGES

- A language L is recursive if there is a Turing machine T deciding L.
- A language L is Turing-decidable if there is a Turing machine T deciding L.
- Example:

  $\{0^n10^n \mid n \geq 0\}$ is a recursive language.

CLOSURE PROPERTIES OF THE CLASS OF RECURSIVE LANGUAGES

CLOSURE PROPERTY UNDER COMPLEMENTATION

Theorem: Let L be a recursive language over Σ. Then, $\overline{L}$ is recursive.

Proof:

Let L be a recursive language over Σ.

Then, there exists a TM T computing χ_L.

Construct a tape TM M computing $\chi_{\overline{L}}$. as follows:

$\rightarrow T \rightarrow T_{moveRight} \xrightarrow{0} T_{write1}$

1

T_{write0}

Then, $\overline{L}$ is recursive.

CLOSURE PROPERTY UNDER UNION

Theorem: Let L_1 and L_2 be recursive languages over Σ. Then, $L_1 \cup L_2$ is recursive.

Proof:

Let L_1 and L_2 be recursive languages over Σ.

Then, there exist TM's T_1 and T_2 computing χ_{L1} and χ_{L2}, respectively.

Construct a 2-tape TM M as follows:

$\rightarrow T_{copyTape1ToTape2} \rightarrow T_1 \rightarrow T_{moveRight} \xrightarrow{0} T_{copyTape2ToTape1} \rightarrow T_2$

1

CLOSURE PROPERTY UNDER UNION

$\rightarrow T_{copyTape1ToTape2} \rightarrow T_1 \rightarrow T_{moveRight} \xrightarrow{0} T_{copyTape2ToTape1} \rightarrow T_2$

1

If the input w is not in L_1 and L_2, $\chi_{L1}(w)$ and $\chi_{L2}(w)=0$. Thus, both T_1 and T_2 must run, and M halts with output 0.

If the input w is in L_1, $\chi_{L1}(w)=1$. Thus, M halts with output 1.

If the input w is not in L_1 but is in L_2, $\chi_{L1}(w)=0$ and $\chi_{L2}(w)=1$. Thus, M halts with output 1.

That is, M computes characteristic function of χ_L.

Then, $L_1 \cup L_2$ is recursive.

CLOSURE PROPERTY UNDER INTERSECTION

<u>Theorem:</u> Let L_1 and L_2 be recursive languages over Σ. Then, $L_1 \cap L_2$ is recursive.

Proof:

Let L_1 and L_2 be recursive languages over Σ.

Then, there exist TM's T_1 and T_2 computing χ_{L1} and χ_{L2}, respectively.

Construct a 2-tape TM M as follows:

$\rightarrow T_{copyTape1ToTape2} \rightarrow T_1 \rightarrow T_{moveRight} \xrightarrow{1} T_{copyTape2ToTape1} \rightarrow T_2$

0

CLOSURE PROPERTY UNDER INTERSECTION

$\rightarrow T_{copyTape1ToTape2} \rightarrow T_1 \rightarrow T_{moveRight} \xrightarrow{1} T_{copyTape2ToTape1} \rightarrow T_2$

0

If the input w is in $L_1 \cap L_2$, $\chi_{L1}(w)$ and $\chi_{L2}(w)=1$. Thus, M halts with output 1.

If the input w is not in L_1, $\chi_{L1}(w)=0$. Thus, M halts with output 0.

If the input w is in L_1 but is not in L_2, $\chi_{L1}(w)=1$ and $\chi_{L2}(w)=0$. Thus, M halts with output 0.

That is, M computes characteristic function of $\chi_{L1 \cap L2}$.

Then, $L_1 \cap L_2$ is recursive.