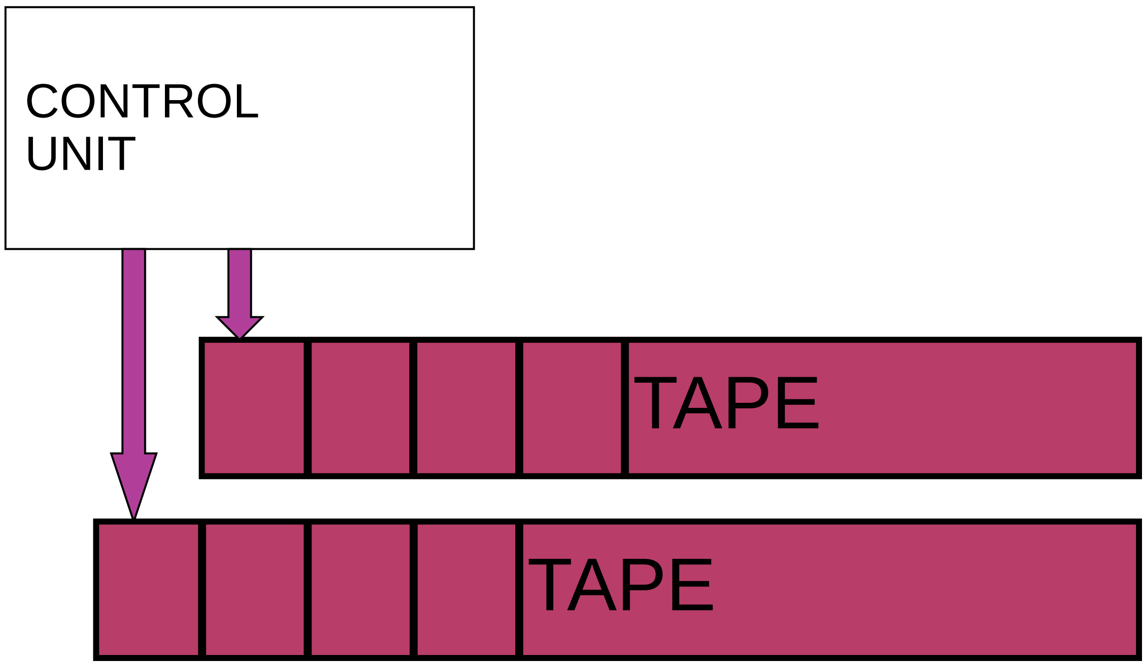
COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- Multitape TM

MULTITAPE TM

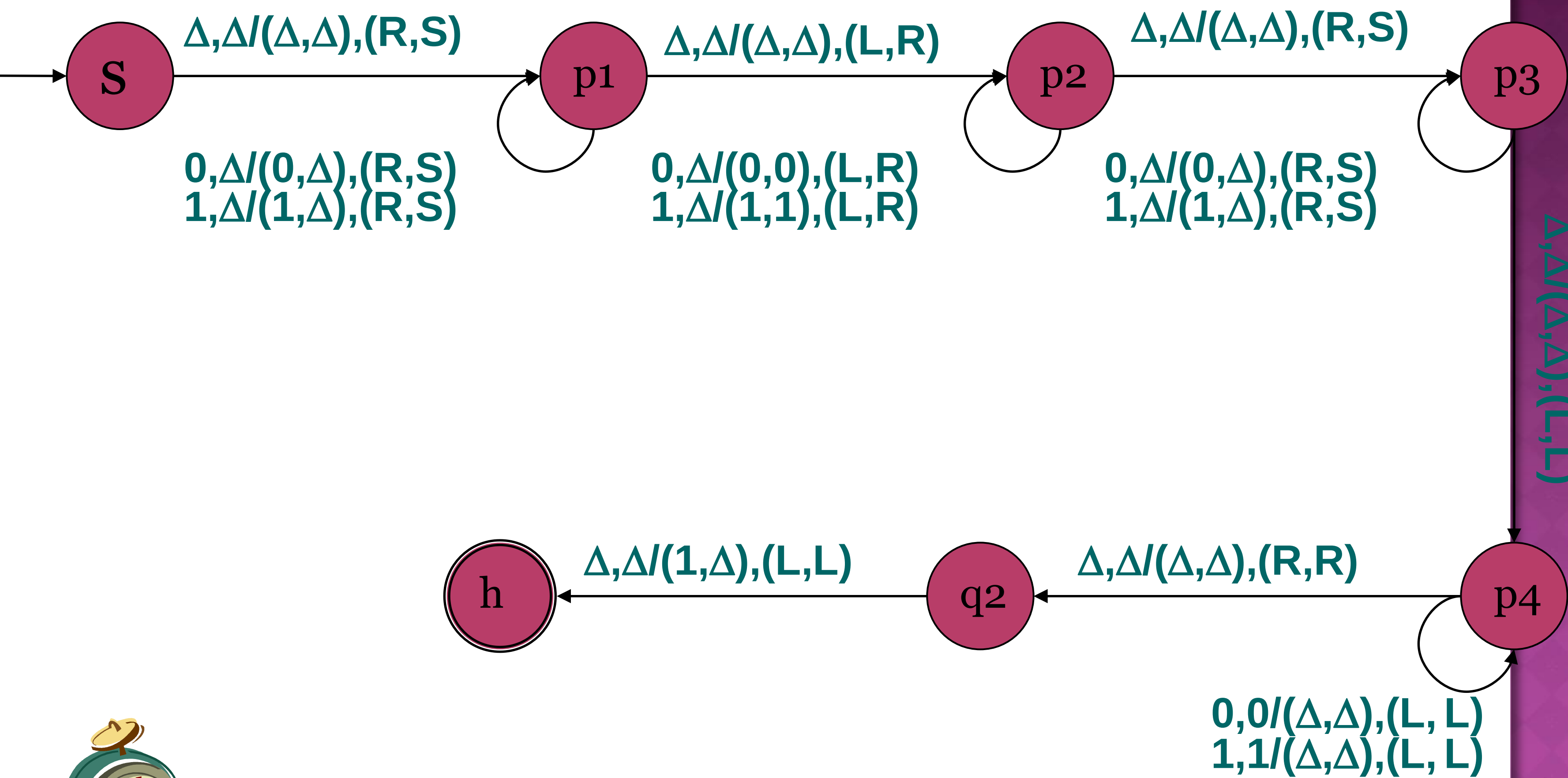
- TM with more than one tape.
- Each tape has its own tape head.
- Each tape is independent.



2-TAPE TURING MACHINE

- a quintuple $(Q, \Sigma, \Gamma, \delta, s)$, where
  - the set of states Q is finite, and does not contain the halt state h,
  - the input alphabet Σ is a finite set of symbols, not including the blank symbol Δ,
  - the tape alphabet Γ is a finite set of symbols containing Σ, but not including the blank symbol Δ,
  - the start state s is in Q, and
  - the transition function δ is a partial function from $Q \times (\Gamma \cup \{\Delta\})^2 \to Q \cup \{h\} \times (\Gamma \cup \{\Delta\})^2 \times \{\mathrm{L}, \mathrm{R}, \mathrm{S}\}^2$.

EXAMPLE OF 2-TAPE TURING MACHINE



EQUIVALENCE OF 2-TAPE TM AND SINGLE-TAPE TM

Theorem:

For any 2-tape TM T, there exists a single-tape TM M such that for any string α in Σ^*:

- if T halts on α with β on its tape, then M halts on α with β on its tape, and
- if T does not halt on α, then M does not halt on α.

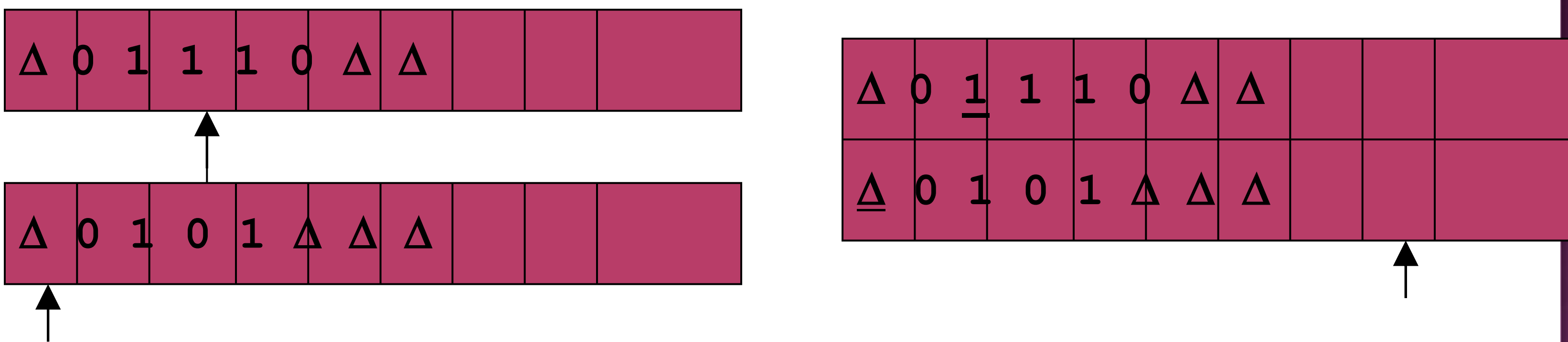
HOW 1-TAPE TM SIMULATES 2-TAPE TM

- Marking the position of each tape head in the content of the tape
- Encode content of 2 tapes on 1 tape
  - When to convert 1-tape symbol into 2-tape symbol

cannot be done all at once because the tape is infinite

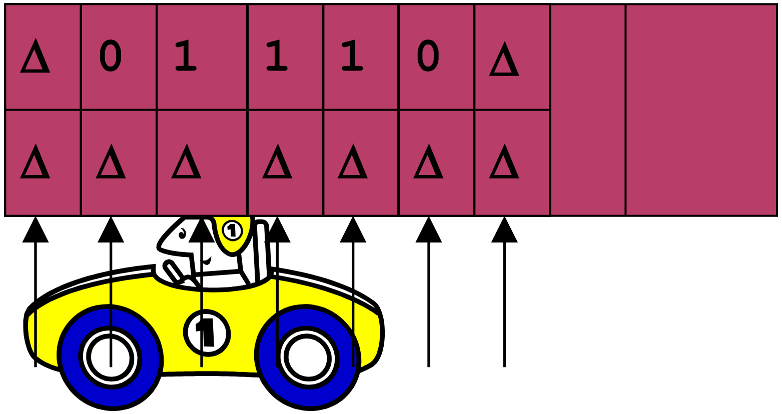
- Construct 1-tape TM simulating a transition in 2-tape TM
- Convert the encoding of 2-tape symbols back to 1-tape symbols

ENCODING 2 TAPES IN 1 TAPE

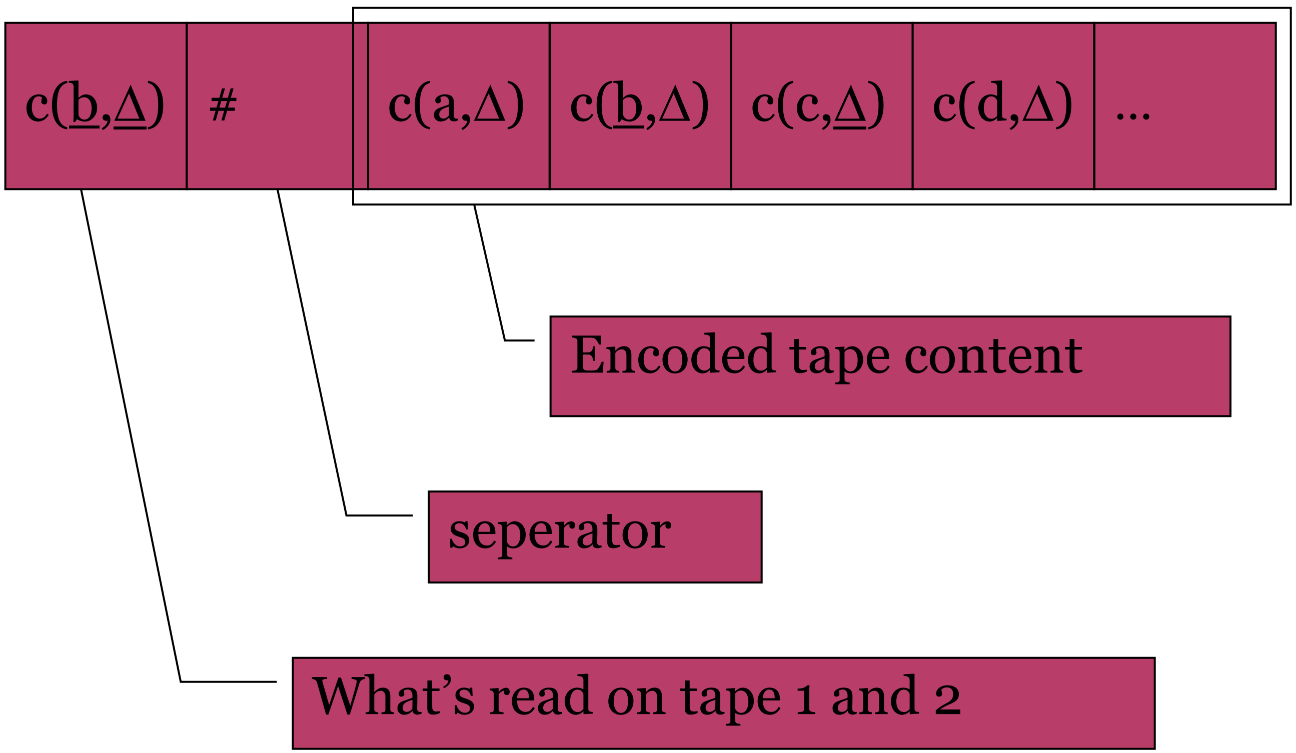


- New alphabet contains:
  - old alphabet
  - encoding of a symbol on tape 1 and a symbol on tape 2
  - encoding of a symbol on tape 1 pointed by its tape head and a symbol on tape 2
  - encoding of a symbol on tape 1 and a symbol on tape 2 pointed by its tape head
  - encoding of a symbol on tape 1 pointed by its tape head and a symbol on tape 2 pointed by its tape head

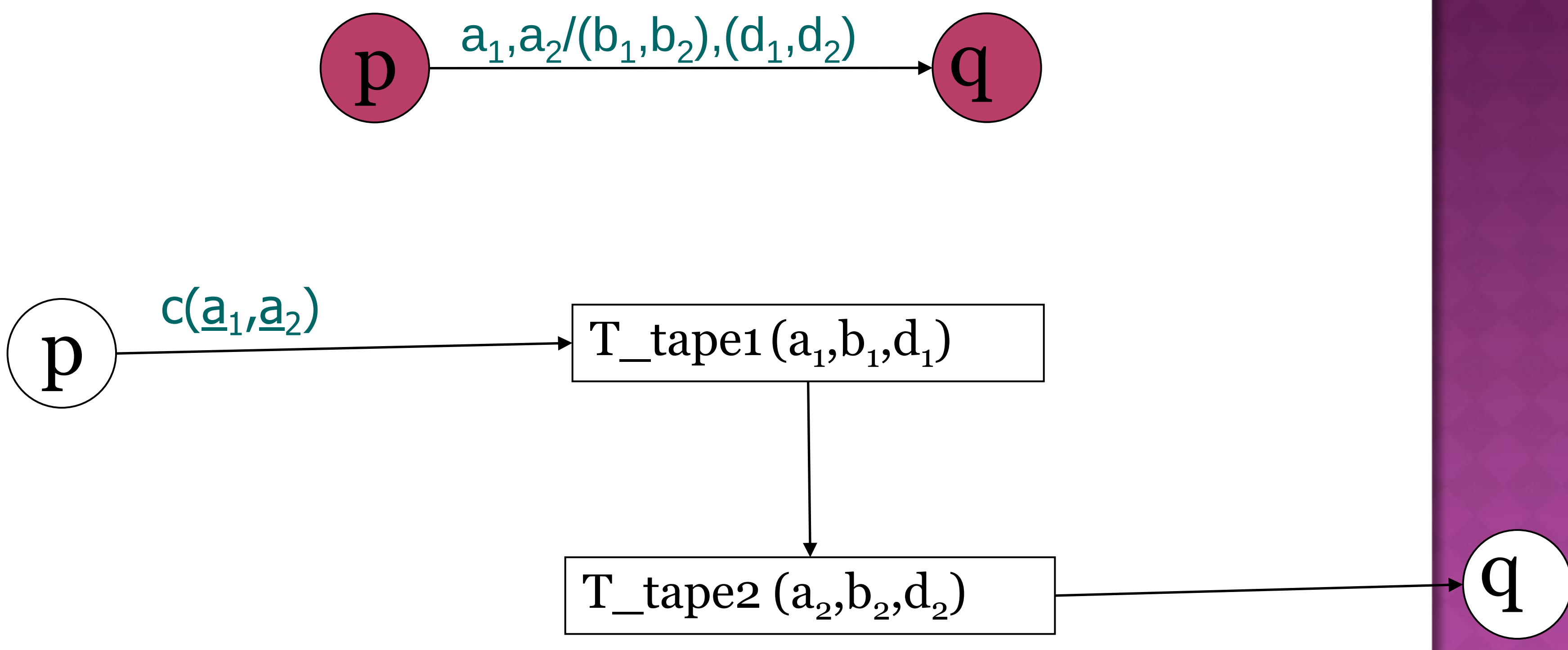
HOW THE TAPE CONTENT IS CHANGED



TAPE FORMAT

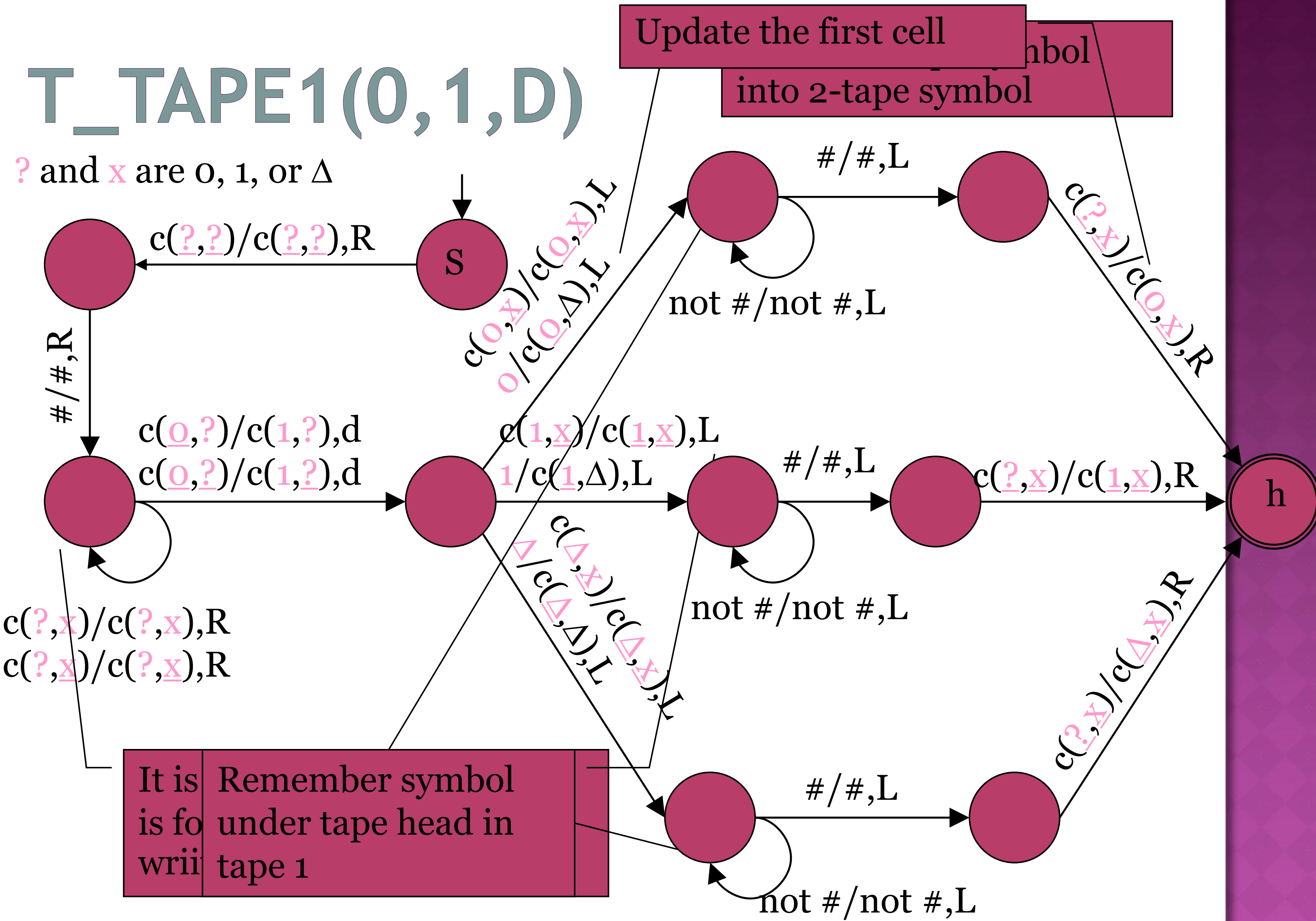

c($\underline{b}$,$\underline{\Delta}$)
#
c(a,Δ)
c($\underline{b}$,Δ)
c(c,$\underline{\Delta}$)
c(d,Δ)
…
Encoded tape content
seperator
What's read on tape 1 and 2

SIMULATING TRANSITIONS IN 2-TAPE TM IN 1-TAPE TM

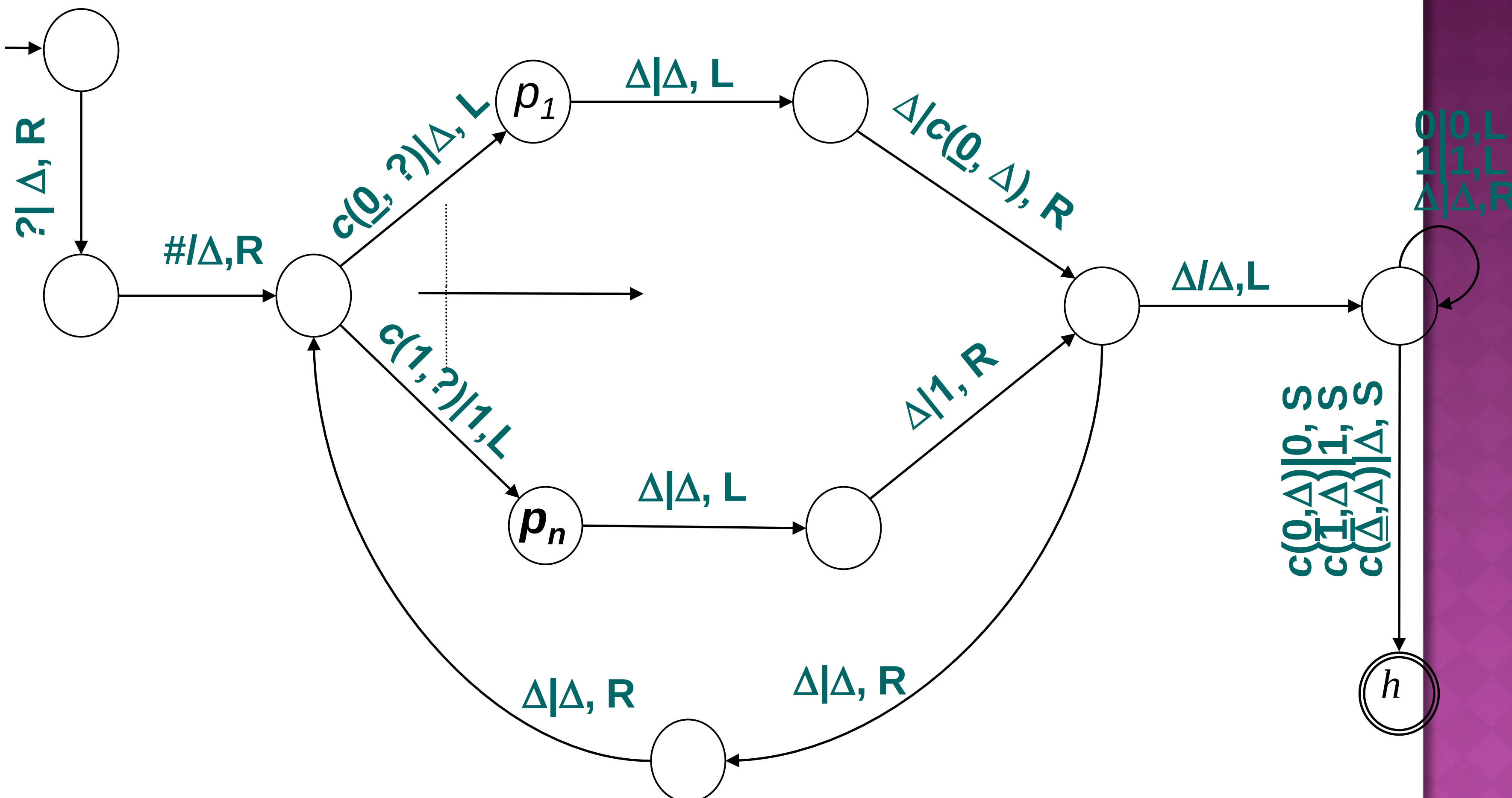


T_TAPE1(0,1,D)

? and x are 0, 1, or Δ

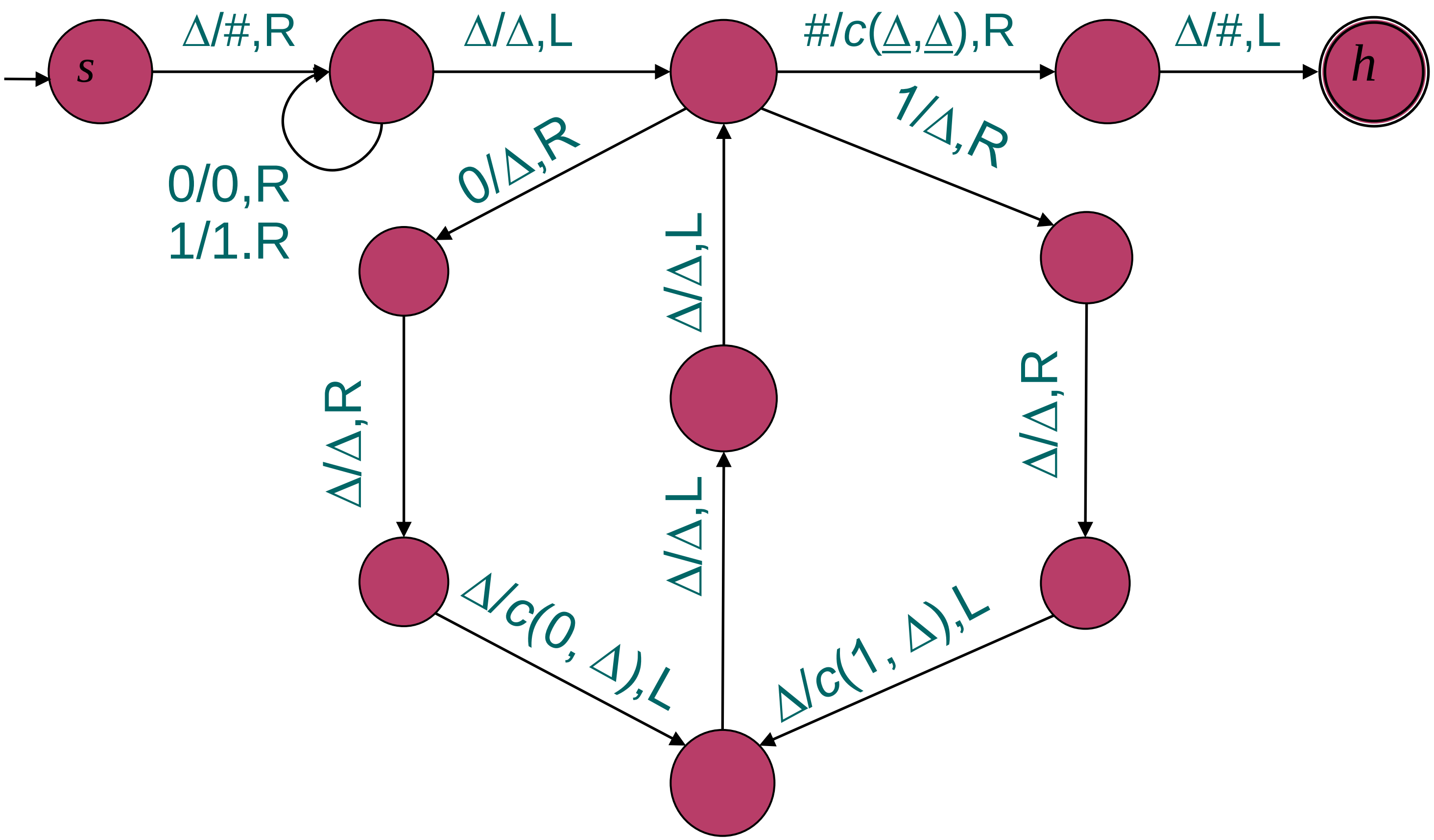


T CLEANUP

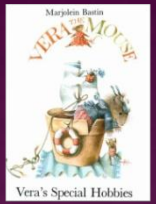


?| Δ, R
#/Δ,R
c(0̲, ?)|Δ, L
c(1, ?)|1,L
p1
pn
Δ|Δ, L
Δ|Δ, L
Δ/c(0̲, Δ), R
Δ|1, R
Δ/Δ,L
0|0,L
1|1,L
Δ|Δ,R
c(0,Δ)|0, S
c(1̲,Δ)|1, S
c(Δ̲,Δ)|Δ, S
Δ|Δ, R
Δ|Δ, R
h

T_{ENCODE}



s
Δ/#,R
0/0,R
1/1.R
Δ/Δ,L
#/c(Δ,Δ),R
Δ/#,L
h
0/Δ,R
1/Δ,R
Δ/Δ,R
Δ/Δ,L
Δ/Δ,L
Δ/Δ,R
Δ/c(0, Δ),L
Δ/c(1 , Δ),L



Marjolein Bastin
VERA THE MOUSE
Vera's Special Hobbies

EQUIVALENCE OF 2-TAPE TM AND SINGLE-TAPE TM

Proof:

Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be a 2-tape TM.

We construct a 1-tape TM $M=(K, \Sigma, \Gamma', \delta', s')$ such that

- $\Gamma' = \Gamma \cup \{c(a,b) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{c(\underline{a},b) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{c(a,\underline{b}) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{c(\underline{a},\underline{b}) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{\#\}$

We need to prove that:

- if T halts on α with output β, then M halts on α with output β, and
- if T does not halt on α, then M does not halt on α

IF T HALTS ON α WITH OUTPUT β

IF T DOES NOT HALT ON α

- If T loops, then M loops.
- If T hangs in a state p, M hangs somewhere from p to the next state.