COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- TM accepting a language

TM ACCEPTING A LANGUAGE

- Definition

  Let $T=(Q, \Sigma, \Gamma, \delta, s)$ be a TM, and $w \in \Sigma^*$.

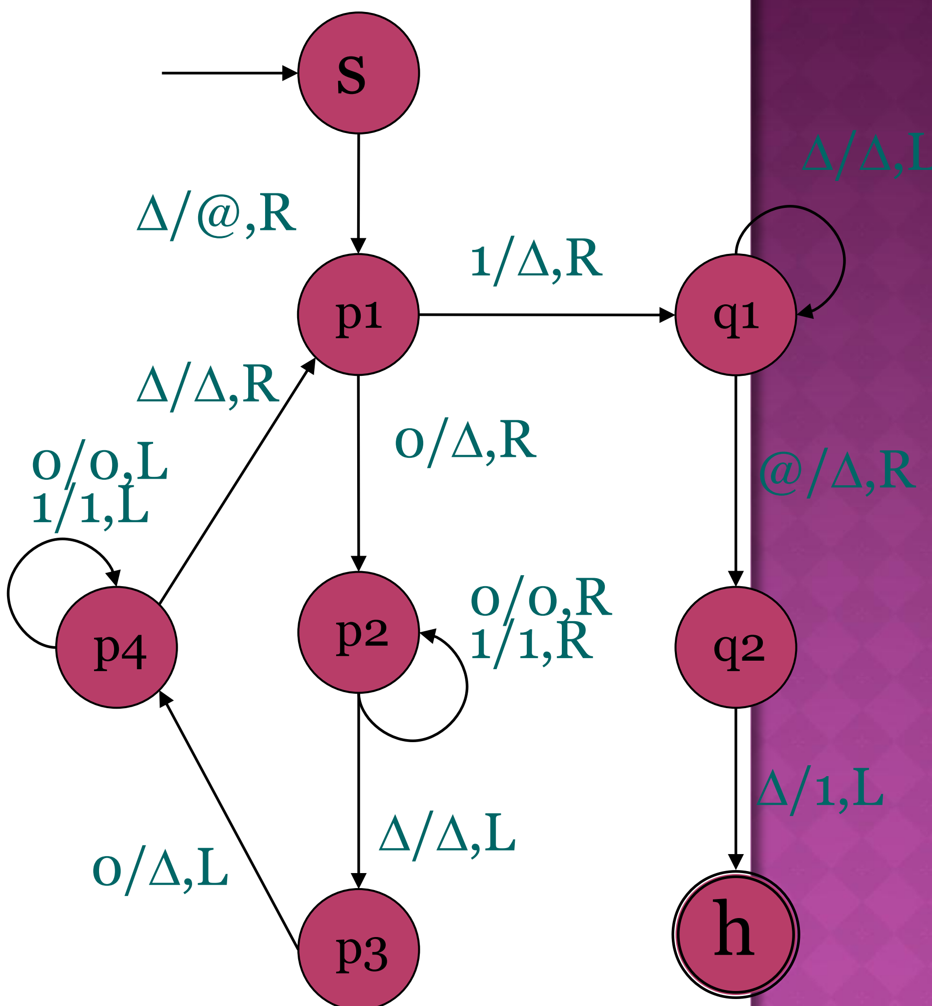
  T accepts w if $(s, \varepsilon, \Delta, w) \mid\!-_T^* (h, \varepsilon, \Delta, 1)$.

  The language accepted by a TM T, denoted by $L(T)$, is the set of strings accepted by T.

EXAMPLE OF LANGUAGE ACCEPTED BY A TM

$L(T)=\{0^n10^n \mid n\geq 0\}$

- T halts on 0^n10^n
- T hangs on $0^{n+1}10^n$ at p3
- T hangs on 0^n10^{n+1} at q1
- T hangs on $0^n\ 1^2\ 0^n$ at q1



TM COMPUTING A FUNCTION

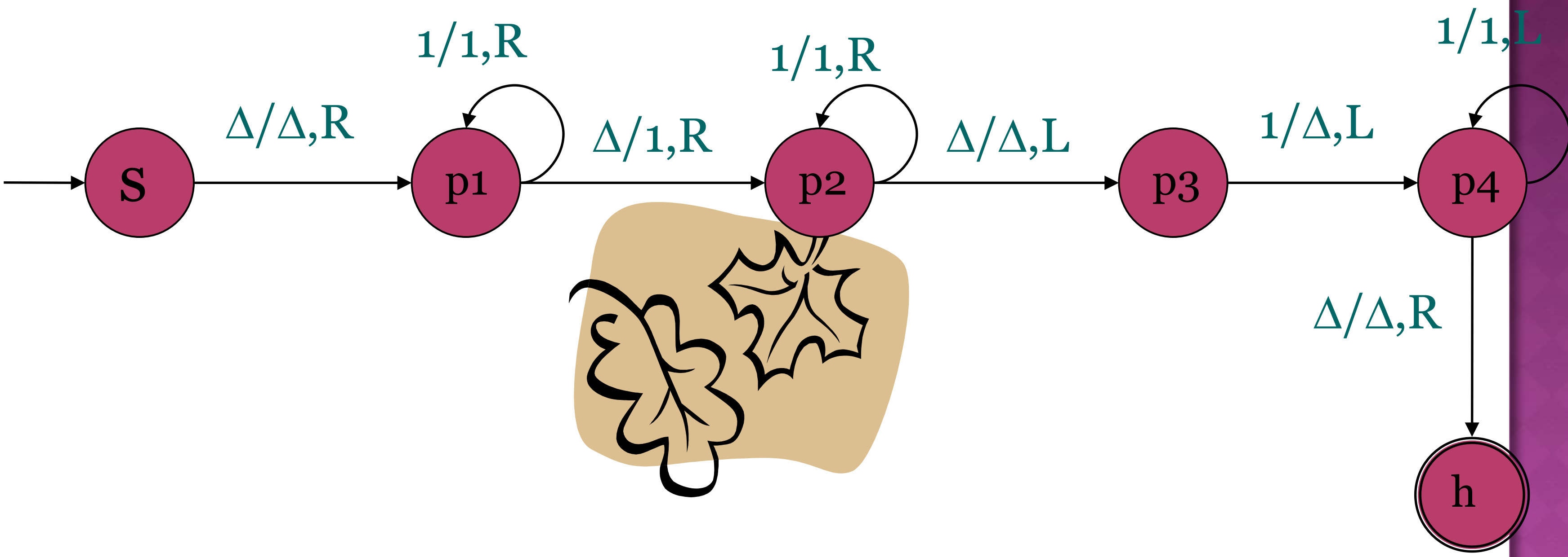
- Definition

  Let $T=(Q, \Sigma, \Gamma, \delta, s)$ be a TM, and f be a function from Σ^* to Γ^*.

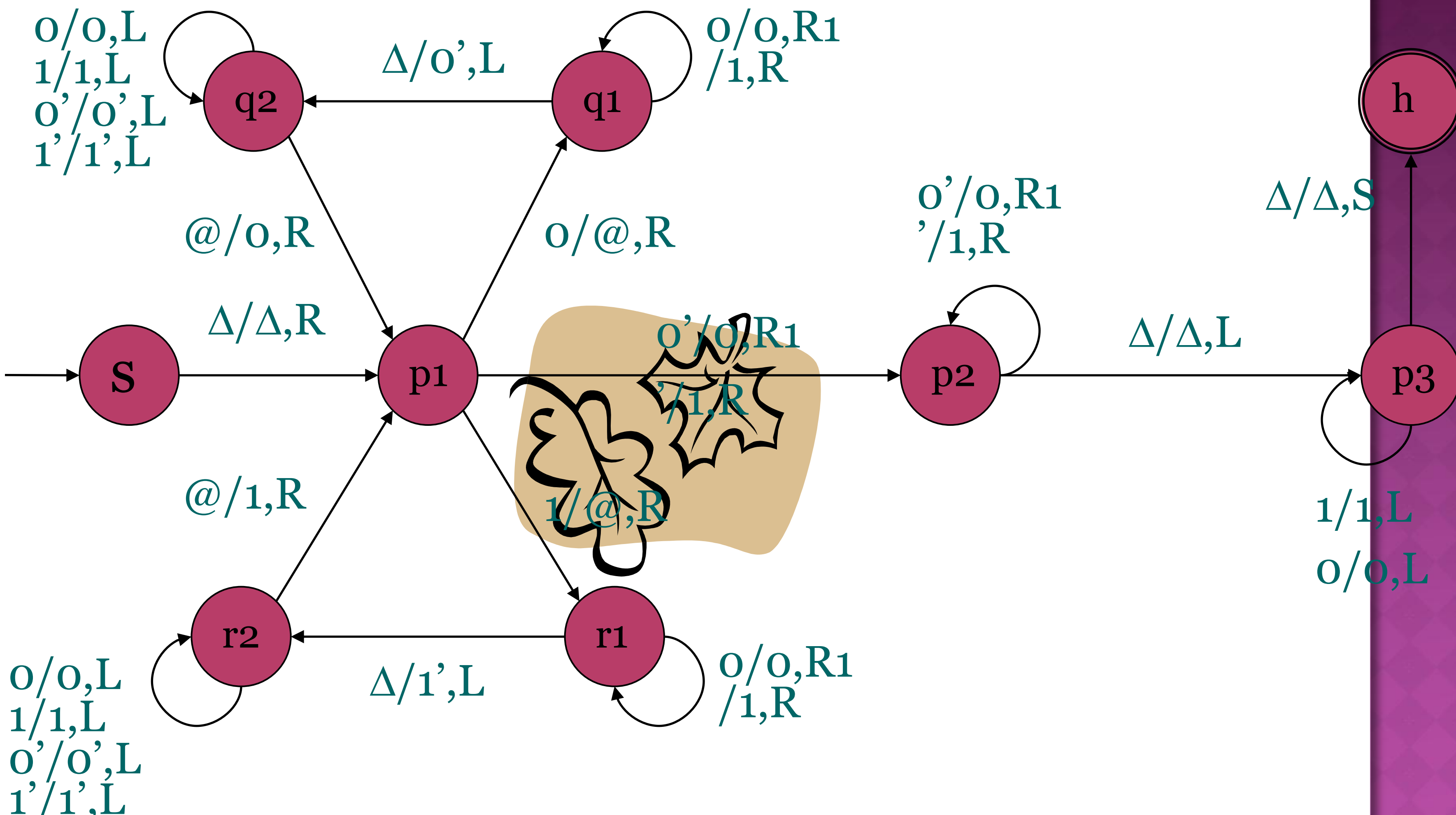
  T computes f if, for any string w in Σ^*,

  $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, f(w))$.

EXAMPLE OF TM COMPUTING FUNCTION



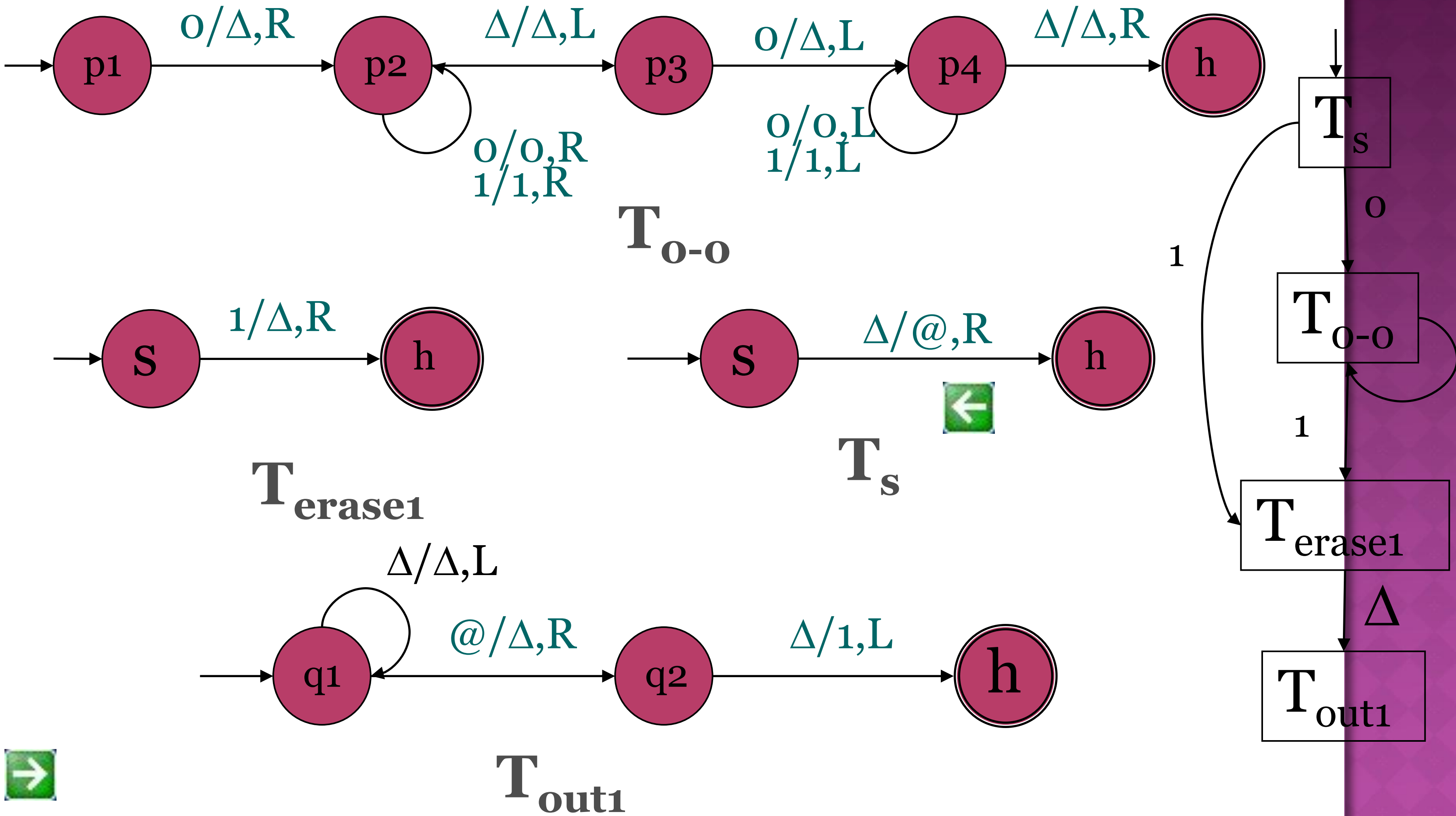
EXAMPLE OF TM COMPUTING FUNCTION



COMPOSITE TM

- Let $T1$ and $T2$ be TM's.
- $T1 \rightarrow T2$ means executing $T1$ until $T1$ halts and then executing $T2$.
- $T1 \xrightarrow{a} T2$ means executing $T1$ until $T1$ halts and if the symbol under the tape head when $T1$ halts is a then executing $T2$.

EXAMPLE OF COMPOSITE TM



NONDETERMINISTIC TM

- An NTM starts working and stops working in the same way as a DTM.
- Each move of an NTM can be nondeterministic.

EACH MOVE IN AN NTM

- reads the symbol under its tape head
- According to the *transition relation* on the symbol read from the tape and its current state, the TM choose one move nondeterministically to:
  - write a symbol on the tape
  - move its tape head to the left or right one cell or not
  - changes its state to the *next state*

HOW TO DEFINE NONDETERMINISTIC TM (NTM)

- a quintuple $(Q, \Sigma, \Gamma, \delta, s)$, where
  - the set of states Q is finite, and does not contain halt state h,
  - the input alphabet Σ is a finite set of symbols, not including the blank symbol Δ,
  - the tape alphabet Γ is a finite set of symbols containing Σ, but not including the blank symbol Δ,
  - the start state s is in Q, and
  - the transition f^n

CONFIGURATION OF AN NTM

Definition

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be an TM.

  A configuration of T is an element of $Q \times \Gamma^* \times \Gamma \times \Gamma^*$

- Can be written as
  - $(q,l,a,$
  - $(q,l \cdot \underline{a} \cdot r)$

string to the left of tape head

symbol under tape head

string to the right of tape head

YIELD THE NEXT CONFIGURATION

Definition

- Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be an NTM, and $(q_1, \alpha_1\underline{a_1}\beta_1)$ and $(q_2, \alpha_2\underline{a_2}\beta_2)$ be two configurations of T.

  We say $(q_1, \alpha_1\underline{a_1}\beta_1)$ yields $(q_2, \alpha_2\underline{a_2}\beta_2)$ in one step, denoted by $(q_1, \alpha_1\underline{a_1}\beta_1) \vdash_T (q_2, \alpha_2\underline{a_2}\beta_2)$, if

  - $(q_2, a_2, \mathrm{S}) \in \delta(q_1, a_1)$, $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$,
  - $(q_2, b, \mathrm{R}) \in \delta(q_1, a_1)$, $\alpha_2 = \alpha_1 b$ and $\beta_1 = a_2\beta_2$,

NTM ACCEPTING A LANGUAGE/COMPUTING A FUNCTION

- Definition

Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be an NTM.

Let $w \in \Sigma^*$ and f be a function from Σ^* to Γ^*.

T accepts w if $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, 1)$.

The language accepted by a TM T, denoted by $L(T)$, is the set of strings accepted by T.

T computes f if, for any string w in Σ^*, $(s, \varepsilon, \Delta, w) \vdash_T^* (h, \varepsilon, \Delta, f(w))$.

EXAMPLE OF NTM

- Let $L=\{ww \mid w \in \{0,1\}^*\}$