COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- Simplifications of Context-Free Grammars
- Reduced Forms
- Removal of Useless Symbols

A SUBSTITUTION RULE

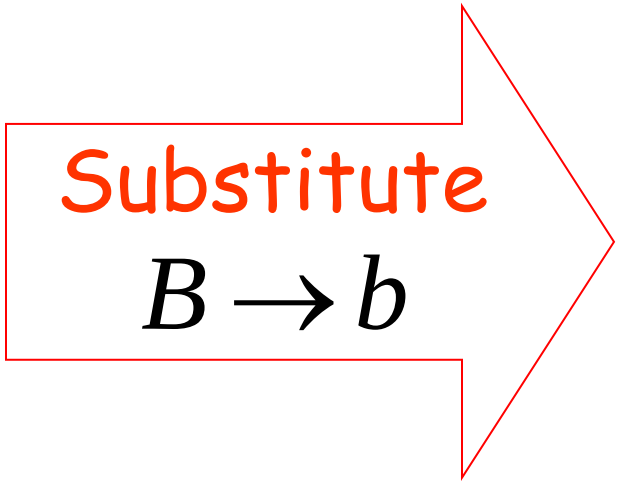
$S \to aB$

$A \to aaA$

$A \to abBc$

$B \to aA$

$B \to b$



Equivalent grammar

$S \to aB \mid ab$

$A \to aaA$

$A \to abBc \mid abbc$

$B \to aA$

A SUBSTITUTION RULE

$$S \to aB \mid ab$$

$$A \to aaA$$

$$A \to abBc \mid abbc$$

$$B \to aA$$

Substitute

$$B \to aA$$

$$S \to \cancel{aB} \mid ab \mid aaA$$

$$A \to aaA$$

$$A \to \cancel{abBc} \mid abbc \mid abaAc$$

Equivalent grammar

In general:

$$A \to xBz$$

$$B \to y_1$$

Substitute

$$B \to y_1$$

$$A \to xBz \mid xy_1z$$

equivalent grammar

NULLABLE VARIABLES

λ – production : $A \to \lambda$

Nullable Variable: $A \Rightarrow \ldots \Rightarrow \lambda$

REMOVING NULLABLE VARIABLES

Example Grammar:

$$S \rightarrow aMb$$

$$M \rightarrow aMb$$

$$M \rightarrow \lambda$$

Nullable variable

Final Grammar

$S \rightarrow aMb$

$M \rightarrow aMb$

~~$M \rightarrow \lambda$~~

Substitute

$M \rightarrow \lambda$

$S \rightarrow aMb$

$S \rightarrow ab$

$M \rightarrow aMb$

$M \rightarrow ab$

UNIT-PRODUCTIONS

Unit Production: $A \to B$

(a single variable in both sides)

REMOVING UNIT PRODUCTIONS

Observation:

$$A \to A$$

Is removed immediately

Example Grammar:

$$S \rightarrow aA$$

$$A \rightarrow a$$

$$A \rightarrow B$$

$$B \rightarrow A$$

$$B \rightarrow bb$$

$S \rightarrow aA$

$A \rightarrow a$

~~$A \rightarrow B$~~

$B \rightarrow A$

$B \rightarrow bb$

Substitute

$A \rightarrow B$

$S \rightarrow aA \mid aB$

$A \rightarrow a$

$B \rightarrow A \mid B$

$B \rightarrow bb$

$S \rightarrow aA \mid aB$

$A \rightarrow a$

$B \rightarrow A \mid B$

$B \rightarrow bb$

Remove

$B \rightarrow B$

$S \rightarrow aA \mid aB$

$A \rightarrow a$

$B \rightarrow A$

$B \rightarrow bb$

$S \rightarrow aA \mid aB$

$A \rightarrow a$

~~$B \rightarrow A$~~

$B \rightarrow bb$

Substitute

$B \rightarrow A$

$S \rightarrow aA \mid aB \mid aA$

$A \rightarrow a$

$B \rightarrow bb$

Remove repeated productions

$S \rightarrow aA \mid aB \mid \cancel{aA}$

$A \rightarrow a$

$B \rightarrow bb$

Final grammar

$S \rightarrow aA \mid aB$

$A \rightarrow a$

$B \rightarrow bb$

USELESS PRODUCTIONS

$$S \to aSb$$

$$S \to \lambda$$

$$S \to A$$

$$A \to aA$$ Useless Production

Some derivations never terminate…

$$S \Rightarrow A \Rightarrow aA \Rightarrow aaA \Rightarrow \ldots \Rightarrow aa\ldots aA \Rightarrow \ldots$$

Another grammar:

$$S \to A$$

$$A \to aA$$

$$A \to \lambda$$

$$B \to bA$$

Useless Production

Not reachable from S

In general:

contains only terminals

if $S \Rightarrow \ldots \Rightarrow xAy \Rightarrow \ldots \Rightarrow w$

$w \in L(G)$

then variable A is useful

otherwise, variable A is useless

A production $A \to x$ is useless
if any of its variables is useless

$S \to aSb$

$S \to \lambda$

Variables	Productions
$S \to A$	useless
useless $A \to aA$	useless
useless $B \to C$	useless
useless $C \to D$	useless

REMOVING USELESS PRODUCTIONS

Example Grammar:

$$S \rightarrow aS \mid A \mid C$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$C \rightarrow aCb$$

First: find all variables that can produce strings with only terminals

$S \to aS \mid A \mid C$

$A \to a$

$B \to aa$

$C \to aCb$

Round 1: $\{A, B\}$

$S \to A$

Round 2: $\{A, B, S\}$

Keep only the variables
that produce terminal symbols: $\{A, B, S\}$

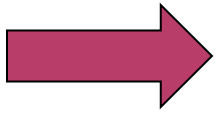
(the rest variables are useless)

$S \to aS \mid A \mid C$

$A \to a$

$B \to aa$

$C \to aCb$



$S \to aS \mid A$

$A \to a$

$B \to aa$

Remove useless productions

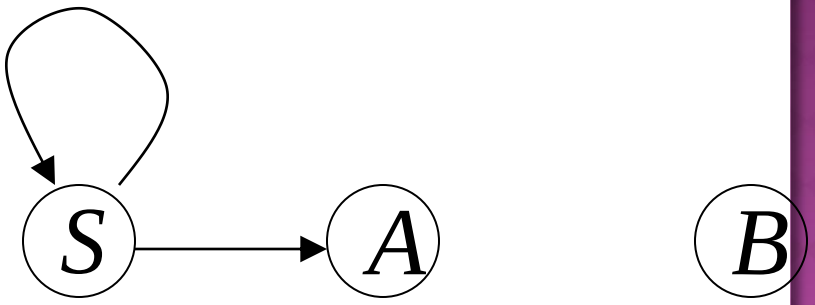
Second: Find all variables reachable from S

Use a Dependency Graph

$S \to aS \mid A$

$A \to a$

$B \to aa$



not reachable

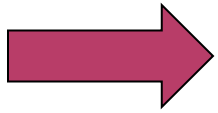
Keep only the variables reachable from S

(the rest variables are useless)

$S \rightarrow aS \mid A$

$A \rightarrow a$

~~$B \rightarrow aa$~~



Final Grammar

$S \rightarrow aS \mid A$

$A \rightarrow a$

Remove useless productions

REMOVING ALL

- **Step 1:** Remove Nullable Variables
- **Step 2:** Remove Unit-Productions
- **Step 3:** Remove Useless Variables