COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- The Myhill-Nerode Theorem

ISOMORPHISM OF DFAS

- $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$, $N = (Q_N, S, \delta_N, s_N, F_N)$: two DFAs
- M and N are said to be isomorphic if there is a (structure-preserving) bijection $f: Q_M \to Q_N$ s.t.
  - $f(s_M) = s_N$
  - $f(\delta_M(p,a)) = \delta_N(f(p),a)$ for all $p \in Q_M$, $a \in \Sigma$
  - $p \in F_M$ iff $f(p) \in F_N$.
- I.e., M and N are essentially the same machine up to renaming of states.
- Facts:
  - 1. Isomorphic DFAs accept the same set.
  - 2. if M and N are any two DFAs w/o inaccessible states accepting the same set, then the quotient automata $M/\approx$ and $N/\approx$ are isomorphic
  - 3. The DFA obtained by the minimization algorithm (lec. 14) is the minimal DFA for the set it accepts, and this DFA is unique up to isomorphism.

MYHILL-NERODE RELATIONS

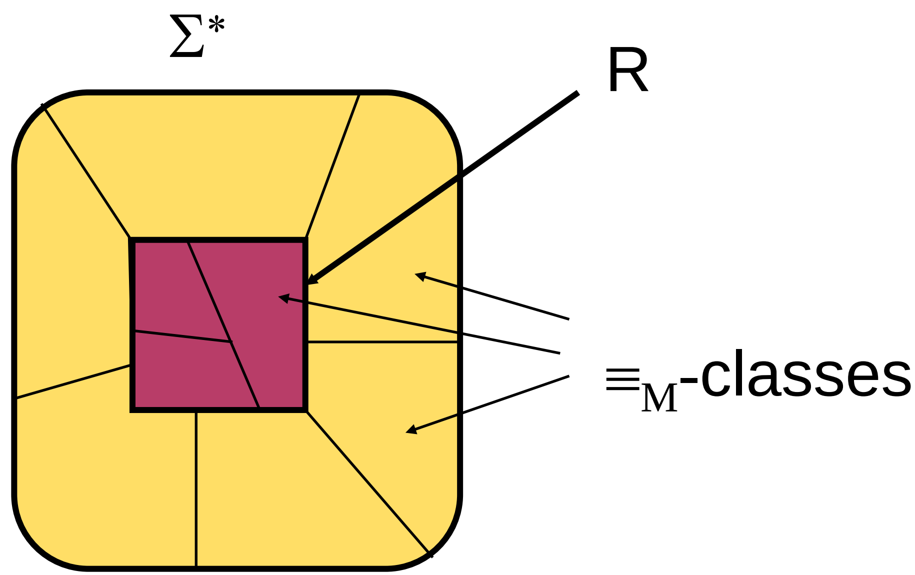
- R: a regular set, M=(Q, Σ, δ,s,F): a DFA for R w/o inaccessible states.
- M induces an equivalence relation $\equiv_M$ on Σ^* defined by
  - $x \equiv_M y$ iff $\Delta(s,x) = \Delta(s,y)$.
  - i.e., two strings x and y are equivalent iff it is indistinguishable by running M on them (i.e., by running M with x and y as input, respectively, from the initial state of M.)
- Properties of $\equiv_M$:
  - 0. $\equiv_M$ is an equivalence relation on Σ^*.

    (cf: $\approx$ is an equivalence relation on states)
  - 1. $\equiv_M$ is a right congruence relation on Σ^*: i.e., for any $x,y \in \Sigma^*$ and $a \in \Sigma$, $x \equiv_M y \Rightarrow xa \equiv_M ya$.
  - pf: if $x \equiv_M y \Rightarrow \Delta(s,xa) = \delta(\Delta(s,x),a) = \delta(\Delta(s,y),a) = \Delta(s, ya)$

    $\Rightarrow xa \equiv_M ya$.

PROPERTIES OF THE MYHILL-NERODE RELATIONS

- Properties of $\equiv_M$:
  - 2. $\equiv_M$ refines R. I.e., for any $x,y \in \Sigma^*$,
  - $x \equiv_M y \Rightarrow x \in R$ iff $y \in R$
  - pf: $x \in R$ iff $\Delta(s,x) \in F$ iff $\Delta(s,y) \in F$ iff $y \in R$.
  - Property 2 means that every $\equiv_M$-class has either all its elements in R or none of its elements in R. Hence R is a union of some $\equiv_M$-classes.
  - 3. It is of finite index, i.e., it has only finitely many equivalence classes.
  - (i.e., the set $\{ [x]_{\equiv_M} \mid x \in \Sigma^* \}$
  - is finite.
  - pf: $x \equiv_M y$ iff $\Delta(s,x) = \Delta(s,y) = q$
  - for some $q \in Q$. Since there
  - are only $|Q|$ states, hence
  - Σ^* has $|Q|$ $\equiv_M$-classes



DEFINITION OF THE MYHILL-NERODE RELATION

- $\equiv$: an equivalence relation on Σ^*,
  R: a language over Σ^*.
- $\equiv$ is called an Myhill-Nerode relation for R if it satisfies property 1~3. i.e., it is a right congruence of finite index refining R.
- Fact: R is regular iff it has a Myhill-Nerode relation.
  - (to be proved later)
  - 1. For any DFA M accepting R, $\equiv_M$ is a Myhill-Nerode relation for R.
  - 2. If $\equiv$ is a Myhill-Nerode relation for R then there is a DFA $M_\equiv$ accepting R.
  - 3. The constructions $M \to \equiv_M$ and $\equiv \to M_\equiv$ are inverse up to isomorphism of automata. (i.e. $\equiv = \equiv_{M_\equiv}$ and $M = M_{\equiv_M}$)

FROM $\equiv$ TO $M_\equiv$

- R: a language over Σ, $\equiv$: a Myhill-Nerode relation for R;
  - the $\equiv$-class of the string x is $[x]_\equiv =_{def} \{y \mid x \equiv y\}$.
  - Note: Although there are infinitely many strings, there are only finitely many $\equiv$ -classes. (by property of finite index)
- Define DFA $M_\equiv = (Q,\Sigma,\delta,s,F)$ where
  - $Q = \{[x] \mid x \in \Sigma^*\}$, $s = [\varepsilon]$,
  - $F = \{[x] \mid x \in R\}$, $\delta([x],a) = [xa]$.
- Notes:
  - 0: $M_\equiv$ has $|Q|$ states, each corresponding to an $\equiv$ -class of $\equiv$. Hence the more classes $\equiv$ has, the more states $M_\equiv$ has.
  - 1. By right congruence of $\equiv$, δ is well-defined, since, if $y,z \in [x] \Rightarrow y \equiv z \equiv x \Rightarrow ya \equiv za \equiv xa \Rightarrow ya, za \in [xa]$
  - 2. $x \in R$ iff $[x] \in F$.
  - pf: =>: by definition of $M_\equiv$;
  - <=: $[x] \in F \Rightarrow \exists y$ s.t. $y \in R$ and $x \equiv y \Rightarrow x \in R$. (property 2)

$M \to \equiv_M$ AND $\equiv \to M_\equiv$ ARE INVERSES

Lemma 15.1: $\Delta([x],y) = [xy]$

pf: Induction on $|y|$. Basis: $\Delta([x],\varepsilon) = [x] = [x\varepsilon]$.

Ind. step: $\Delta([x],ya) = \delta(\Delta([x],y),a) = \delta([xy],a) = [xya]$. QED

Theorem 15.2: $L(M_\equiv) = R$.

pf: $x \in L(M_\equiv)$ iff $\Delta([\varepsilon],x) \in F$ iff $[x] \in F$ iff $x \in R$. QED

Lemma 15.3: $\equiv$: a Myhill-Nerode relation for R, M: a DFA for R w/o inaccessible states, then

1. if we apply the construction $\equiv \to M_\equiv$ to $\equiv$ and then apply $M \to \equiv_M$ to the result, the resulting relation $\equiv_{M_\equiv}$ is identical to $\equiv$.
2. if we apply the construction $M \to \equiv_M$ to M and then apply $\equiv \to M_\equiv$ to the result, the resulting relation $M_{\equiv_M}$ is identical to M.

Pf: (of lemma 15.3) (1) Let $M_{\equiv}$ =(Q,Σ,δ,s,F) be the DFA constructed as described above. then for any x,y in Σ^*,

$x \equiv_{M\equiv} y$ iff $\Delta([\varepsilon], x) = \Delta([\varepsilon],y)$ iff $[x] = [y]$ iff $x \equiv y$.

(2) Let M = (Q, Σ ,δ,s,F) and let $M\equiv_M$ = (Q', Σ , δ',s',F'). Recall that

- $[x] = \{y \mid y \equiv_M x\} = \{y \mid \Delta(s,y) = \Delta(s,x) \}$
- $Q' = \{[x] \mid x \in \Sigma^*\}$, $s' = [\varepsilon]$, $F' = \{[x] \mid x \in R\}$
- $\delta'([x], a) = [xa]$.

Now let f:Q'-> Q be defined by $f([x]) = \Delta(s,x)$.

- 1. By def., $[x] = [y]$ iff $\Delta(s,x) = \Delta(s,y)$, so f is well-defined and 1-1. Since M has no inaccessible state, f is onto.
- 2. $f(s') = f([\varepsilon]) = \Delta(s, \varepsilon) = s$
- 3. $[x] \in F' \iff x \in R \iff \Delta(s,x) \in F \iff f([x]) \in F$.
- 4. $f(\delta'([x],a)) = f([xa]) = \Delta(s,xa) = \delta(\Delta(s,x),a) = \delta(f([x]), a)$
- By 1~4, f is an isomorphism from $M\equiv_M$ to M. QED

RELATIONS B/T DFAS AND MYHILL-NERODE RELATIONS

Theorem 15.4: R: a regular set over Σ. Then up to isomorphism of FAs, there is a 1-1 correspondence b/t DFAs w/o inaccessible states accepting R and Myhill-Nerode relations for R.

- I.e., Different DFAs accepting R correspond to different Myhill-Nerode relations for R, and vice versa.
- We now show that there exists a coarsest Myhill-Neorde relation $\equiv_R$ for any R, which corresponds to the unique minimal DFA for R.

Def 16.1: $\equiv_1$, $\equiv_2$: two relations. If $\equiv_1 \subseteq \equiv_2$ (i.e., for all x,y, $x \equiv_1 y \Rightarrow x \equiv_2 y$) we say $\equiv_1$ refines $\equiv_2$.

Note:1. If $\equiv_1$ and $\equiv_2$ are equivalence relations, then $\equiv_1$ refines $\equiv_2$ iff every $\equiv_1$-class is included in a $\equiv_2$-class.

2. The refinement relation on equivalence relations is a partial order. (since $\subseteq$ is ref, transitive and antisymmetric).

THE REFINEMENT RELATION

Note:

3. If , $\equiv_1 \subseteq \equiv_2$,we say $\equiv_1$ is the finer and $\equiv_2$ is the coarser of the two relations.

4. The finest equivalence relation on a set U is the identity relation $I_U = \{(x,x) \mid x \in U\}$

5. The coarsest equivalence relation on a set U is universal relation $U^2 = \{(x,y) \mid x, y \in U\}$

Def. 16.1: R: a language over Σ (possibly not regular).
Define a relation $\equiv_R$ over Σ^* by

$$x \equiv_R y \text{ iff for all } z \in \Sigma^* \ (xz \in R \Leftrightarrow yz \in R)$$

i.e., x and y are related iff whenever appending the same string to both of them, the resulting two strings are either both in R or both not in R.

PROPERTIES OF $\equiv_R$

Lemma 16.2: Properties of $\equiv_R$:

- 0. $\equiv_R$ is an equivalence relation over Σ^*.
- 1. $\equiv_R$ is right congruent
- 2. $\equiv_R$ refines R.
- 3. $\equiv_R$ the coarsest of all relations satisfying 0,1 and 2.
- [4. If R is regular => $\equiv_R$ is of finite index.]

Pf: (0) : trivial; (4) immediate from (3) and theorem 15.2.

(1) $x \equiv_R y$ => for all $z \in \Sigma^*$ ($xz \in R$ <=> $yz \in R$)

=> $\forall a \, \forall w$ ($xaw \in R$ <=> $yaw \in R$)

=> $\forall a$ ($xa \equiv_R ya$)

(2) $x \equiv_R y$ => ($x \in R$ <=> $y \in R$)

(3) Let $\equiv$ be any relation satisfying 0~2. Then

$x \equiv y$ => $\forall z \; xz \equiv yz$ --- by ind. on $|z|$ using property (1)

=> $\forall z$ ($xz \in R$ <=> $yz \in R$) --- by (2) => $x \equiv_R y$.

MYHILL-NERODE THEOREM

Thorem16.3: Let R be any language over Σ. Then the following statements are equivalent:

(a) R is regular;

(b) There exists a Myhill-Nerode relation for R;

(c) the relation $\equiv_R$ is of finite index.

pf: (a) =>(b) : Let M be any DFA for R. The construction $M \rightarrow \equiv_M$ produces a Myhill-Nerode relation for R.

(b) => (c): By lemma 16.2, any Myhill-Nerode relation for R is of finite index and refines R => $\equiv_R$ is of finite index.

(c)=>(a): If $\equiv_R$ is of finite index, by lemma 16.2, it is a Myhill-Nerode relation for R, and the construction $\equiv \rightarrow M_{\equiv}$ produce a DFA for R.

RELATIONS B/T $\equiv_R$ AND COLLAPSED MACHINE

Note: 1. Since $\equiv_R$ is the coarsest Myhill-Nerode relation for a regular set R, it corresponds to the DFA for R with the fewest states among all DFAs for R.

(i.e., let M = (Q,...) be any DFA for R and M = (Q',...) the DFA induced by $\equiv_R$, where Q' = the set of all $\equiv_R$-classes

==> |Q| = | the set of $\equiv_M$-classes | >= | the set of $\equiv_R$ -classes |

= |Q'|.

Fact: M=(Q,S,s,d,F): a DFA for R that has been collapsed (i.e., M = M/$\approx$). Then $\equiv_R = \equiv_M$ (hence M is the unique DFA for R with the fewest states).

pf: $x \equiv_R y$ iff $\forall z \in \Sigma^*$ $(xz \in R \Leftrightarrow yz \in R)$

iff $\forall z \in \Sigma^*$ $(\Delta(s,xz) \in F \Leftrightarrow \Delta(s,yz) \in F)$

iff $\forall z \in \Sigma^*$ $(\Delta(\Delta(s,x),z) \in F \Leftrightarrow \Delta(\Delta(s,y),z) \in F)$

iff $\Delta(s,x) \approx \Delta(s,y)$ iff $\Delta(s,x) = \Delta(s,y)$ -- since M is collapsed

iff $x \equiv_M y$ Q.E.D.

AN APPLICATION OF THE MYHILL-NERODE RELATION

- Can be used to determine whether a set R is regular by determining the number of $\equiv_R$ -classes.
- Ex: Let $A = \{a^n b^n \mid n \geq 0 \}$.
  - If $k \neq m \Rightarrow a^k$ not $\equiv_A a^m$, since $a^k b^k \in A$ but $a^m b^k \notin A$.

    Hence $\equiv_A$ is not of finite index => A is not regular.
  - In fact $\equiv_A$ has the following $\equiv_A$-classes:
  - $G_k = \{a^k\}, k \geq 0$
  - $H_k = \{a^{n+k} b^n \mid n \geq 1 \}, k \geq 0$
  - $E = \Sigma^* - \bigcup_{k \geq 0} (G_k \cup H_k) = \Sigma^* - \{a^m b^n \mid m \geq n \geq 0 \}$

UNIQUENESS OF MINIMAL NFAS

- Problem: Does the conclusion that minimal DFA accepting a language is unique apply to NFA as well ?

Ans : ?

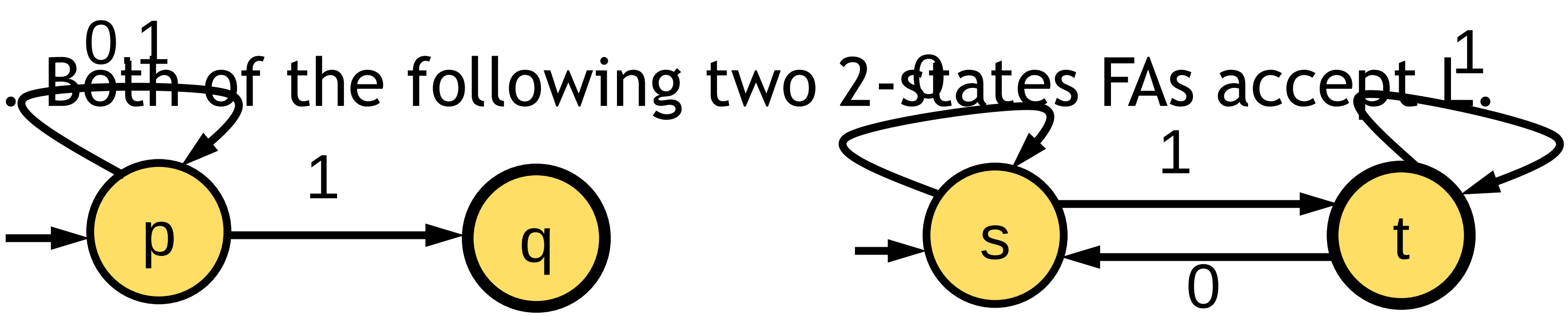
MINIMAL NFAS ARE NOT UNIQUE UP TO ISOMORPHISM

- Example: let L = { x1 | x ∈ {0,1} }*

1. What is the minimum number k of states of all FAs accepting L ?

Analysis : k ≠ 1. Why ?

2. Both of the following two 2-states FAs accept L.



COLLAPSING NFAS

- Minimal NFAs are not unique up to isomorphism
- Part of the Myhill-Nerode theorem generalize to NFAs based on the notion of *bisimulation*.
- *Bisimulation*:

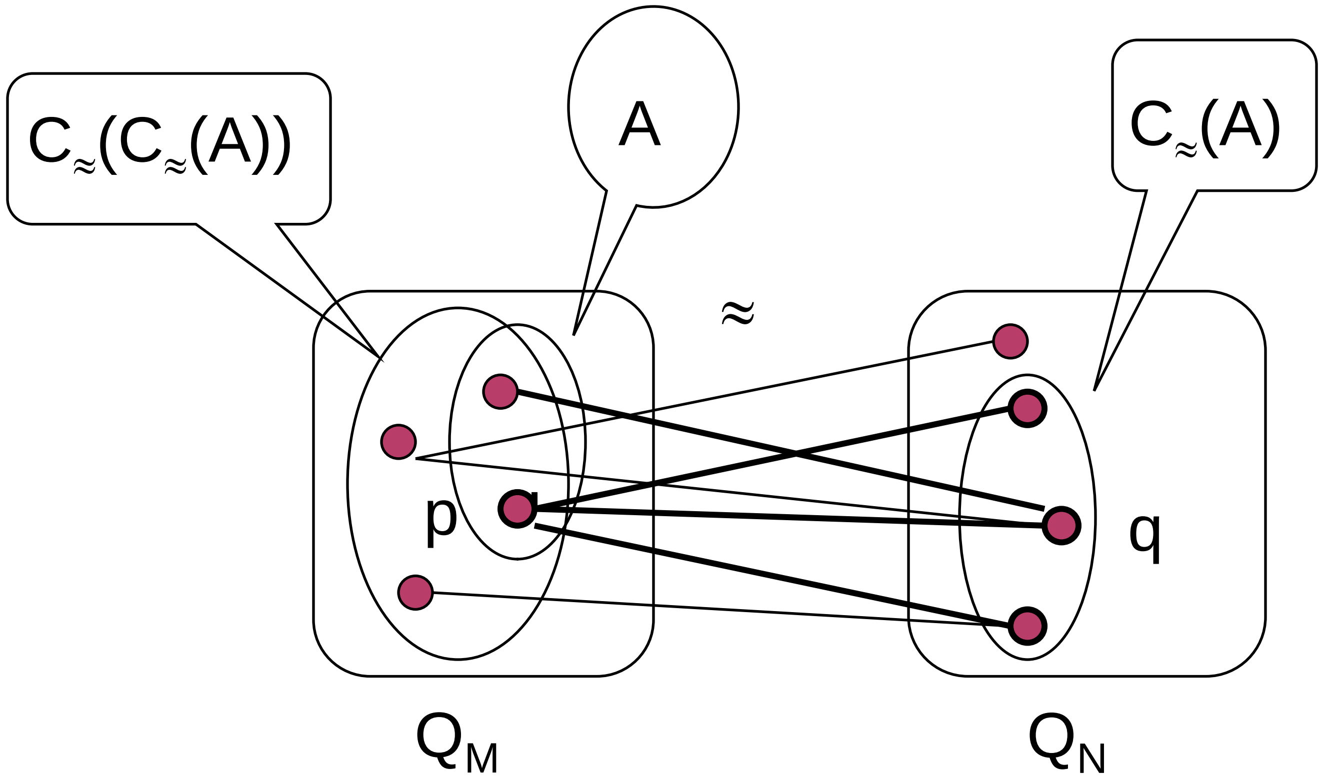
Def: $M=(Q_M,\Sigma, \delta_M,S_M,F_M)$, $N=(Q_N,\Sigma,\delta_N,S_N,F_N)$: two NFAs,

$\approx$: a binary relation from Q_M to Q_N.

- For $B \subseteq Q_N$, define $C_\approx(B) = \{p \in Q_M \mid \exists q \in B \ \ p \approx q \}$
- For $A \subseteq Q_M$, define $C_\approx(A) = \{q \in Q_N \mid \exists P \in A \ \ p \approx q \}$

Extend $\approx$ to subsets of Q_M and Q_N as follows:

- $A \approx B <=>_{def} A \subseteq C_\approx(B)$ and $B \subseteq C_\approx(A)$
- iff $\forall p \in A \ \exists q \in B$ s.t. $p \approx q$ and $\forall q \in B \ \exists p \in A$ s.t. $p \approx q$



$C_{\approx}(C_{\approx}(A))$
A
$C_{\approx}(A)$
$\approx$
p
q
Q_M
Q_N

BISIMULATION

- Def B.1: A relation $\approx$ is called a bisimulation if
  - 1. $S_M \approx S_N$
  - 2. if $p \approx q$ then $\forall a \in \Sigma$, $\delta_M(p,a) \approx \delta_N(q,a)$
  - 3. if $p \approx q$ then $p \in F_M$ iff $q \in F_N$.
- M and N are *bisimilar* if there exists a bisimulation between them.
- For each NFA M, the *bisimilar class* of M is the family of all NFAs that are bisimilar to M.
- Properties of bisimulaions:
  1. Bisimulation is symmetric: if $\approx$ is a bisimulation b/t M and N, then its reverse $\{(q,p) \mid p \approx q\}$ is a bisimulation b/t N and M.
  2. Bisimulation is transitive: $M \approx_1 N$ and $N \approx_2 P \Rightarrow M \approx_1 \approx_2 P$
  3. The union of any nonempty family of bisimulation b/t M and N is a bisimulation b/t M and N.

PROPERTIES OF BISIMULATIONS

Pf: 1,2: direct from the definition.

(3): Let $\{\approx_i \mid i \in I\}$ be a nonempty indexed set of bisimulations b/t M and N. Define $\approx =_{def} \bigcup_{i \in I} \approx_i$.

Thus $p \approx q$ means $\exists i \in I\ p \approx_i q$.

1. Since I is not empty, $S_M \approx_i S_N$ for some $i \in I$, hence $S_M \approx S_N$
2. If $p \approx q \Rightarrow \exists i \in I\ p \approx_i q \Rightarrow \delta_M(p,a) \approx_i \delta_N(q,a) \Rightarrow \delta_M(p,a) \approx \delta_N(q,a)$
3. If $p \approx q \Rightarrow p \approx_i q$ for some $i \Rightarrow (p \in F_M \Leftrightarrow q \in F_N)$

Hence $\approx$ is a bisimulation b/t M and N.

Lem B.3: $\approx$: a bisimulation b/t M and N. If $A \approx B$, then for all x in Σ^*, $\Delta(A,x) \approx \Delta(B,x)$.

pf: by induction on $|x|$. Basis: 1. $x = \varepsilon \Rightarrow \Delta(A,\varepsilon) = A \approx B = \Delta(B,\varepsilon)$.

2. $x = a$: since $A \subseteq C_{\approx}(B)$, if $p \in A \Rightarrow \exists q \in B$ with $p \approx q$. $\Rightarrow \delta_M(p,a) \subseteq C_{\approx}(\delta_N(q,a)) \subseteq C_{\approx}(\Delta_N(B,a))$. $\Rightarrow$ $\Delta_M(A,a) = \bigcup_{p \in A} \delta_M(p,a) \subseteq C_{\approx}(\Delta_N(B,a))$.

By a symmetric argument, $\Delta_N(B,a) \subseteq C_{\approx}(\Delta_M(A,a))$.

So $\Delta_M(A,a) \approx \Delta_N(B,a))$.

BISIMILAR AUTOMATA ACCEPT THE SAME SET.

3. Ind. case: assume $\Delta_M(A,x) \approx \Delta_N(B,x)$. Then

$\Delta_M(A,xa) = \Delta_M(\Delta_M(A,x), a) \approx \Delta_N(\Delta_N(B,x),a) = \Delta_N(B,xa)$. Q.E.D.

Theorem B.4: Bisimilar automata accept the same set.

Pf: assume $\approx$: a bisimulation b/t two NFAs M and N.

Since $S_M \approx S_N$ => $\Delta_M(S_M,x) \approx \Delta_N(S_N,x)$ for all x.

Hence for all x, $x \in L(M)$ <=> $\Delta_M(S_M, x) \cap F_M \neq \{\}$ <=> $\Delta_N(S_N,x) \cap F_N \neq \{\}$ <=> $x \in L(N)$. Q.E.D.

Def: $\approx$: a bisimulation b/t two NFAs M and N

The support of $\approx$ in M is the states of M related by $\approx$ to some state of N, i.e., $\{p \in Q_M \mid p \approx q \text{ for some } q \in Q_N\} = C_{\approx}(Q_N)$.

AUTOBISIMULATION

Lem B.5: A state of M is in the support of all bisimulations involving M iff it is accessible.

Pf: Let $\approx$ be any bisimulation b/t M and another FA.

By def B.1(1), every start state of M is in the support of $\approx$.

By B.1(2), if p is in the support of $\approx$, then every state in $\delta(p,a)$ is in the support of $\approx$. It follows by induction that every accessible state is in the support of $\approx$.

Conversely, since the relation B.3 = $\{(p,p) \mid p \text{ is accessible}\}$ is a bisimulation from M to M and all inaccessible states of M are not in the support of B.3. It follows that no inaccessible state is in the support of all bisimulations. Q.E.D.

Def. B.6: An autobisimulation is a bisimlation b/t an automaton and itself.

PROPERTY OF AUTOBISIMULATIONS

Theorem B.7: Every NFA M has a coarsest autobisimulation $\equiv_M$, which is an equivalence relation.

Pf: let B be the set of all autobisimulations on M.

B is not empty since the identity relation $I_M = \{(p,p) \mid p \text{ in } Q\}$ is an autobisimulation.

1. let $\equiv_M$ be the union of all bisimualtions in B. By Lem B.2(3), $\equiv_M$ is also a bisimualtion on M and belongs to B. So $\equiv_M$ is the largest (i.e., coarsest) of all relations in B.
2. $\equiv_M$ is ref. since for all state p $(p,p) \in I_M \subseteq \equiv_M$.
3. $\equiv_M$ is sym. and tran. by Lem B.2(1,2).
4. By 2,3, $\equiv_M$ is an equivalence relation on Q.

FIND MINIMAL NFA BISIMILAR TO A NFA

- $M = (Q,\Sigma,\delta,S,F)$: a NFA.
- Since accessible subautomaton of M is bisimilar to M under the bisimulation B.3, we can assume wlog that M has no inaccessible states.
- Let $\equiv$ be $\equiv_M$, the maximal autobisimulation on M.
  for p in Q, let $[p] = \{q \mid p \equiv q\}$ be the $\equiv$-class of p, and
  let $\ll$ be the relation relating p to its $\equiv$-class [p], i.e.,
  $$\ll \subseteq Qx2^Q =_{def} \{(p,[p]) \mid p \text{ in } Q\}$$
  for each set of states $A \subseteq Q$, define $[A] = \{[p] \mid p \text{ in } A\}$. Then

Lem B.8: For all $A,B \subseteq Q$,

- 1. $A \subseteq C_\equiv(B)$ iff $[A] \subseteq [B]$, 2. $A \equiv B$ iff $[A] = [B]$, 3. $A \ll [A]$

pf:1. $A \subseteq C_\equiv(B)$ $\Leftrightarrow \forall p$ in A $\forall$ q in B s.t. $p \equiv q$ $\Leftrightarrow$ $[A] \subseteq [B]$

2. Direct from 1 and the fact that $A \equiv B$ iff $A \subseteq C_\equiv(B)$ and $B \subseteq C_\equiv(A)$

3. $p \in A \Rightarrow p \in [p] \in [A]$, $B \in [A] \Rightarrow \exists\, p \in A$ with $p \ll [p] = B$.

MINIMAL NFA BISIMILAR TO AN NFA (CONT'D)

- Now define M' = {Q', S, d', S',F'} = $M/\equiv$ where
  - Q' = [Q] = {[p] | $p \in Q$},
  - S' = [S] = {[p] | $p \in S$} , F' = [F] = {[p] | $p \in F$} and
  - $\delta'([p],a) = [\delta(p,a)]$,
  - Note that δ' is well-defined since
    $[p] = [q] \Rightarrow p \equiv q \Rightarrow \delta(p,a) \equiv \delta(q,a) \Rightarrow [\delta(p,a)] = [\delta(q,a)]$
    $\Rightarrow \delta'([p],a) = \delta'([q],a)$

Lem B.9: The relation « is a bisimulation b/t M and M'.

pf: 1. By B.8(3): $S \subseteq [S] = S'$.

2. If p « [q] => $p \equiv q \Rightarrow \delta(p,a) \equiv \delta(q,a)$
   $\Rightarrow [\delta(p,a)] = [\delta(q,a)] \Rightarrow \delta(p,a)$ « $[\delta(p,a)] = [\delta(q,a)]$.

3. if $p \in F \Rightarrow [p] \in [F] = F'$ and
   if $[p] \in F' = [F] \Rightarrow \exists q \in F$ with $[q] = [p] \Rightarrow p \equiv q \Rightarrow p \in F$.

By theorem B.4, M and M' accept the same set.

AUTOBISIMULATION

Lem B.10: The only autobisimulation on M' is the identity relation =.

Pf: Let ~ be an autobisimulation of M'. By Lem B.2(1,2), the relation « ~ » is a bisimulation from M to itself.

1. Now if there are $[p] \neq [q]$ (hence not $p \equiv q$) with [p] ~ [q]

=> p « [p] ~ [q] » q => p « ~ » q => « ~ » $\not\subset \equiv$, a contradiction !.

On the other hand, if [p] not~ [p] for some [p] => for any [q], [p] not~ [q] (by 1. and the premise)

=> p not (« ~ ») q for any q (p « [p] [q] » q)

=> p is not in the support of « ~ »

=> p is not accessible, a contradiction.

QUOTIENT AUTOMATA ARE MINIMAL FAS

- Theorem B11: M: an NFA w/t inaccessible states, $\equiv$: maximal autobisimulation on M. Then M' = M /$\equiv$ is the minimal automata bisimilar to to M and is unique up to isomorphism.

pf: N: any NFA bisimilar to M w/t inaccessible states.

N' = N/ $\equiv_N$ where $\equiv_N$ is the maximal autobisimulation on N.

=> M' bisimiar to M bisimilar to N bisimiar to N'.

Let $\approx$ be any bisimulation b/t M' and N'.

Under $\approx$, every state p of M' has at least on state q of N' with p $\approx$ q and every state q of N' has exactly one state p of M' with p $\approx$ q.

O/w p $\approx$ q $\approx^{-1}$ p' $\neq$ p => $\approx \approx^{-1}$ is a non-identity autobisimulation on M, a contradiciton!.

Hence $\approx$ is 1-1. Similarly, $\approx^{-1}$ is 1-1 => $\approx$ is 1-1 and onto and hence is an isomorphism b/t M' and N'. Q.E.D.

ALGORITHM FOR COMPUTING MAXIMAL BISIMULATION

- a generalization of that of Lec 14 for finding equivalent states of DFAs

The algorithm: Find maximal bisimulation of two NFAs M and N

- 1. write down a table of all pairs (p,q) of states, initially
- unmarked
- 2. mark (p,q) if $p \in F_M$ and $q \notin F_N$ or vice versa.
- 3. repeat until no more change occur: if (p,q) is unmarked and if for some $a \in \Sigma$, either
  $\exists p' \in \delta_M(p,a)$ s.t. $\forall q' \in \delta_N(q,a)$, (p',q') is marked, or
  $\exists q' \in \delta_N(q,a)$ s.t. $\forall p' \in \delta_M(p,a)$, (p',q') is marked,
  then mark (p,q).
- 4. define $p \equiv q$ iff (p,q) are never marked.
- 5. If $S_M \equiv S_N \Rightarrow \equiv$ is the maximal bisimulation
- o/w M and N has no bisimulation.