COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- Applications of Pumping Lemma
- Closure Property

Observation:

- Every language of finite size has to be regular

(we can easily construct an NFA
that accepts every string in the language)

Therefore, every non-regular language
has to be of infinite size

(contains an infinite number of strings)

Suppose you want to prove that
An infinite language L is not regular

1. Assume the opposite: L is regular

2. The pumping lemma should hold for L

3. Use the pumping lemma to obtain a contradiction

4. Therefore, L is not regular

Explanation of Step 3: How to get a contradiction

1. Let m be the critical length for L

2. Choose a particular string $w \in L$ which satisfies the length condition $|w| \geq m$

3. Write $w = xyz$

4. Show that $w' = xy^i z \notin L$ for some $i \neq 1$

5. This gives a contradiction, since from pumping lemma $w' = xy^i z \in L$

Note: It suffices to show that only one string $w \in L$ gives a contradiction

You don't need to obtain contradiction for every $w \in L$

Example of Pumping Lemma application

Theorem: The language $L = \{a^n b^n : n \geq 0\}$

is not regular

Proof: Use the Pumping Lemma

$$L = \{a^n b^n : n \geq 0\}$$

Assume for contradiction
that L is a regular language

Since L is infinite
we can apply the Pumping Lemma

$$L = \{a^n b^n : n \geq 0\}$$

Let m be the critical length for L

Pick a string w such that: $w \in L$

and length $|w| \geq m$

We pick $w = a^m b^m$

From the Pumping Lemma:

we can write $w = a^m b^m = x\ y\ z$

with lengths $|x\,y| \le m, \quad |y| \ge 1$

$$w = xyz = a^m b^m = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...a}_{z}}^{m}\overbrace{b...b}^{m}$$

Thus: $y = a^k, \quad 1 \le k \le m$

$$x\,y\,z = a^m b^m \qquad y = a^k, \quad 1 \le k \le m$$

From the Pumping Lemma: $x\,y^i\,z \in L$

$$i = 0, 1, 2, \ldots$$

Thus: $x\,y^2\,z \in L$

$$x\,y\,z = a^m b^m \qquad y = a^k, \quad 1 \le k \le m$$

From the Pumping Lemma: $x\,y^2\,z \in L$

$$xy^2z = \overbrace{\underbrace{a...a}_{x}\underbrace{a...a}_{y}\underbrace{a...a}_{y}a...a}^{m+k}\overbrace{b...b}^{m} \in L$$

with $z = a...ab...b$

Thus: $a^{m+k} b^m \in L$

$$a^{m+k} b^m \in L \qquad k \geq 1$$

BUT: $L = \{a^n b^n : n \geq 0\}$

$$\Downarrow$$

$$a^{m+k} b^m \notin L$$

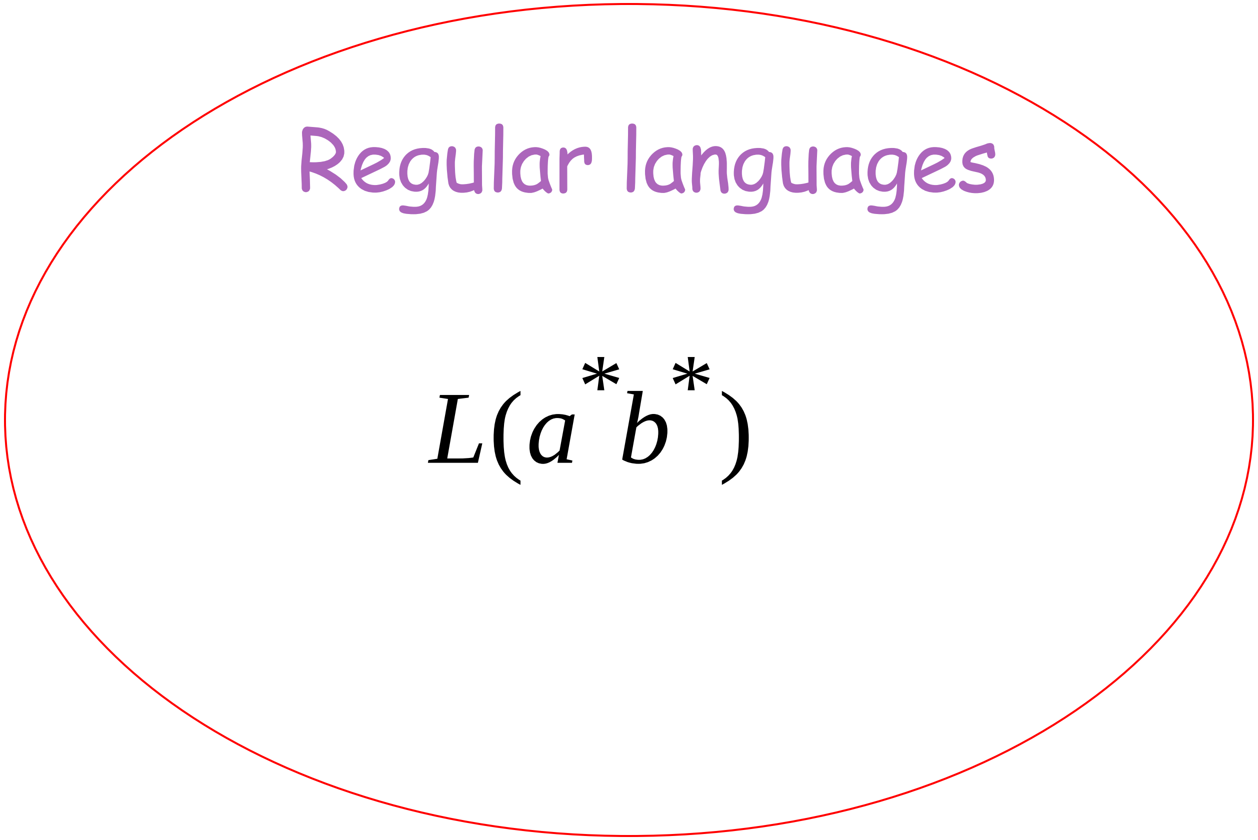
CONTRADICTION!!!

Therefore: Our assumption that L is a regular language is not true

Conclusion: L is not a regular language

END OF PROOF

Non-regular language $\{a^n b^n : \ \ n \geq 0\}$



USING CLOSURE PROPERTY

Let ∇ be a binary operation on languages and the class of regular languages is closed under ∇. (∇ can be $\cup$, $\cap$, or -)

- If L_1 and L_2 are regular, then $L_1 \nabla L_2$ is regular.
- If $L_1 \nabla L_2$ is not regular, then L_1 or L_2 are not regular.
- If $L_1 \nabla L_2$ is not regular but L_2 is regular, then L_1 is not regular.

PROVE THAT $\{W \in \{0,1\}^* \mid$ THE NUMBER OF 0'S AND 1'S IN W ARE EQUAL$\}$ IS NOT REGULAR

Let $L=\{w \in \{0,1\}^* \mid$ the number of 0's and 1's in w are equal$\}$.

Let $R= \{0^i1^i \mid i \geq 0\}$.

$R = 0^*1^* \cap L$

We already prove that R is not regular.

But 0*1* is regular.

Then, L is not regular.

USING CLOSURE PROPERTY

Let ∇ be a unary operation on a language and the class of regular languages is closed under ∇.

(∇ can be complement or *)

- If L is regular, then ∇L is regular.
- If ∇L is not regular, then L is not regular.

THE NUMBER OF 0'S AND 1'S IN W ARE NOT EQUAL} IS NOT REGULAR

Let L = $\{w \in \{0,1\}^* \mid$ the number of 0's and 1's in w are not equal$\}$.

Let R = $\overline{L}$ = $\{w \in \{0,1\}^* \mid$ the number of 0's and 1's in w are equal$\}$.

We already prove that R is not regular.

Then, L is not regular.