COURSE: THEORY OF AUTOMATA COMPUTATION

TOPICS TO BE COVERED

- Introduction
- Why do we study Theory of Computation ?
- Importance of Theory of Computation
- Languages
- Languages and Problems

WHAT IS COMPUTATION ?

- Sequence of mathematical operations ?
  - What are, and are not, mathematical operations?
- Sequence of well-defined operations
  - How many operations ?
    - The fewer, the better.
  - Which operations ?
    - The simpler, the better.

WHAT DO WE STUDY IN THEORY OF COMPUTATION ?

- What is computable, and what is not ?
- Basis of
  - Algorithm analysis
  - Complexity theory
- What a computer can and cannot do
- Are you trying to write a non-existing program?
  - Can you make your program more efficient?

WHAT DO WE STUDY IN COMPLEXITY THEORY ?

- What is easy, and what is difficult, to compute ?

- What is easy, and what is hard for computers to do?
- Is your cryptograpic scheme safe?

APPLICATIONS IN COMPUTER SCIENCE

- Analysis of algorithms
- Complexity Theory
- Cryptography
- Compilers
- Circuit design

HISTORY OF THEORY OF COMPUTATION

- 1936 Alan Turing invented the *Turing machine*, and proved that there exists an *unsolvable problem*.
- 1940's Stored-program computers were built.
- 1943 McCulloch and Pitts invented *finite automata*.
- 1956 Kleene invented *regular expressions* and proved the equivalence of regular expression and finite automata.

HISTORY OF THEORY OF COMPUTATION

- 1956 Chomsky defined *Chomsky hierarchy*, which organized languages recognized by different automata into hierarchical classes.
- 1959 Rabin and Scott introduced *nondeterministic finite automata* and proved its equivalence to (deterministic) finite automata.
- 1950's-1960's More works on languages, grammars, and compilers

HISTORY OF THEORY OF COMPUTATION

- 1965 Hartmantis and Stearns defined *time complexity*, and Lewis, Hartmantis and Stearns defined *space complexity*.
- 1971 Cook showed the first *NP-complete problem*, the *satisfiability* proobblem.
- 1972 Karp Showed many other NP-complete problems.

HISTORY OF THEORY OF COMPUTATION

- 1976 Diffie and Helllman defined *Modern Cryptography* based on NP-complete problems.

- 1978 Rivest, Shamir and Adelman proposed a public-key encryption scheme, *RSA*.

ALPHABET AND STRINGS

- An *alphabet* is a finite, non-empty set of symbols.
  - $\{0,1\}$ is a binary alphabet.
  - $\{A, B, \ldots, Z, a, b, \ldots, z\}$ is an English alphabet.
- A *string* over an alphabet Σ is a sequence of any number of symbols from Σ.
  - 0, 1, 11, 00, and 01101 are strings over $\{0, 1\}$.
  - *Cat*, *CAT*, and *compute* are strings over the English alphabet.

EMPTY STRING

- An *empty string*, denoted by ε, is a string containing no symbol.
  - ε is a string over any alphabet.

LENGTH

- The length of a string x, denoted by $length(x)$, is the number of positions of symbols in the string.

  Let $\Sigma = \{a, b, \ldots, z\}$

  $length(automata) = 8$

  $length(computation) = 11$

  $length(\varepsilon) = 0$

- $x(i)$, denotes the symbol in the i^{th} position of a string x, for $1 \leq i \leq length(x)$.

STRING OPERATIONS

- Concatenation
- Substring
- Reversal

CONCATENATION

- The concatenation of strings x and y, denoted by $x{\cdot}y$ or $x\,y$, is a string z such that:
  - $z(i) = x(i)$ for $1 \le i \le length(x)$
  - $z(i) = y(i)$ for $length(x)<i\le length(x)+length(y)$

- Example
  - $automata{\cdot}computation = automatacomputation$

CONCATENATION

The concatenation of string x for n times, where $n \geq 0$, is denoted by x^n

- $x^0 = \varepsilon$
- $x^1 = x$
- $x^2 = x\ x$
- $x^3 = x\ x\ x$
- $\ldots$

SUBSTRING

Let x and y be strings over an alphabet Σ

The string x is a substring of y if there exist strings w and z over Σ such that $y = w\,x\,z$.

- ε is a substring of every string.
- For every string x, x is a substring of x itself.

Example

- ε, *comput* and *computation* are substrings of *computation*.

REVERSAL

Let x be a string over an alphabet Σ

The reversal of the string x, denoted by x^r, is a string such that

- if x is ε, then x^r is ε.
- If a is in Σ, y is in Σ^* and $x = a\,y$, then $x^r = y^r\,a$.

EXAMPLE OF REVERSAL

$$
\begin{aligned}
& (automata)^r \\
= & (utomata)^r\, a \\
= & (tomata)^r\, ua \\
= & (omata)^r\, tua \\
= & (mata)^r\, otua \\
= & (ata)^r\, motua \\
= & (ta)^r\, amotua \\
= & (a)^r\, tamotua \\
= & (\varepsilon)^r\, atamotua \\
= & \; atamotua
\end{aligned}
$$

Σ^*

- The set of strings created from any number (0 or 1 or …) of symbols in an alphabet Σ is denoted by Σ^*.
- That is, $\Sigma^* = \cup_{i=0}^{\infty} \Sigma^i$
  - Let $\Sigma = \{0, 1\}$.
  - $\Sigma^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, \ldots\}$.

Σ^+

- The set of strings created from at least one symbol (1 or 2 or …) in an alphabet Σ is denoted by Σ^+.
- That is, $\Sigma^+ = \cup_{i=1}^{\infty} \Sigma^i$

  $= \cup_{i=0..\infty} \Sigma^i - \Sigma^0$

  $= \cup_{i=0..\infty} \Sigma^i - \{\varepsilon\}$
- Let $\Sigma = \{\boldsymbol{0}, \boldsymbol{1}\}$. $\Sigma^+ = \{\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{00}, \boldsymbol{01}, \boldsymbol{10}, \boldsymbol{11}, \boldsymbol{000}, \boldsymbol{001}, \boldsymbol{010}, \boldsymbol{011}, \ldots\}$.

  Σ^* and Σ^+ are infinite sets.

LANGUAGES

- A language over an alphabet Σ is a set of strings over Σ.
  - Let $\Sigma = \{0, 1\}$ be the alphabet.
  - $L_e = \{\omega \in \Sigma^* \mid$ the number of 1's in ω is even$\}$.
  - $\varepsilon, 0, 00, 11, 000, 110, 101, 011, 0000, 1100, 1010, 1001, 0110, 0101, 0011, \ldots$ are in L_e

OPERATIONS ON LANGUAGES

- Complementation
- Union
- Intersection
- Concatenation
- Reversal
- Closure

COMPLEMENTATION

Let L be a language over an alphabet Σ.

The complementation of L, denoted by $\overline{L}$, is Σ^*-L.

Example:

Let $\Sigma = \{0, 1\}$ be the alphabet.

$L_e = \{\omega\in\Sigma^* \mid$ the number of 1's in ω is even$\}$.

$\overline{L_e} = \{\omega\in\Sigma^* \mid$ the number of 1's in ω is not even$\}$.

$\overline{L_e} = \{\omega\in\Sigma^* \mid$ the number of 1's in ω is odd$\}$.

UNION

Let L_1 and L_2 be languages over an alphabet Σ.

The union of L_1 and L_2, denoted by $L_1 \cup L_2$, is $\{x \mid x$ is in L_1 or $L_2\}$.

Example:

$\{x \in \{0,1\}^* \mid x$ begins with $0\} \cup \{x \in \{0,1\}^* \mid x$ ends with $0\}$

$= \{x \in \{0,1\}^* \mid x$ begins or ends with $0\}$

INTERSECTION

Let L_1 and L_2 be languages over an alphabet Σ.

The intersection of L_1 and L_2, denoted by $L_1 \cap L_2$, is $\{ x \mid x$ is in L_1 and $L_2\}$.

Example:

$\{ x \in \{0,1\}^* \mid x$ begins with $0\} \cap \{ x \in \{0,1\}^* \mid x$ ends with $0\}$

$= \{ x \in \{0,1\}^* \mid x$ begins and ends with $0\}$

CONCATENATION

Let L_1 and L_2 be languages over an alphabet Σ.

The concatenation of L_1 and L_2, denoted by $L_1 \cdot L_2$, is $\{w_1 \cdot w_2 \mid w_1$ is in L_1 and w_2 is in $L_2\}$.

Example

$\{ x \in \{0,1\}^* \mid x$ begins with $0\} \cdot \{x \in \{0,1\}^* \mid x$ ends with $0\}$

$= \{ x \in \{0,1\}^* \mid x$ begins and ends with 0 and $length(x) \geq 2\}$

$\{ x \in \{0,1\}^* \mid x$ ends with $0\} \cdot \{x \in \{0,1\}^* \mid x$ begins with $0\}$

$= \{ x \in \{0,1\}^* \mid x$ has 00 as a substring$\}$

REVERSAL

Let L be a language over an alphabet Σ.

The reversal of L, denoted by L^r, is $\{w^r \mid w$ is in $L\}$.

Example

$\{x \in \{0,1\}^* \mid x$ begins with 0$\}^r$

$= \{x \in \{0,1\}^* \mid x$ ends with 0$\}$

$\{x \in \{0,1\}^* \mid x$ has 00 as a substring$\}^r$

$= \{x \in \{0,1\}^* \mid x$ has 00 as a substring$\}$

KLEENE'S CLOSURE

Let L be a language over an alphabet Σ.

The Kleene's closure of L, denoted by L^*, is $\{x \mid$ for an integer $n \geq 0$ $x = x_1 x_2 \ldots x_n$ and $x_1, x_2, \ldots, x_n$ are in $L\}$.

That is, $L^* = \cup_{i=0}^{\infty} L^i$

Example: Let $\Sigma = \{0,1\}$ and

$L_e = \{\omega \in \Sigma^* \mid$ the number of 1's in ω is even$\}$

$L_e^* = \{\omega \in \Sigma^* \mid$ the number of 1's in ω is even$\}$

$(\overline{L}_e)^* = \{\omega \in \Sigma^* \mid$ the number of 1's in ω is odd$\}^*$

$= \{\omega \in \Sigma^* \mid$ the number of 1's in $\omega > 0\}$

CLOSURE

Let L be a language over an alphabet Σ.

The closure of L, denoted by L^+, is $\{ x \mid$ for an integer $n \geq 1$, $x = x_1x_2\ldots x_n$ and $x_1, x_2, \ldots, x_n$ are in $L\}$

That is, $L^+ = \cup_{i=1}^{\infty} L^i$

Example:

Let $\Sigma = \{0, 1\}$ be the alphabet.

$L_e = \{\omega \in \Sigma^* \mid$ the number of 1's in ω is even$\}$

$L_e^+ = \{\omega \in \Sigma^* \mid$ the number of 1's in ω is even$\} = L_e^*$

OBSERVATION ABOUT CLOSURE

$L^{+} = L^{*} - \{\varepsilon\}$?

Example:

L = {$\omega \in \Sigma^*$ | the number of 1's in ω is even}

L^{+} = {$\omega \in \Sigma^*$ | the number of 1's in ω is even} = L_e^*

Why?

$L^{*} = L^{+} \cup \{\varepsilon\}$?

LANGUAGES AND PROBLEMS

- Problem
  - Example: What are prime numbers > 20?
- Decision problem
  - Problem with a YES/NO answer
  - Example: Given a positive integer n, is n a prime number > 20?
- Language
  - Example: $\{n \mid n$ is a prime number $> 20\}$

    $= \{23, 29, 31, 37, \ldots\}$

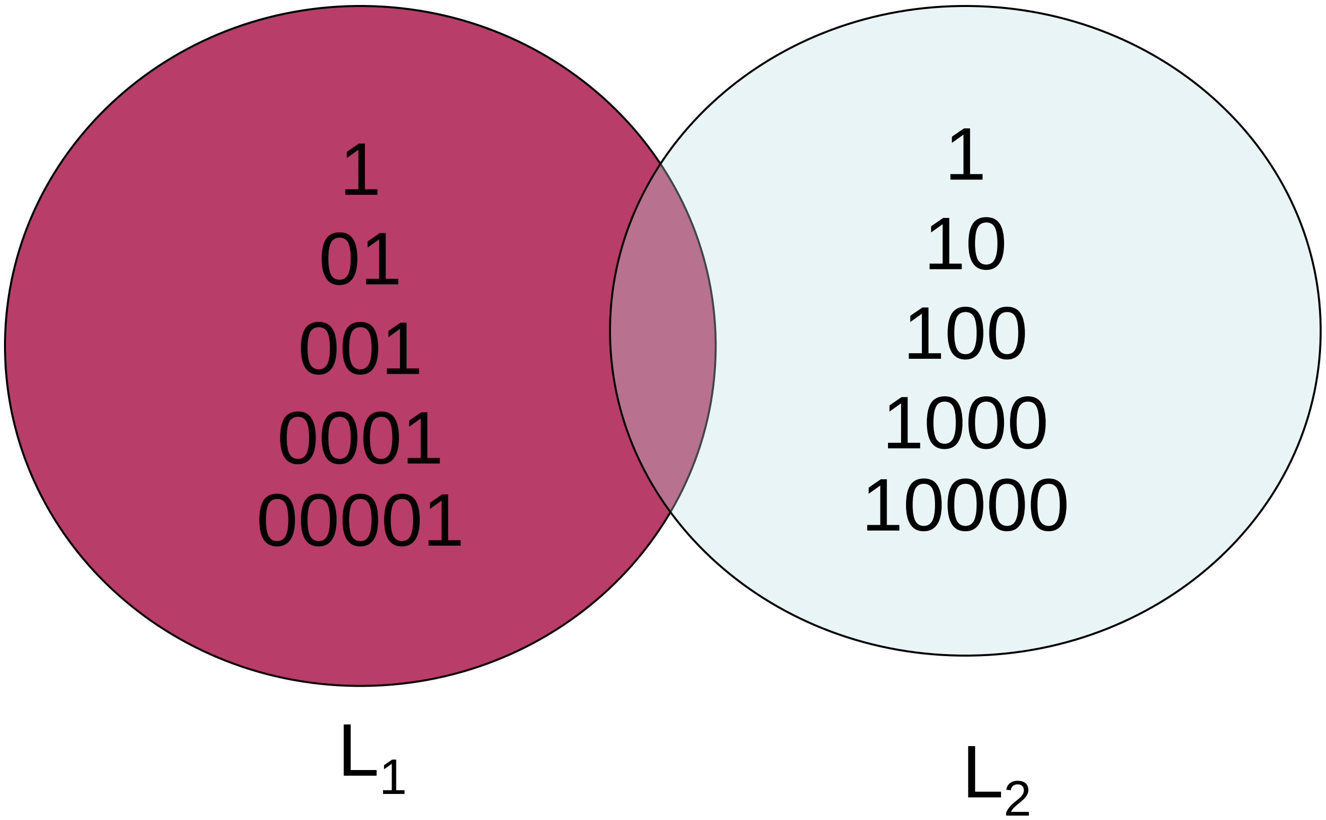
LANGUAGE RECOGNITION AND PROBLEM

- A problem is represented by a set of strings of the input whose answer for the corresponding problem is "YES".
- a string is in a language = the answer of the corresponding problem for the string is "YES"
  - Let "Given a positive integer n, is n a prime number > 20?" be the problem P.
  - If a string represents an integer i in $\{m \mid m$ is a prime number > 20$\}$, then the answer for the problem P for $n = i$ is true.

COMMON MISCONCEPTION

Beware

A LANGUAGE IS A SET.



AND, THERE IS ALSO A SET OF LANGUAGES.