Digital Signal Processing- Lecture 20

Topics to be covered:

- Up sampling & Down sampling

Multirate Digital Signal Processing

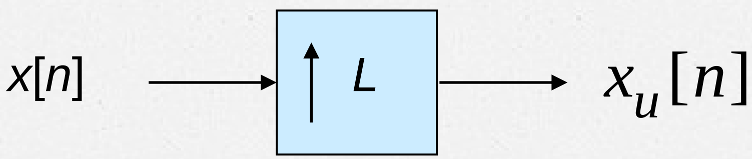
Basic Sampling Rate Alteration Devices

- *Up-sampler* - Used to increase the sampling rate by an integer factor
- *Down-sampler* - Used to decrease the sampling rate by an integer factor

Up-Sampler

Time-Domain Characterization

- An up-sampler with an *up-sampling factor* L, where L is a positive integer, develops an output sequence with a sampling rate that is L times larger than that of the input sequence $x[n]$ $x_u[n]$
- Block-diagram representation



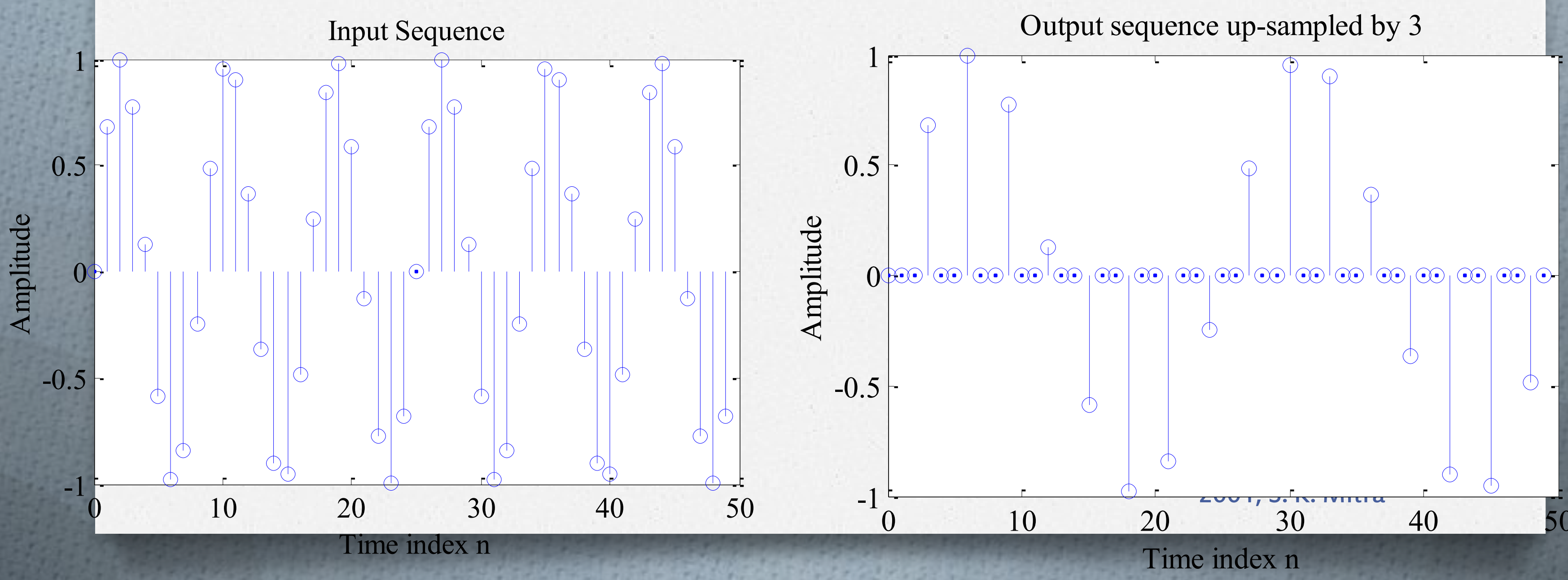
Up-Sampler

- Up-sampling operation is implemented by inserting $L-1$ equidistant zero-valued samples between two consecutive samples of $x[n]$
- Input-output relation

$$x_u[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \cdots \\ 0, & \text{otherwise} \end{cases}$$

Up-Sampler

- Figure below shows the up-sampling by a factor of 3 of a sinusoidal sequence with a frequency of 0.12 Hz obtained using Program 10_1



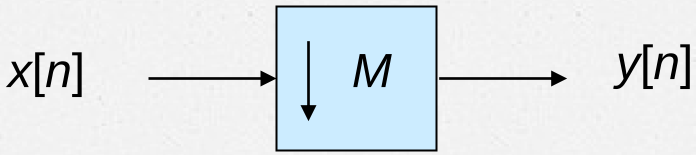
Up-Sampler

- In practice, the zero-valued samples inserted by the up-sampler are replaced with appropriate nonzero values using some type of filtering process
- Process is called ***interpolation*** and will be discussed later

Down-Sampler

Time-Domain Characterization

- An down-sampler with a ***down-sampling factor*** M, where M is a positive integer, develops an output sequence $y[n]$ with a sampling rate that is $(1/M)$-th of that of the input sequence $x[n]$
- Block-diagram representation



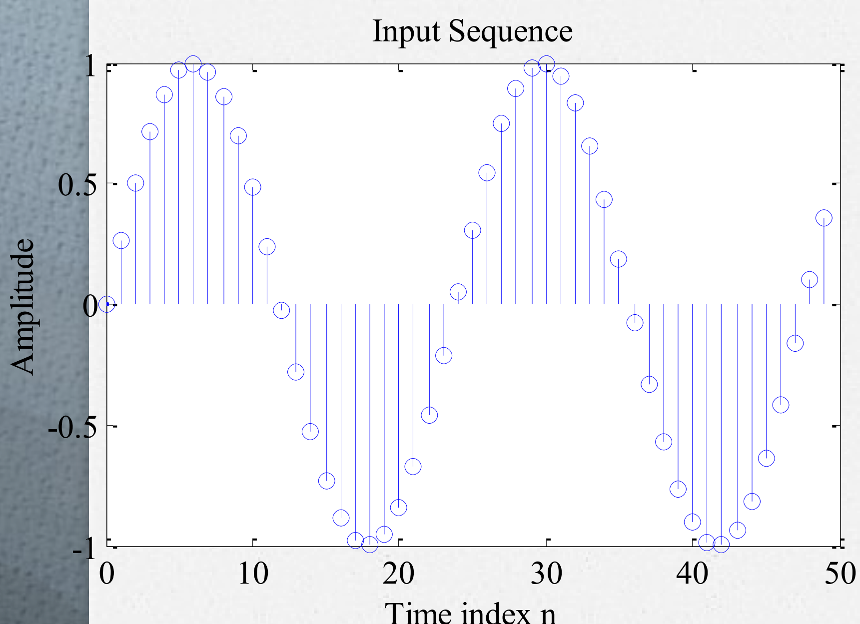
Down-Sampler

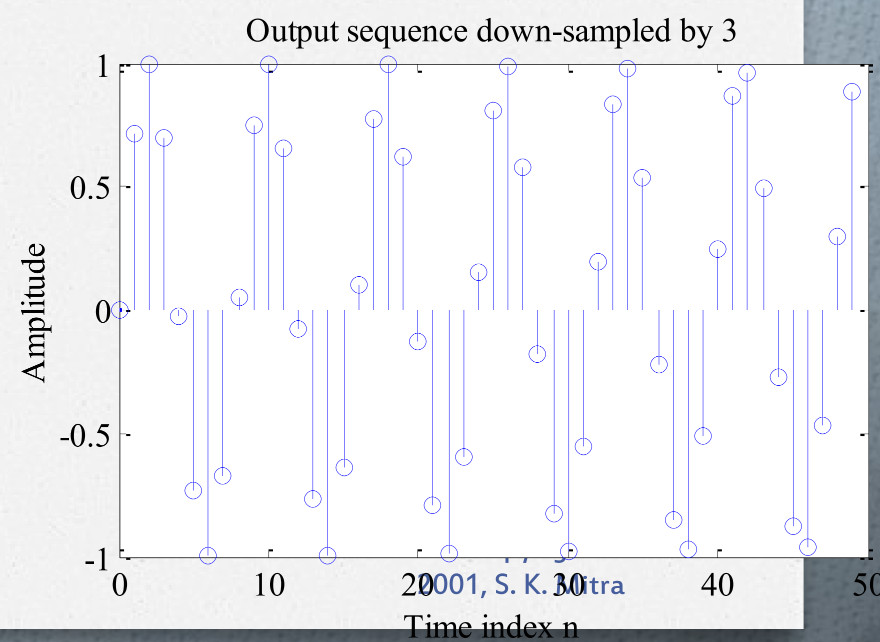
- Down-sampling operation is implemented by keeping every M-th sample of $x[n]$ and removing $M-1$ in-between samples to generate $y[n]$
- Input-output relation

$$y[n] = x[nM]$$

Down-Sampler

- Figure below shows the down-sampling by a factor of 3 of a sinusoidal sequence of frequency 0.042 Hz obtained using Program 10_2





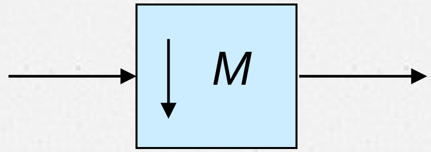
Basic Sampling Rate Alteration Devices

- Sampling periods have not been explicitly shown in the block-diagram representations of the up-sampler and the down-sampler
- This is for simplicity and the fact that the ***mathematical theory of multirate systems*** can be understood without bringing the sampling period T or the sampling frequency F_T into the picture

Down-Sampler

- Figure below shows explicitly the time-dimensions for the down-sampler

$$x[n] = x_a(nT)$$



$$y[n] = x_a(nMT)$$

Input sampling frequency

$$F_T = \frac{1}{T}$$

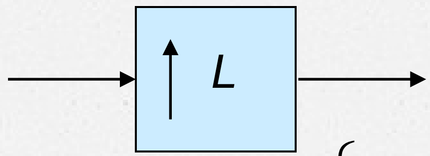
Output sampling frequency

$$F_T' = \frac{F_T}{M} = \frac{1}{T'}$$

Up-Sampler

- Figure below shows explicitly the time-dimensions for the up-sampler

$$x[n] = x_a(nT)$$



$y[n]$

$$= \begin{cases} x_a(nT/L), & n = 0, \pm L, \pm 2L, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Input sampling frequency

$$F_T = \frac{1}{T}$$

Output sampling frequency

$$F_T' = LF_T = \frac{1}{T'}$$

Basic Sampling Rate Alteration Devices

- The *up-sampler* and the *down-sampler* are *linear* but *time-varying discrete-time systems*
- We illustrate the time-varying property of a down-sampler
- The time-varying property of an up-sampler can be proved in a similar manner

Basic Sampling Rate Alteration Devices

- Consider a factor-of-*M* down-sampler defined by
- Its output $y_1[n]$ for an input $x_1[n] = x[n - n_0]$ is then given by

$$y[n] = x[nM]$$

- From the input-output relation of the down-sampler we obtain

$$y_1[n] = x_1[Mn] = x[Mn - n_0]$$

$$y[n - n_0] = x[M(n - n_0)]$$

$$= x[Mn - Mn_0] \neq y_1[n]$$