Digital Signal Processing- Lecture 12

Topics to be covered:

- Inverse Z transform

IIR vs FIR Filters

- FIR filters often employed in problems where linear phase required.
- When *phase distortion tolerable*, *IIR* are usually *favoured*
  - Typically require *less parameters* to achieve sharp cutoff filters.
  - Thus for given response specification, *lower computational complexity/less memory* (despite FFT cannot be used)
- Main problems of IIR filters.
  - *Difficult design.*

IIR Filters Design from an Analogue Prototype

- Given filter specifications, direct determination of filter coefficients is too complex
- Well-developed design methods exist for analogue low-pass filters
- Almost all methods rely on converting an analogue filter to a digital one

Analogue filter Rational Transfer Function

Assume an analog filter can be described by a rational transfer function ($\{\alpha_k\}$ and $\{\beta_k\}$ real-valued)

$$H_a(s) = \frac{\sum_{k=0}^{M} \beta_k s^k}{\sum_{k=0}^{N} \alpha_k s^k}$$

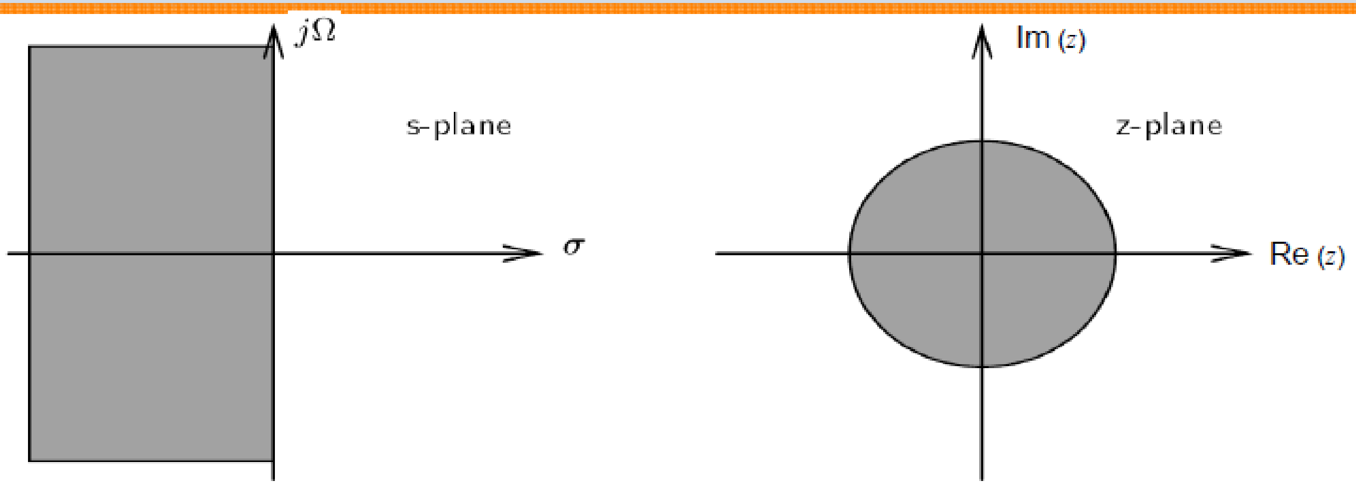
where $H_a(s)$ is the Laplace transform of the impulse response $h_a(t)$

$$H_a(s) = \int_{-\infty}^{\infty} h_a(t)\, e^{-st} dt.$$

In the time domain, it means that the input $x(t)$ and the output $y(t)$ are related by

$$\sum_{k=0}^{M} \alpha_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} \beta_k \frac{d^k x(t)}{dt^k}.$$

Analogue to Digital Conversion



$$H_c(s) \leftrightarrow H(z)$$

- Analogue filters stable if poles on left half of the s-plane / Digital filters stable if poles inside unit circle

➡ Left half of the s-plane should map inside the unit circle in the z-plane.

- The $j\Omega$ axis in the s-plane should map the unit circle in the z-plane; i.e.

Impulse Invariant method

Start with suitable analogue transfer function $h_c(t)$ and discretize it

$$h(n) \triangleq h_c(nT) \text{ where } T = 1/F_s \text{ sampling period.}$$

Sampling in time $\Leftrightarrow$ Periodic repetition in frequency

$$H(\omega) = F_s \sum_{k=-\infty}^{\infty} H_c[(\omega - 2\pi k) F_s]$$

where $\omega = 2\pi f$ and $f = F/F_s$ is the normalized frequency

Impulse Invariant method: Steps

1. Compute the Inverse Laplace transform to get impulse response of the analogue filter

2. Sample the impulse response (quickly enough to avoid aliasing problem)

3. Compute z-transform of resulting sequence

Example 1 – Impulse Invariant Method

Consider first order analogue filter

$$H_c(s) = \frac{s}{s+\alpha} = 1 - \frac{\alpha}{s+\alpha}$$

Corresponding impulse response is

$$h_c(t) = \underbrace{\delta(t)} - \alpha e^{-\alpha t} v(t)$$

The presence of delta term prevents sampling of impulse response which thus cannot be defined

Fundamental problem: high-pass and band-stop filters have functions with numerator and denominator polynomials of the same degree and

Example 2 – Impulse Invariant Method

Consider an analogue filter

$$H_c(s) = \frac{C}{s - \alpha}$$

Step 1. Impulse response of the analogue filter

$$h_c(t) = Ce^{\alpha t}$$

Step 2. Sample the impulse response

$$h(n) = Ce^{\alpha nT}$$

Step 3. Compute z-transform

$$H(z) = \frac{C}{1 - \epsilon^{\alpha T} z^{-1}}$$