Digital Signal Processing- Lecture 5

Topics to be covered:

- **FIR Filter**

FIR Digital Filter, IIR Digital Filter

- Let {h[n]}: impulse response

  {x(n)}: input,

  {y(n)}: output

- Finite impulse response (FIR) filter:

$$y(n)=\sum_{j=0}^{J-1}h(j)x(n-j)$$

- Infinite impulse response (IIR) filter

$$y(n)=\sum_{i=1}^{P}a(i)y(n-i)+\sum_{k=0}^{Q}b(k)x(n-k)$$

$$\frac{Y(z)}{X(z)}=\frac{\sum_{m=0}^{Q}b(m)z^{-m}}{1+\sum_{k=1}^{P}a(k)z^{-k}}=\frac{B(z)V(z)}{A(z)V(z)}$$

FIR Digital Filter, IIR Digital Filter

- Impulse input:

  if $x(n)=\delta(n)$, $y(n)=h(n)$ is the impulse response that has finite extent.

- Computation is the same as convolution.

- The length of $\{y(n)\}$ may be infinite!
- Stability concerns:
  - The magnitude of $y(n)$ may become infinity even if all $x(n)$ are finite!
  - coefficient values,
  - quantization error

FIR Digital Filter, IIR Digital Filter

- FIR filter can be implemented using direct form or fast convolution methods like FFT ,hence STABLE.
- Realized by Non-Recursive methods.

- IIR filters are often factored into products (cascade realization) or sum (parallel realization) of 1^{st} order or 2^{nd} order sections due to numerical concerns(Manual Calculation only possible)
- Realised by Recursive(Feedback) methods.

FIR Digital Filter, IIR Digital Filter

- They have LINEAR PHASE.
- Less susceptible to Noise.
- To design we have

a)Park Mc Clellan's method.

b)Fourier Series method.

c)Frequency Sampling OR Inverse Fourier Transform method.

d)Window technique.

E.g. Rectangular,Hamming,Hanning,Bartlett,Blackmann ,Kaiser Windows.

e)Minimax or Optimal Filter Design.

- They don't have linear phase & hence are used at places where phase distortion is tolerable.
- More susceptible to Noise.
- To design we have

a)Impulse Invarience method.

b)Bilinear Transformation method.

c)Backward difference method.

FIR Digital Filter, IIR Digital Filter

- Storage Requirements & Arithmetic operation is more here.
- Greater Flexibility to control the shape of their Magnitude response & Realization Efficiency.

- Storage Requirements & Arithmetic operation is less.
- Less Flexibility to control the shape of their Magnitude response.
- Often derived from analog filters

Linear Phase Response

$h(n) = \pm h(N-1-n)$……….Only with FIR, Not with IIR or Analog Filters.

Where N =length of impulse response.

Used in Speech related Applications, Data Transmission Applications

e.g.

1.For h(n)={2,1,1,2} find if it obeys Linear Phase response….???

2.For h(n)={-3,2,1,-2,3} find if it obeys Linear Phase response….???

Magnitude Characteristics & Order of FIR Filters

filter is to be designed.

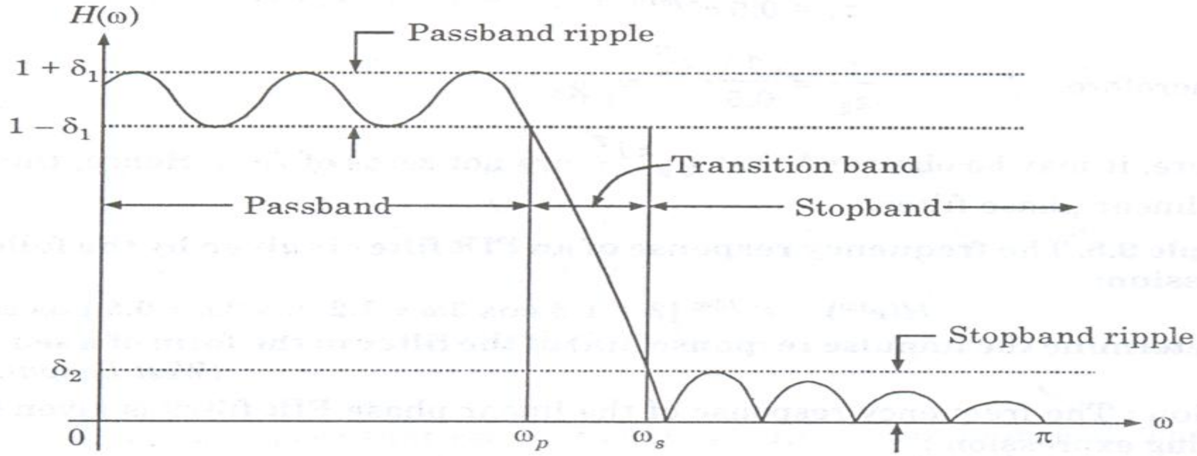


Fig. 9.3. *Magnitude specifications used for FIR filter design.*

The magnitude response given in figure 9.3 can be expressed mathematically as under:

$$\left.\begin{array}{ll} 1-\delta_1 \leq |H(\omega)| \leq 1+\delta_1 & \text{for} \quad 0 \leq \omega \leq \omega_p \\ 0 \leq |H(w)| \leq \delta_2 & \text{for} \quad \omega_s \leq \omega \leq \pi \end{array}\right\} \quad \ldots(9.23)$$

The approximate empirical formula for order N is given by

$$N = \frac{-10\log_{10}(\delta_1 . \delta_2) - 15}{14\Delta f} \quad \ldots(9.24)$$

Here $\Delta f = \dfrac{\omega_s - \omega_p}{2\pi}$ is the transition band.

or $\Delta f = f_s - f_p$, where $\omega_s = 2\pi f_s$ and $\omega_p = 2\pi f_p$

and length of the filter *i.e.*, $M = N$.

FIR Filter Design: Rectangular Window

- Let w(n)=Rectangular Window Function,
- Where
- w(n)=1 $0 \le n \le M-1$

hd(n)=Infinite Input Sequence(Arbitrary),&

h(n)=Finite Truncated Impulse Response.

Then

$h(n)=hd(n) \times w(n)$

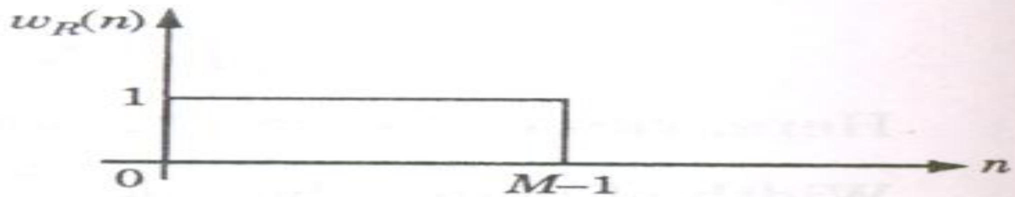


Fig. 9.4 *Rectangular window.*

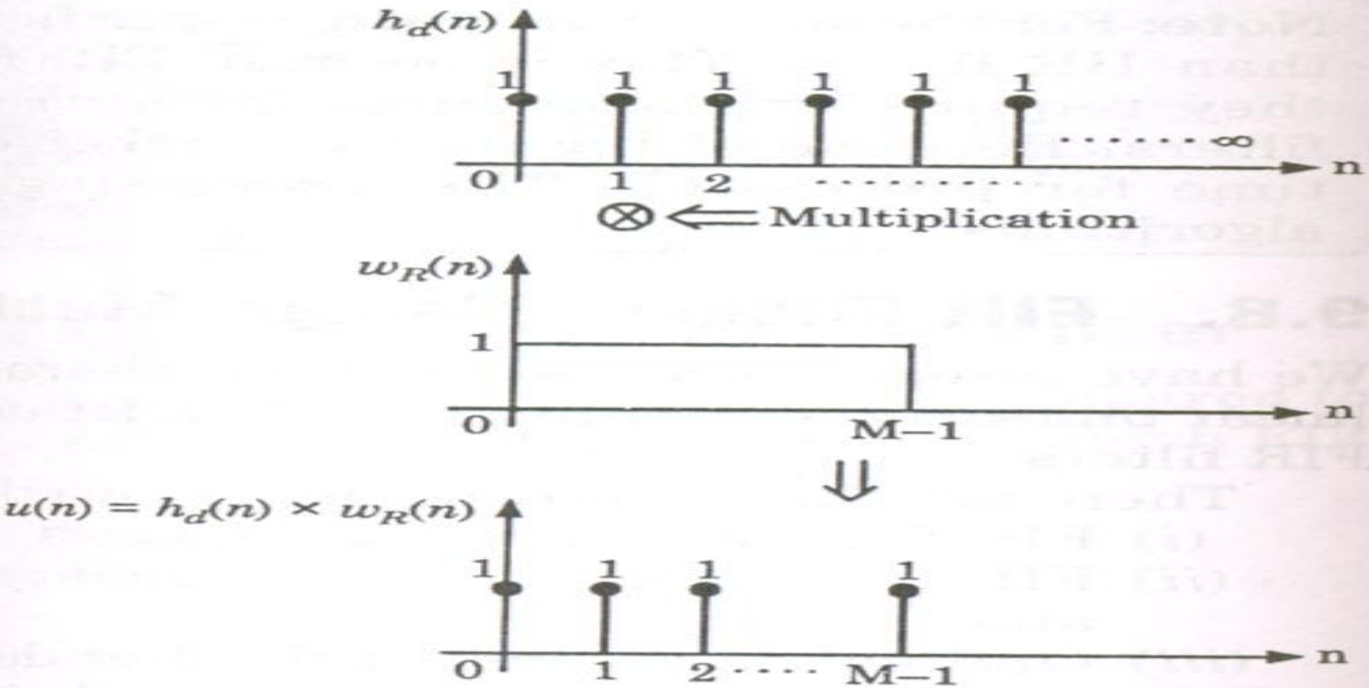


Fig. 9.5. *Truncation process*

for $n = 0, 1, \ldots, M-1$

Gibbs Phenomenon:Ringing Effect/Oscillatory Behaviour due to Sidelobes(generated owing to the sharp cut-off/abrupt discontinuity) in the Frequency Response of the window Function

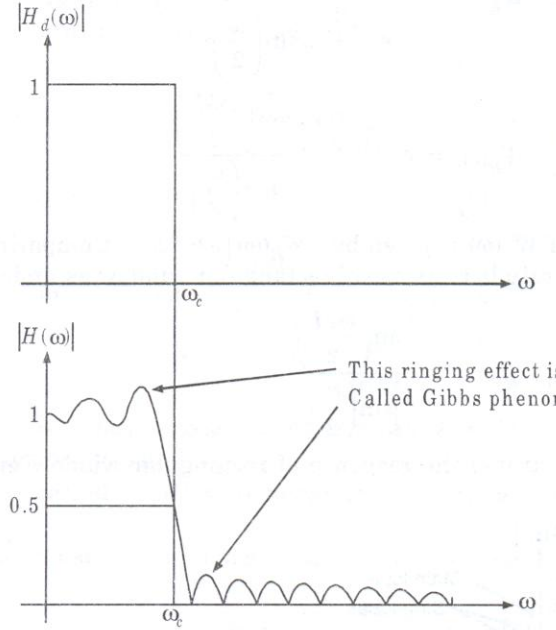
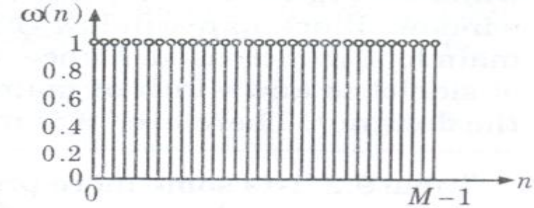
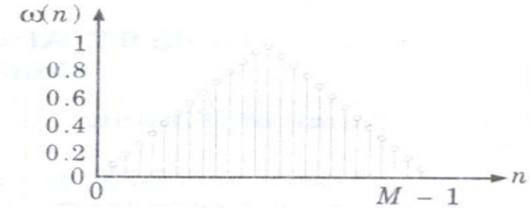
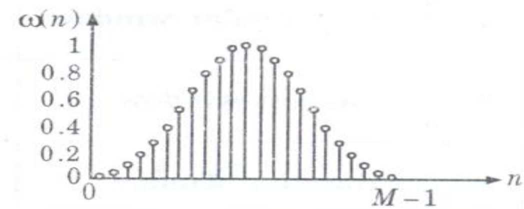
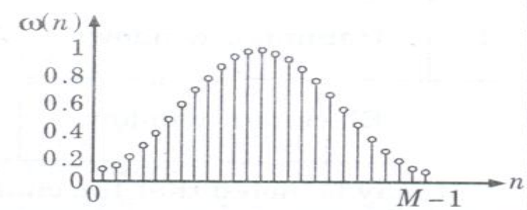


Fig. 9.6. *(a) The desired frequency response $H_d(\omega)$ (b) The frequency response of FIR filter obtained by windowing. It has smoothing and ringing effect because of windowing.*

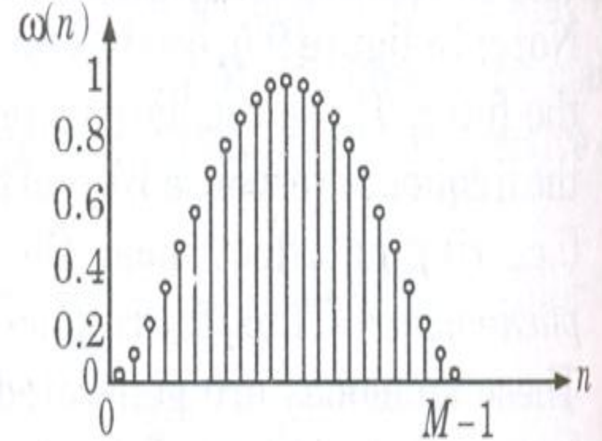
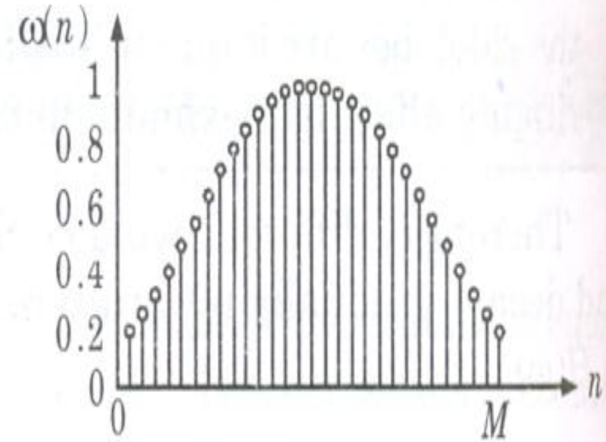
Various other window functions

Table 9.1 Various window functions and their corresponding shapes

Sr. No.	Name of Window	Time-domain sequence, $\omega(n)$, $0 \le n \le M-1$	Shape of window function
1.	Rectangular	1	
2.	Bartlett (triangular)	$1 - \dfrac{2\left\lvert n - \dfrac{M-1}{2} \right\rvert}{M-1}$	
3.	Blackman	$0.42\text{–}05 \cos \dfrac{2\pi n}{M-1} + 0.08 \cos \dfrac{4\pi n}{M-1}$	
4.	Hanning	$0.54 - 0.46 \cos \dfrac{2\pi n}{M-1}$	

(Contd..)

Various other window functions

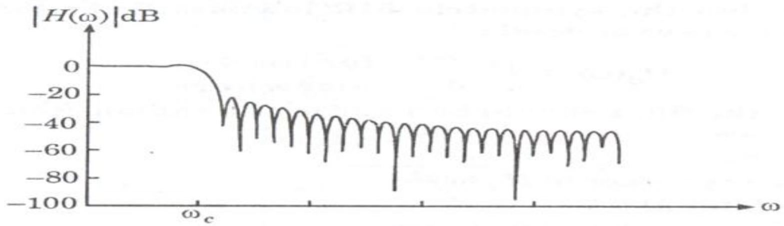
5.	Hanning	$\frac{1}{2}\left(1-\cos\frac{2\pi n}{M-1}\right)$	
6.	Kaiser	$\frac{I_0\left\{\beta\left[1-\left(\frac{n-\alpha}{\alpha}\right)^2\right]^{\frac{1}{2}}\right\}}{I_0(\beta)}$	

Comparative Study for Trade Off between Attenuation of Side lobes & Transition Width of main Lobe.

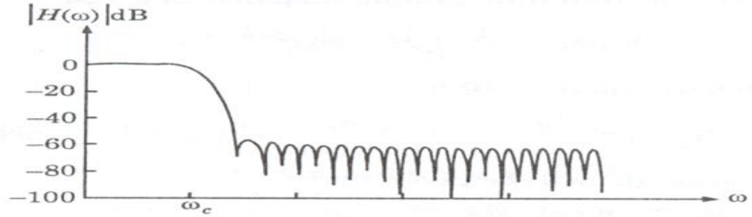
Table 5.2 Attenuation and Transition Widths of Commonly Used Windows

S. No.	Name of Window	Transition width of the main lobe	Minimum stopband attenuation	Relative amplitude of sidelobe
1.	Rectangular window	$\frac{4\pi}{M+1}$	– 21 dB	– 13 dB
2.	Bartlett window	$\frac{8\pi}{M}$	– 25 dB	– 25 dB
3.	Hanning window	$\frac{8\pi}{M}$	– 44 dB	– 31 dB
4.	Hamming window	$\frac{8\pi}{M}$	– 53 dB	– 41 dB
5.	Blackman window	$\frac{12\pi}{M}$	– 74 dB	– 57 dB

It may be noted that the characteristics of Kaiser window have not been mentioned

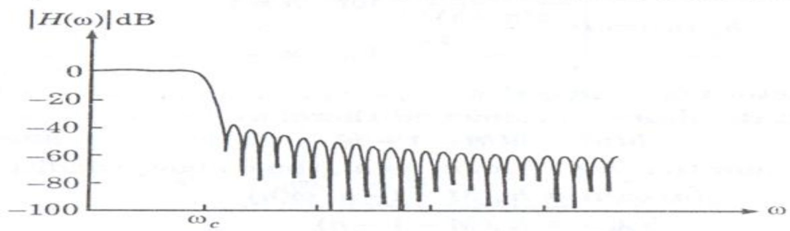


(a) Magnitude response of FIR lowpass filter designed using rectangular window. It has significant sidelobes.



(b) Magnitude response of FIR lowpass filter designed using Hamming window. It has reduced sidelobes but slightly increased transition band

(c) Magnitude response of FIR lowpass filter designed using Blackman window. It has very small sidelobes but increased width of main lobe.



(d) Magnitude response of FIR lowpass filter designed using Kaiser window. It has reduced sidelobes & transition band is also narrow

Fig. 9.7. *Magnitude response of lowpass FIR filter designed by various windows.*