Digital Signal Processing- Lecture 4

Topics to be covered:

- **Discrete time system**
- **Discrete time LTI System**

DISCRETE TIME SIGNALS AND SYSTEMS

2 Discrete-Time Signals and Systems

2.2 Discrete-Time Systems

2.2.1 Input-Output Description of Systems

***Ex.** Determine the response of the following systems to*

$$x(n) = \{ \ldots 0, 0, -3, -2, -1, \underset{\uparrow}{0}, 1, 2, 3, 0, 0, \ldots \}$$

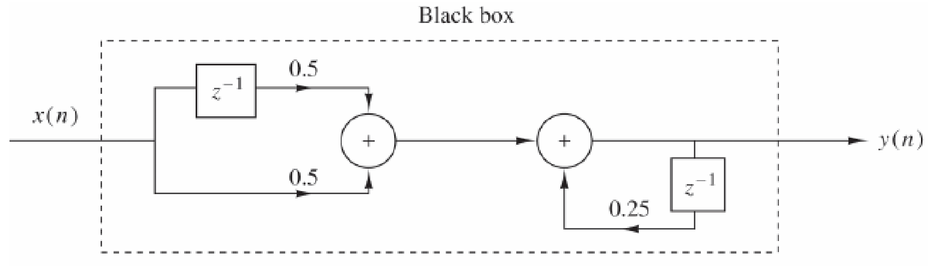
1. $y(n) = x(n)$
2. $y(n) = x(n-1)$
3. $y(n) = x(n+1)$
4. $y(n) = [x(n+1) + x(n) + x(n-1)]/3$
5. $y(n) = \text{median}\,[x(n+1), x(n), x(n-1)]$
6. $y(n) = x(n) + x(n-1) + x(n-2) + \ldots$

   $= y(n-1) + x(n)$

2.2 Discrete-Time Systems

2.2.2 Block Diagram Representation of Discrete-Time Systems

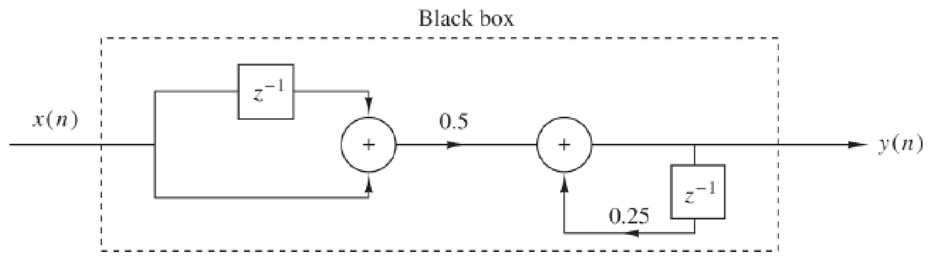
Ex.



(a)

$$y(n) = 0.25\, y(n-1) + 0.5\, x(n) + x(n-1)$$

Initial condition?



(b)

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

1. ***Static (memoryless) versus dynamic***: $y(n) = T[x(n), n]$
2. ***Time-invariant versus time-variant***: $x(n-k) \rightarrow y(n-k)$
3. ***Linear versus nonlinear***: $T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$
4. ***Causal versus noncausal***: $y(n) = F[x(n), x(n-1), x(n-2), \ldots]$
5. ***Stable versus unstable***: $|x(n)| \leqslant M_x < \infty \rightarrow |y(n)| \leqslant M_y < \infty$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

***Ex. Time-invariant versus time-variant*:** $x(n-k) \rightarrow y(n,k) \stackrel{?}{=} y(n-k)$

1. $\boldsymbol{y(n) = x(n) + x(n-1)}$	$y(n,k) = x(n-k) + x(n-k-1) = y(n-k)$
2. $y(n) = nx(n)$	$y(n,k) = n\,x(n-k) \neq y(n-k)$
3. $y(n) = x(-n)$	$y(n,k) = x(-n-k) \neq y(n-k)$
4. $y(n) = x(n)\cos \omega_0 n$	$y(n,k) \neq y(n-k)$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Linear versus nonlinear: $T[a_1 x_1(n) + a_2 x_2(n)] \stackrel{?}{=} a_1 T[x_1(n)] + a_2 T[x_2(n)]$

1. $y(n) = nx(n)$
2. $y(n) = x(n^2)$
3. $y(n) = x^2(n)$
4. $y(n) = Ax(n) + B$
5. $y(n) = e^{x(n)}$

Let $x(n) = a_1 x_1(n) + a_2 x_2(n)$ and

Check $T[x(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Causal versus noncausal: $y(n) \stackrel{?}{=} F[x(n), x(n-1), x(n-2), \ldots]$

1. $y(n) = x(n) + x(n-1)$
2. $y(n) = \sum_{k=-\infty}^{n} x(k)$
3. $y(n) = ax(n)$
4. $y(n) = x(n) + 3x(n+4)$
5. $y(n) = x(n^2)$
6. $y(n) = 2x(n)$
7. $y(n) = x(-n)$

2.2 Discrete-Time Systems

2.2.3 Classification of Discrete-Time Systems

Ex. Stable versus unstable: $|x(n)| \leqslant M_x < \infty \xrightarrow{?} |y(n)| \leqslant M_y < \infty$

$$y(n) = y^2(n-1) + x(n)$$

$$x(n) = \{ \ldots 0, 0, 2, 0, 0, \ldots \}$$

$$\Downarrow$$

$$y(n) = \{ \ldots 0, 0, 2, 2^2, 2^4, 2^8, \ldots \}$$

2.3 Analysis of Discrete-Time LTI Systems

2.3.2 Resolution of a Discrete-Time Signal into Impulses (1/2)

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$