Digital Signal Processing- Lecture 3

Topics to be covered:

- **Energy & Power Signals**
- **Basic operations on the signal**

Energy and Power Signals

The total energy of a continuous time signal x(t) is defined as

$$E_x = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt$$

And its average power is

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt$$

In the case of a discrete time signal x[nT], the total energy of the signal is

$$E_{dx} = T \sum_{n=-\infty}^{\infty} \left| x^2[n] \right|$$

And its average power is defined by

$$P_{dx} = \lim_{N \to \infty} \left(\frac{1}{2N+1} \right) \sum_{n=-N}^{N} \left| x[nT] \right|^2$$

Energy and Power Signals

- A signal is referred to as an energy signal, if and only if the total energy of the signal satisfies the condition
  $0 < E < \infty$

- On the other hand, it is referred to as a power signal, if and only if the average power of the signal satisfies the condition
  $0 < P < \infty$

- An energy signal has zero average power, whereas a power signal has infinite energy.

- Periodic signals and random signals are usually viewed as power signals, whereas signals that are both deterministic and non-periodic are energy signals.

Example1:

Compute the signal energy and signal power for

$x[nT] = (-0.5)^n u(nT)$, T = 0.01 seconds

Solution:

$$E_{dx} = \lim_{N\to\infty} T \sum_{n=-N}^{N} |x(nT)|^2 = 0.01 \sum_{n=0}^{\infty} |(-0.5)^n|^2$$

$$= 0.01 \sum_{n=0}^{\infty} (-0.5)^{2n} = 0.01 \sum_{n=0}^{\infty} 0.25^n$$

$$= 0.01\left[1 + 0.25 + (0.25)^2 + (0.25)^3 + \ldots\ldots\right]$$

$$= \frac{0.01}{1 - 0.25} = 1/75$$

Since E_{dx} is finite, the signal power is zero.

Example2:

Repeat Example1 for $y[nT] = 2e^{j3n}u[nT]$, $T = 0.2$ second.

Solution:

$$P_{dx} = \lim_{N\to\infty}\left(\frac{1}{2N+1}\right)\sum_{n=-N}^{N}\left|y(nT)\right|^2 = \lim_{N\to\infty}\left(\frac{1}{2N+1}\right)\sum_{n=0}^{N}\left|2e^{j3n}\right|^2$$

$$= \lim_{N\to\infty}\left(\frac{1}{2N+1}\right)\sum_{n=0}^{N}2^2 = \lim_{N\to\infty}\frac{4}{2N+1}\sum_{n=0}^{N}1 = \lim_{N\to\infty}\frac{4(N+1)}{2N+1}$$

$$= \lim_{N\to\infty}4\left(\frac{N}{2N+1}+\frac{1}{2N+1}\right) = 4\times\frac{1}{2} = 2$$

Basic Operations on Signals

(a) Operations performed on dependent variables

1. Amplitude Scaling:

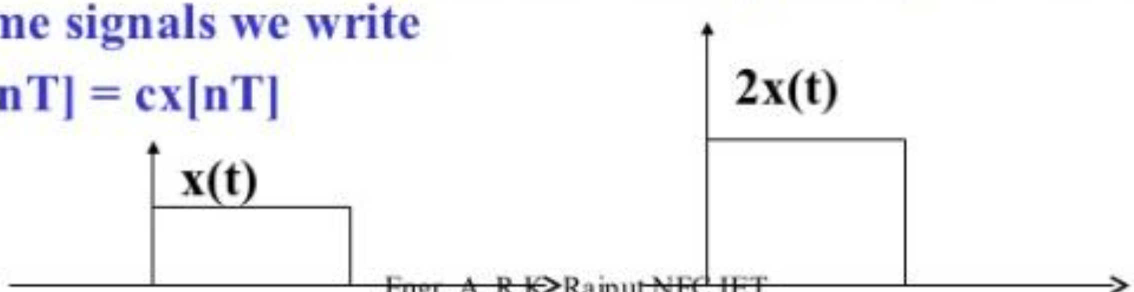
let x(t) denote a continuous time signal. The signal y(t) resulting from amplitude scaling applied to x(t) is defined by

$$y(t) = cx(t)$$

where c is the scale factor.

In a similar manner to the above equation, for discrete time signals we write

$$y[nT] = cx[nT]$$



(b) Operations performed on independent variable

- **Time Scaling:**

  Let $y(t)$ is a compressed version of $x(t)$. The signal $y(t)$ obtained by scaling the independent variable, time t, by a factor k is defined by

  $$y(t) = x(kt)$$

  - if $k > 1$, the signal $y(t)$ is a compressed version of $x(t)$.
  - If, on the other hand, $0 < k < 1$, the signal $y(t)$ is an expanded (stretched) version of $x(t)$.

Example of time scaling