Digital Signal Processing- Lecture 1

Topics to be covered:

- **DSP**
- **DISCRETE-TIME SIGNALS:** Signal classifications
- frequency domain representation
- Time domain representation
- representation of sequences by Fourier transform

What is DSP?

- Digital signal processing (DSP) is the study of signals in a digital representation and the processing methods of these signals.
- DSP and analogue signal processing are subfields of signal processing.
  - Signal processing is the processing, amplification and interpretation of signals and deals with the analysis and manipulation of signals.
- DSP has three major subfields: audio signal, digital image and speech processing.

Why DSP?

- Analogue processing
  - difficult to implement, necessary devices to perform the required operations even may not exist.
  - Inaccurate,
  - Noisy,
  - Small dynamic range,
  - Poor repeatability,
  - Inflexible to changes of algorithm
  - Slow,
  - High cost of storage of analogue waveforms

Signal Sampling

- In order to use an analogue signal on a computer it must be digitized with an analogue to digital converter (ADC) – Signal Sampling.
- Signal Sampling is usually carried out in two stages, discretisation and quantization.
  - Discretisation concerns the process of transferring continuous signals and equations into discrete forms e.g., Zero-Order-Hold (ZOH)
  - Quantisation is the process of approximating a continuous range of values (or a very large set of possible discrete values) by a relatively-small set of discrete symbols or integer values.

Frequency Domain

- Signals are converted from time or space domain to the frequency domain usually through the Fourier transform.
  - The Fourier transform converts the signal information to a magnitude and phase component of each frequency.
  - Often the Fourier transform is converted to the power spectrum, which is the magnitude of each frequency component squared.
- By studying signals in the frequency domain, engineers can get information of which frequencies are present in the input signal and which are missing.

Laplace, Fourier and z-transform

- An ordinary Laplace transform can be written as a special case of a two-sided transform, and since the two-sided transform can be written as the sum of two one-sided transforms.
- The theory of the Laplace-, Fourier-, and z-transforms are at bottom the same subject. However, a different point of view and different characteristic problems are associated with each of these major integral transforms.

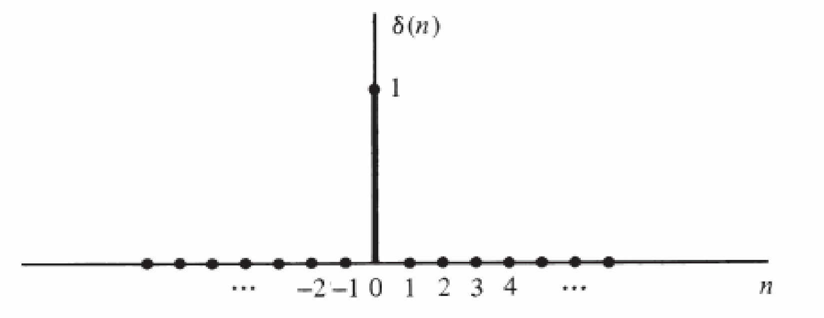
DFT & FFT

- It is impossible to apply the continuous Fourier transform to discrete and probably non-periodic signal, however, the Discrete Fourier transform (DFT) is available for the use with discrete data.
- A fast Fourier transform (FFT) is an efficient algorithm to compute the DFT and its inverse. FFTs are of great importance to a wide variety of applications, e.g., digital signal processing.

2.1 Discrete-Time Signals

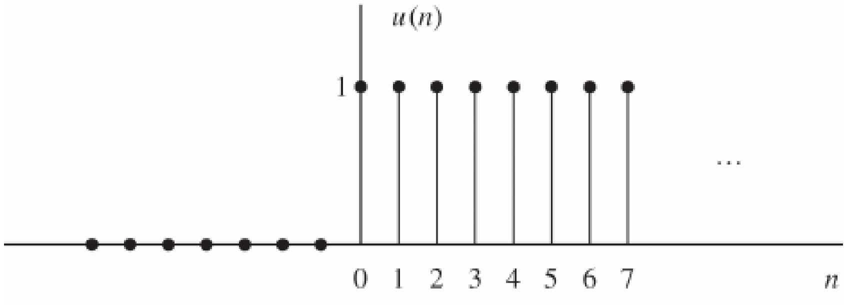
2.1.1 Some Elementary Discrete-Time Signals (1/2)

Unit sample sequence



$$\delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$$

Unit step signal

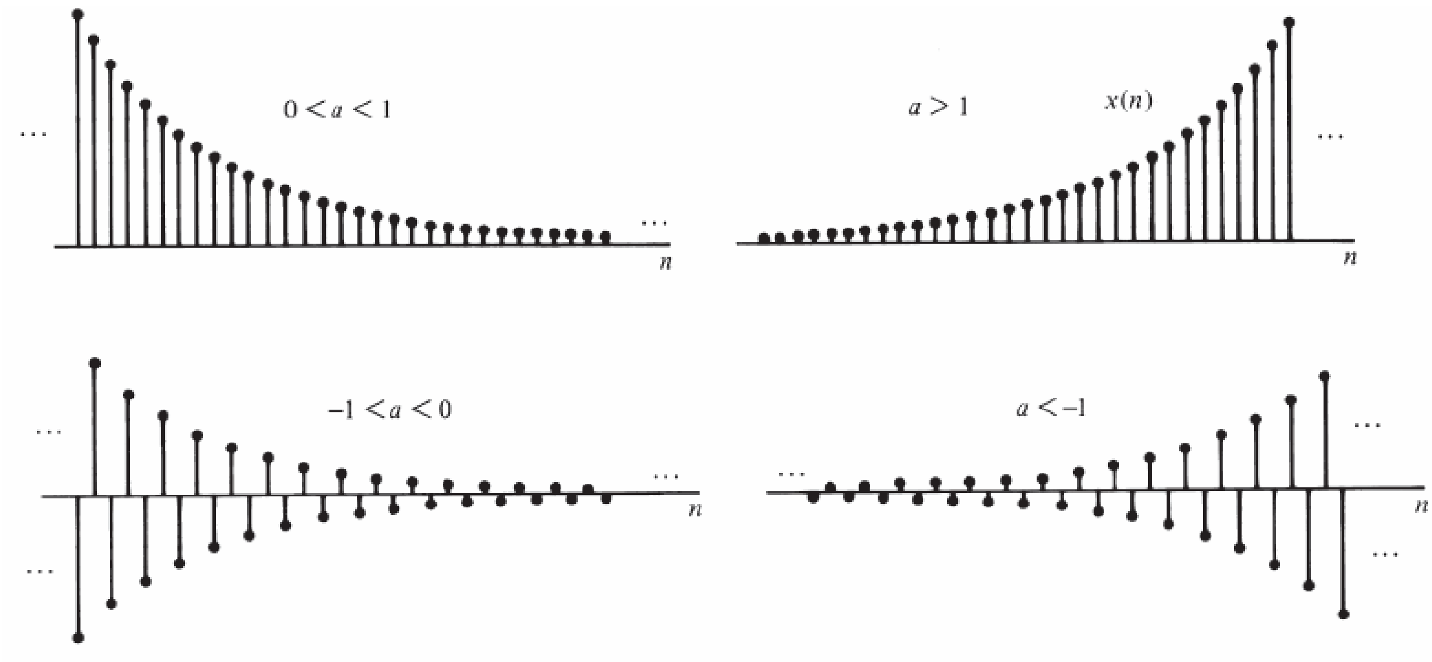


$$u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

2.1 Discrete-Time Signals

2.1.1 Some Elementary Discrete-Time Signals (2/2)

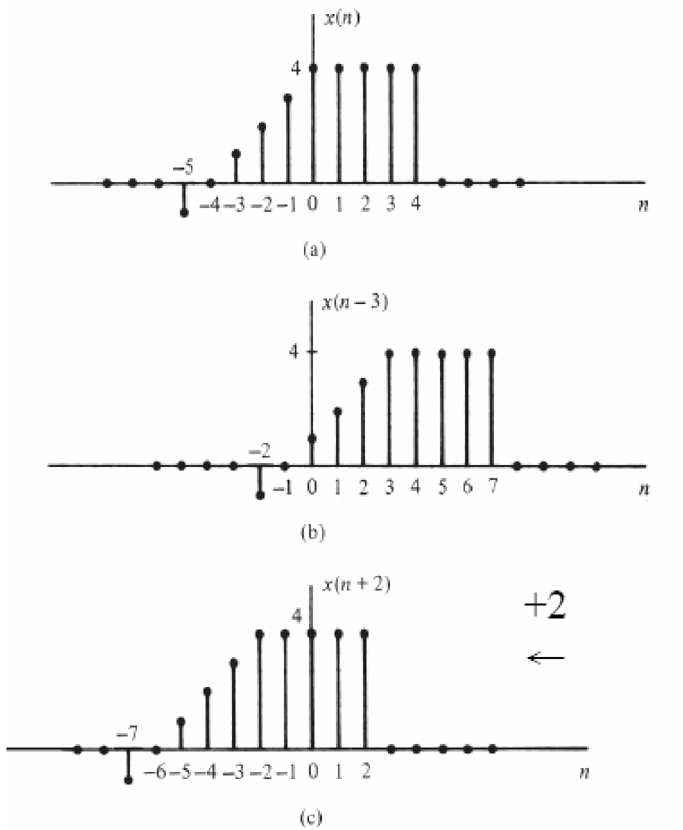
Exponential Signal — $x(n) = a^n$, $-\infty < n < \infty$



2.1 Discrete-Time Signals

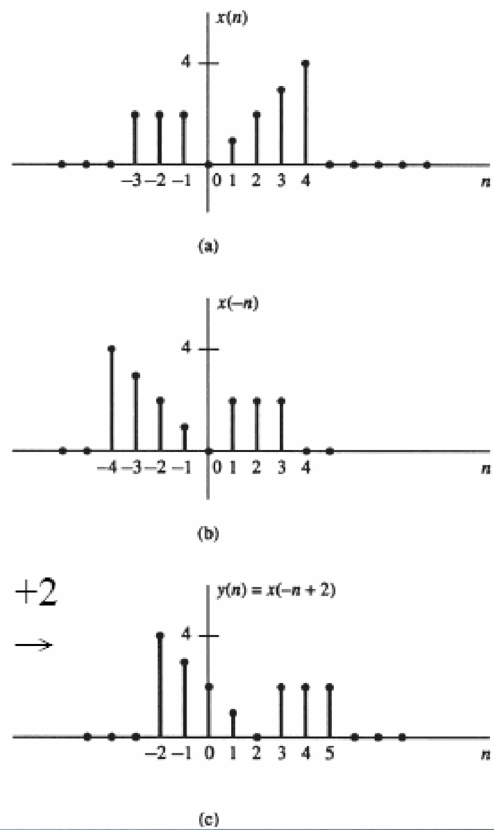
2.1.3 Simple Manipulations of Discrete-Time Signals (1/2)

Transformation of the independent variable (time)



+2 ←

+2 →



2.1 Discrete-Time Signals

2.1.3 Simple Manipulations of Discrete-Time Signals (2/2)

Addition, multiplication, and scaling of sequences

$$y(n) = Ax(n)$$

$$y(n) = x_1(n) + x_2(n)$$

$$y(n) = x_1(n)\ x_2(n)$$

Signal Classification

- Periodic and non-periodic signals
- A signal is periodic if

$$x(t) = x(t + T)$$

where T is the period.

Energy Signal and Power Signal

- Consider applying a time varying voltage $v(t)$ to a resistor,

  the instantaneous average power drops across the resistor is,

$$p(t) = v^2(t)/R$$

or

$$p(t) = i^2(t) \times R$$

Power Signal

- For the purposes of signal classification the average power of a signal is measured over all time, i.e., $T \to \infty$

so

$$average\ power = \lim_{T \to \infty} \left(\frac{1}{T} \int_{t=-T/2}^{T/2} y^2(t)\, dt \right)$$

Energy Signal

- To get energy in Joules (J), it is necessary to integrate over some specified time interval.
- So the energy in a signal between time 0 and time T:

$$energy(T) = \int_{t=0}^{T} y^2(t)\,dt$$

- and thus the total energy is

$$total\ energy = \int_{t=-\infty}^{\infty} y^2(t)\,dt$$

Relation of Power and Energy Signal

- the average power can be expressed as:

$$average\ \ power = \lim_{T \to \infty} \left(\frac{1}{T} \times total\ \ energy \right)$$

- Clearly, because of the $1/T$ factor, if the total energy is finite then the average power is zero.
- Conversely, if the average power is not zero then the total energy is infinite

Energy Signal

- if the resistor is 1 Ω , the power is equal to the square of the voltage or current signal. In general, the instantaneous power of a signal is taken to be the square of the signal:

$$p(t) = y^2(t)$$

- The average power of a signal is its mean value. For instance, over a time period -*T*/2 to *T*/2

$$average\ power = \frac{1}{T} \int_{t=-T/2}^{T/2} y^2(t)\,dt$$

Example – Finite Energy Signal

- Consider a transient signal that starts at t = 0 and decays to zero with an exponential form:

$$y(t) = e^{-t}, \quad t \geq 0$$

$$total\ energy = \int_{t=-\infty}^{\infty} y^2(t)\,dt = \int_{t=0}^{\infty} e^{-2t}\,dt = \frac{1}{2}$$

which is a finite energy signal

Example - Non-zero Average Power Signal

- The average power of a periodic signals can be written as, (WHY?)

$$\bar{P} = \frac{1}{T} \int_{t=0}^{T} y^2(t)\,dt$$

- Let $y(t) = \sin(\omega t)$, where T is the period equals $2\pi/\omega$

$$\bar{P} = \frac{1}{T} \int_{t=0}^{T} \sin^2\left(\frac{2\pi}{T} t\right) dt = \frac{1}{2}$$

Example