Lecture 7

Time Response Analysis

Topics Covered

- Test input signals
- Response of a first-order system
- Performance of a second-order system
- Effects of a third pole and a zero on system response
- Root location and the transient response

continue

- Steady-state error analysis
- Performance indices
- The simplification of linear systems
- Examples and simulation
- Summary

Introduction

- Transient response
- Steady-state response
- Design specifications
- How to get compromise?

3.1 Test input signals

The standard test input signals commonly used are:

- Step input
- Ramp input
- Parabolic input
- Sinusoidal input
- Unit impulse input

continue

Representation of test signals

Input	*time domain*	*frequency domain*
• Step:	$1(t), \quad t \geq 0$	$\frac{1}{s}$
• Ramp:	$t, \qquad t \geq 0$	$\frac{1}{s^2}$
• Parabolic:	$\frac{1}{2}t^2 \quad t \geq 0$	$\frac{1}{s^3}$
• sinusoidal:	$A\sin\omega t$	$\frac{A\omega}{s^2+\omega^2}$

Unit impulse response

Unit impulse: $$\delta(t) = \begin{cases} \dfrac{1}{\varepsilon}, & -\dfrac{\varepsilon}{2} \le t \le \dfrac{\varepsilon}{2} \\ 0, & otherwise \end{cases}$$

System impulse response: $$g(t) = L^{-1}[G(s)]$$

System response is the convolution integral of *g(t)* and *r(t)*:

$$y(t) = \int_{-\infty}^{t} g(t-\tau)r(\tau)d\tau = L^{-1}[G(s)R(s)]$$

continue

Standard test signal

The standard test signals are of the general form:

$$r(t) = t^n$$

And its Laplace transform is:

$$R(s) = \frac{n!}{s^{n+1}}$$

Performance indices

(viewpoint from engineering)

Transient Performance:

- Time delay t_d
- Rise time t_r
- Peak time t_p
- Settling time t_s
- Percent overshoot $\sigma\%$

Steady-state Performance: Steady-state error

3.2 Response of a first-order system

The model of first-order system

$$T\dot{c}(t) + c(t) = r(t)$$

or

$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{Ts+1}$$

For example, temperature or speed control system and water level regulating system.

Response of first-order system

- Unit step response (*No steady-state error*)
- Unit impulse response (⟷ *transfer function*)
- Unit ramp response (*Constant steady-state error*)
- Unit parabolic response (*Infinite steady-state error*)

Important conclusion
(for n-order LTI system)

From above analysis, we can see that impulse response of a system is the 1st-order derivative of step response or 2nd-order derivative of ramp response of the system.

Conclusion:

System response for the derivative of a certain input signal is equivalent to the derivative of the response for this input signal.

3.3 Response and performance of a second-order system

- Model of 2nd-order system

$$T(s)=\frac{Y(s)}{R(s)}=\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$$

- Roots of characteristic equation (Poles)

$$s_{1,2}=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

The response depends on ζ and ω_n

Unit step response of 2nd-order system

- If $\zeta < 0$, 2 positive real-part roots, *unstable*
- If $0 < \zeta < 1$, 2 negative real-part roots, *underdamped*
- If $\zeta = 1$, 2 equal negative real roots, *critically damped*
- If $\zeta > 1$, 2 distinct negative real roots, *overdamped*
- If $\zeta = 0$, 2 complex conjugate roots, *undamped*

Case 1: underdamped

- Oscillatory response
- No steady-state error

Case 2: critically damped

- Mono-incremental response
- No Oscillation
- No steady-state error

Case 3: overdamped

- Mono-incremental response
- slower than critically damped
- No Oscillation
- No steady-state error

Performance evaluation

(underdamped condition)

- Performance indices evaluation

  1 Time delay

  2 Rise time

  3 Peak time

  4 Percent overshoot

  5 Settling time

- An example of performance evaluation

3.4 Effects of a third pole and a zero on 2nd-order system response

- Effect of a third pole
- Effect of a third zero
- Dominant poles

3.5 Root location and transient response

- Characteristic roots (modes)
- Effects of Zeros on response

STEADY STATE ERROR

$$ess = \lim_{s \to 0} sE(s)$$

$$ess = \lim_{s \to 0} sR(s) \frac{1}{1 + G(S)H(S)}$$