Control Systems

Lecture: 6

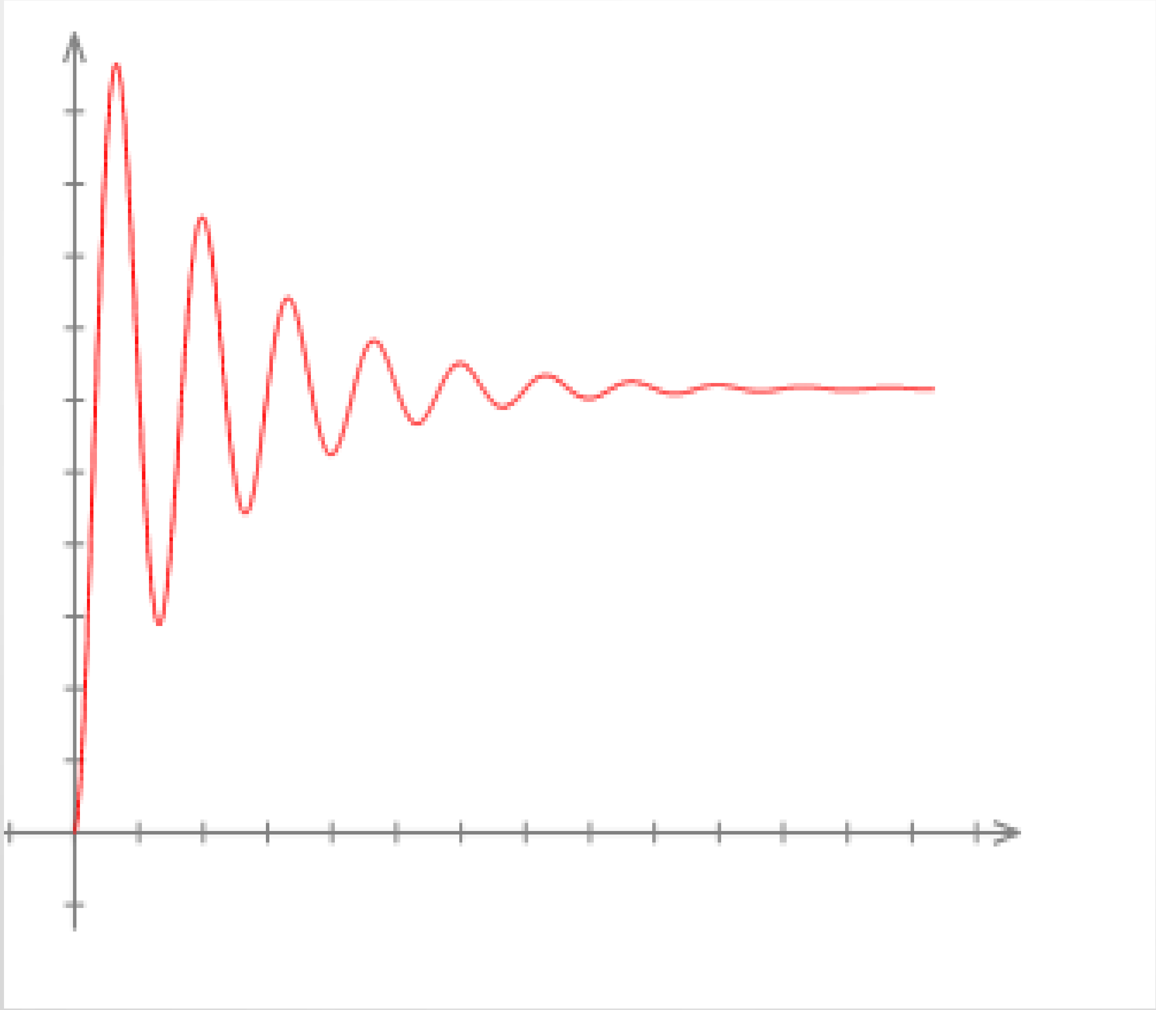
Topics Covered

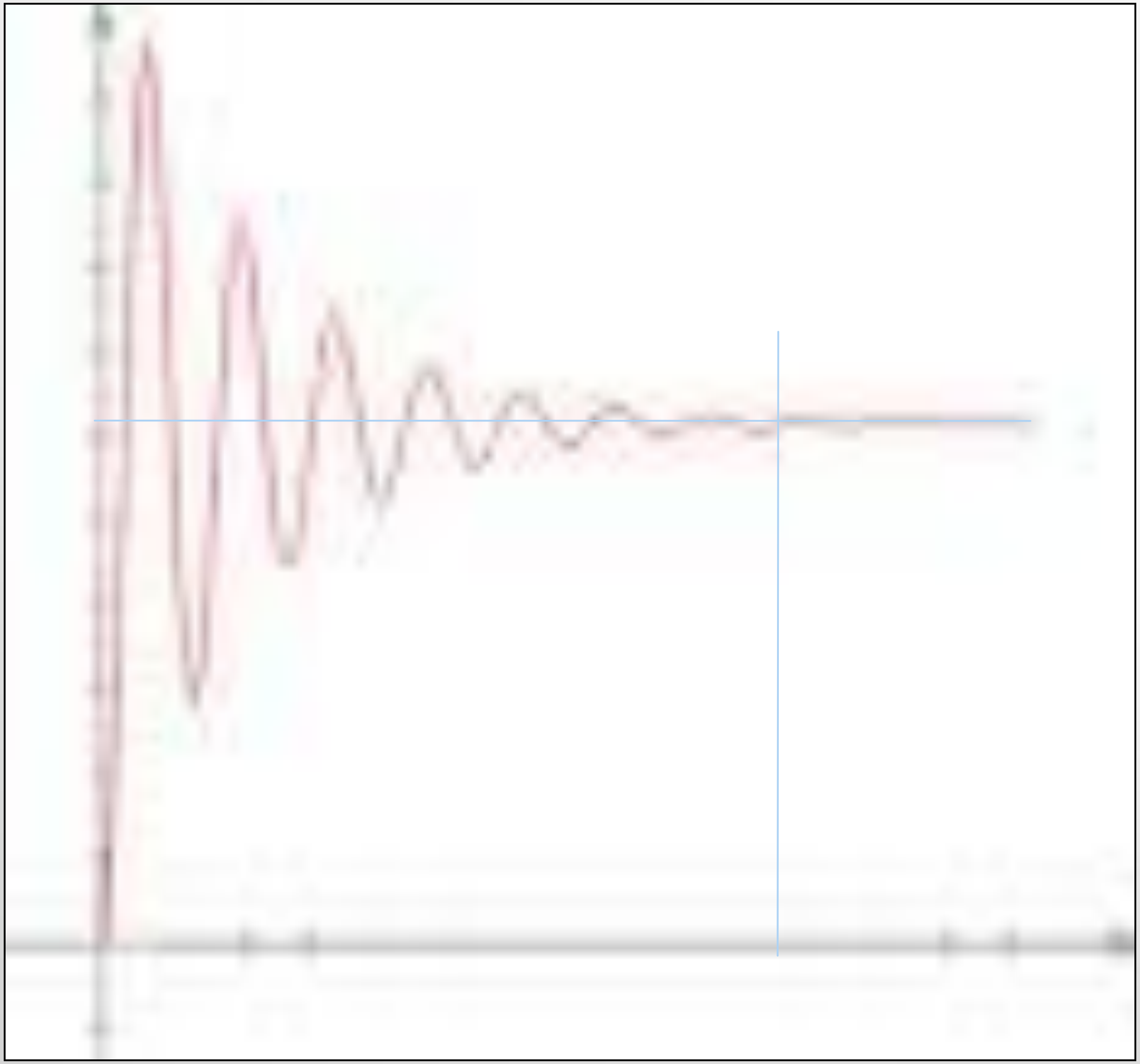
Time domain analysis

Time Domain Analysis

- Time Response:-The response given by the system which is function of the time, to the applied excitation is called a time response of a control system
- The final state achieved by the output is called as steady state
- Output variation during the time it takes to achieve the steady state is called as transient response of the system

- Transient response: the output variation during the time ,it takes to achieve its final value is called as transient Response
- Time required to achieve the final value is called as transient period





TIME RESPONSE OF SECOND ORDER SYSTEM

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$C(s) = R(s)\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$stepi/p--r(t) = d$$

$$R(s) = d/s$$

$$C(s) = \frac{d}{s} * \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = [(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)]$$

$$= d\left[\frac{1}{s} - \frac{s + 2\zeta\omega_n}{[(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)]}\right]$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$d\left[\frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}\right]$$

$$d\left[\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}*\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right]$$

$$taking laplace inverse$$

$$\ell^{-1}C(s)=d\ell^{-1}\left[\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}*\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right]$$

$$c(t)=d\left[1-e^{-\zeta\omega_n t}.\cos\omega_d t-\frac{\zeta\omega_n}{\omega_d}.e^{-\zeta\omega_n t}.\sin\omega_d t\right]$$

$$\omega_d=\omega_n\sqrt{1-\zeta^2}$$

$$c(t)=d\left[1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}.\cos\omega_d t+\zeta.\sin\omega_d t\right]\right]$$

$$c(t)=d\left[1-e^{-\zeta\omega_n t}\left[.\cos\omega_n\sqrt{1-\zeta^2}t+\frac{\zeta}{\sqrt{1-\zeta^2}}.\sin\omega_n\sqrt{1-\zeta^2}t\right]\right]$$

$$c(t) = d\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}.\cos\omega_d t + \zeta.\sin\omega_d t\right]\right]$$

$$\sin\phi = \sqrt{1-\zeta^2} \;\therefore\; \cos\phi = \zeta$$

$$c(t) = d\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sin\underset{\zeta}{\phi}.\cos\omega_d t + \cos\phi.\sin\omega_d t\right]\right]$$

$$c(t) = d\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)\right]$$

$$\omega_d = \omega_d\sqrt{1-\zeta^2}\, and\, \phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

RISE TIME FOR UNDERDAMPED SECOND ORDER SYSTEM

$$c(t) = d\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)\right]$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \varphi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$C(t) = d$$

then

$$1 = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left[(\omega_n\sqrt{1-\zeta^2}.)t_r + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]$$

$$\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left[(\omega_n\sqrt{1-\zeta^2}.)t_r + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right] = 0$$

Rise time

$$\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \text{ is infinite}$$

$$\sin\left[(\omega_n\sqrt{1-\zeta^2})t_r + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right] = 0$$

$$(\omega_n\sqrt{1-\zeta^2})t_r + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) = \pi$$

$$t_r = \frac{\pi-\phi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Maximum over shoot peak time

- The maximum positive deviation of the output

With respect to its desired value Mp

Mp=C(t)max -1

%Mp=C(t)max -1   *100

tp

1

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$$

$$\frac{dc(t)}{dt} = -\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \omega_n \sqrt{1-\zeta^2}\, t \cos(\omega_n \sqrt{1-\zeta^2}\, t + \phi) - \frac{-\zeta\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{(1-\zeta^2)}\, t + \phi)$$

put

$$\frac{dc(t)}{dt} = 0$$

$$\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \{-\omega_n \sqrt{1-\zeta^2} \cos((\omega_n \sqrt{1-\zeta^2})t + \phi) + \zeta\omega_n \sin(\omega_n \sqrt{(1-\zeta^2)}\, t + \phi)\} = 0$$

$$\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} isfinite$$

$$\omega_n \sqrt{1-\zeta^2} \cos[(\omega_n \sqrt{1-\zeta^2})t + \phi] = \zeta\omega_n \sin(\omega_n \sqrt{(1-\zeta^2)}\, t + \phi)$$

$$\tan[\omega_n \sqrt{1-\zeta^2})t + \phi] = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\tan[\omega_n\sqrt{1-\zeta^2})t+\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}]=\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\omega_n\sqrt{1-\zeta^2})t=n\pi------n=0,1,2$$

$$t_p=\frac{\pi}{\omega_n\sqrt{1-\zeta^2})}$$

$$c(t)_{\max}=1-\frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}t_p+\phi)$$

$$c(t)_{\max}=1-\frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}}\sin\left[\omega_n\sqrt{1-\zeta^2}\left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2})}\right)+\phi\right]$$

$$1-\frac{e^{-\zeta\omega_n\left(\frac{\pi}{\omega_n\sqrt{1-\zeta^2})}\right)}}{\sqrt{1-\zeta^2}}\sin(\pi+\phi)$$

$$1-\frac{e^{-\zeta\frac{\pi}{\sqrt{1-\zeta^2})}}}{\sqrt{1-\zeta^2}}\sin(-\phi)$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} \; and \; \sin\phi = \sqrt{1-\zeta^2}$$

$$c(t)_{\max} = 1 + e^{-\zeta\frac{\pi}{\sqrt{1-\zeta^2)}}}$$

$$M_P = C(t)_{\max} - 1$$

$$M_P = e^{-\zeta\frac{\pi}{\sqrt{1-\zeta^2)}}}$$

Maximum overshoot: Mp

$$Mp = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\%Mp = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} * 100$$

$$PEAK \quad TIME$$

$$tp = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

Delay time& rise time

- Time required to reach 50% of the final value in first attempt
- Time required to reach its peak value
- Settling time: the time needed to settle down afore said oscillations within 2% of thedesired value

Time delay

$$td = \frac{1 + 0.7\zeta}{\omega_n}$$

SETTLING TIME Ts

$$\text{Ts} = 4 * \text{time constant}$$

$$= 4 * \frac{1}{\zeta\omega_n}$$

$$Ts = \frac{4}{\zeta\omega_n}$$

STEADY STATE ERROR

$$ess = \lim_{s \to 0} sE(s)$$

- FINAL VALUE THEOREM

$$ess = \lim_{s \to 0} sR(s) \frac{1}{1 + G(S)H(S)}$$

STATIC POSITION ERROR COEFFICIENT

$STEP\ I/P$

$$ess = \lim_{s \to 0} sR(s) \frac{1}{1 + G(S)H(S)}$$

$$R(s) = \frac{1}{s}$$

$$ess = \lim_{s \to 0} s.\frac{1}{s}.\frac{1}{1 + G(S)H(S)} = \frac{1}{1 + \lim_{s \to 0} G(S)H(S)}$$

$$put Kp = \lim_{s \to 0} G(S)H(S)$$

Kp is called positional error coefficient

$$ess = .\frac{1}{1 + Kp}$$

STATIC VELOCITY ERROR COEFFICIENT

$$ess = \lim_{s \to 0} R(s) \frac{1}{1 + G(S)H(S)}$$

unit ramp i/p

$$R(s) = \frac{1}{s^2}$$

$$ess = \lim_{s \to 0} s.\frac{1}{s^2}.\frac{1}{1 + G(S)H(S)} = \lim_{s \to 0} \frac{1}{s + sG(S)H(S)}$$

$$= \lim_{s \to 0} \frac{1}{sG(S)H(S)}$$

$$putKv = \lim_{s \to 0} sG(S)H(S)$$

Kv is called velocity error coefficient

$$ess = \frac{1}{Kv}$$

STATIC ACCELERATION ERROR COEFFICIENT

$$ess = \lim_{s \to 0} sR(s) \frac{1}{1 + G(S)H(S)}$$

unit PARABOLIC i/p

$$R(s) = \frac{1}{s^3}$$

$$ess = \lim_{s \to 0} s.\frac{1}{s^3}.\frac{1}{1 + G(S)H(S)} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(S)H(S)}$$

$$= \lim_{s \to 0} \frac{1}{s^2 G(S)H(S)}$$

$$putKa = \lim_{s \to 0} s^2 G(S)H(S)$$

Ka is called acceleration error coefficient

$$ess = \frac{1}{Ka}$$

STEADY STATE ERROR

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$= \lim_{s \to 0} \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

- Type.o system: UNIT STEP

$$\therefore K_p = K$$

$$e_{ss} = \frac{1}{1+K_p}$$

$$e_{ss} = \frac{1}{1+K}$$

STEADY STATE ERROR

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$= \lim_{s\to 0} s\frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

- Type 0 system UNIT RAMP

$$\therefore K_v = 0$$

$$e_{ss} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{o} = \infty$$

STEADY STATE ERROR

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$= \lim_{s\to 0} s^2 \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

- Type o system UNIT parabola

$$\therefore K_a = 0$$

$$e_{ss} = \frac{1}{K_a}$$

$$e_{ss} = \frac{1}{o} = \infty$$

STEADY STATE ERROR

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)--}{s(1+sT_1)(1+sT_2)--}$$

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0} \frac{K(1+sT_a)(1+sT_b)--}{s(1+sT_1)(1+sT_2)--}$$

- Type 1 system UNIT STEP

$$K_p = \infty$$

$$e_{ss} = \frac{1}{1+K_p}$$

$$e_{ss} = \frac{1}{1+\infty}$$

$$e_{ss} = 0$$

STEADY STATE ERROR

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$= \lim_{s\to 0} s\,\frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

- Type 0 system UNIT RAMP

$$\therefore K_v = 0$$

$$e_{ss} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{o} = \infty$$

STEADY STATE ERROR

$$G(s)H(s) = \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$= \lim_{s\to 0} s^2 \frac{K(1+sT_a)(1+sT_b)--}{(1+sT_1)(1+sT_2)--}$$

- Type o system UNIT parabola

$$\therefore K_a = 0$$

$$e_{ss} = \frac{1}{K_a}$$

$$e_{ss} = \frac{1}{o} = \infty$$