Control Systems

Lecture: 3

Steady State Response

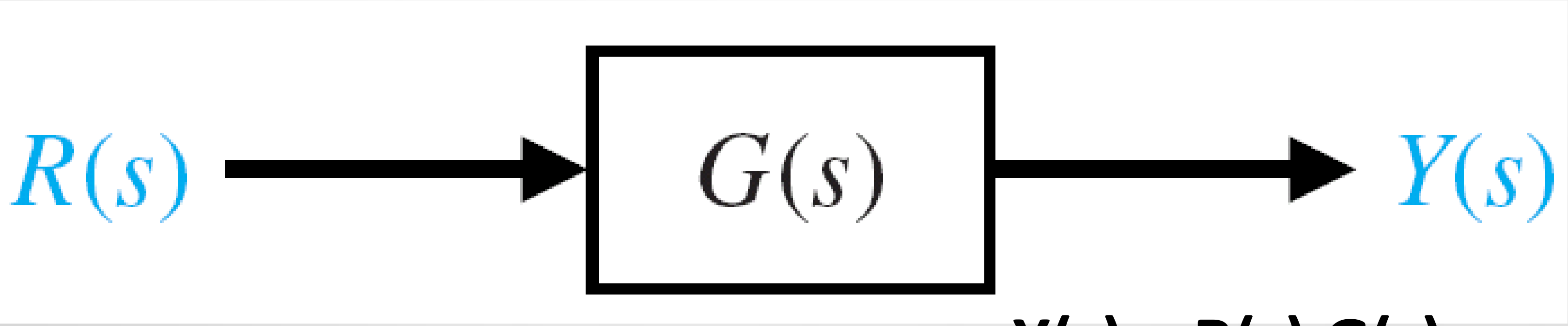
Topics Covered

Steady State response analysis

Steady-state response

- **If the steady-state response of the output does not agree with the steady-state of the input exactly, the system is said to have a steady-state error.**

- **It is a measure of system accuracy when a specific type of input is applied to a control system.**



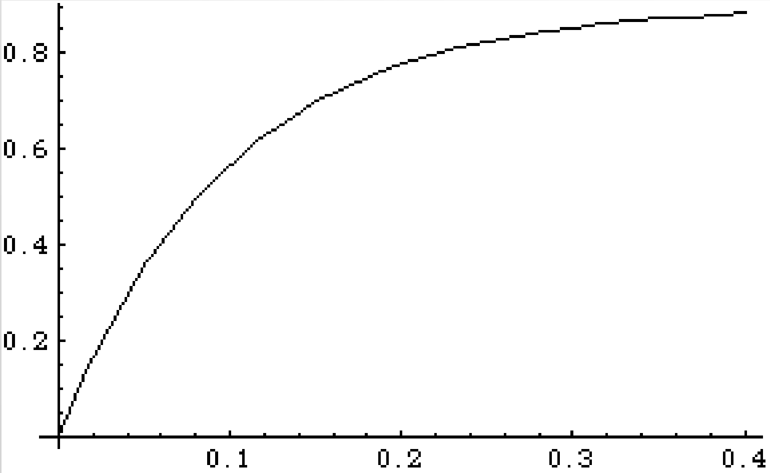
$Y(s) = R(s)\, G(s)$

Steady-state error

$T(s) = 9/(s + 10)$

$Y(s) = 9/s(s+10)$

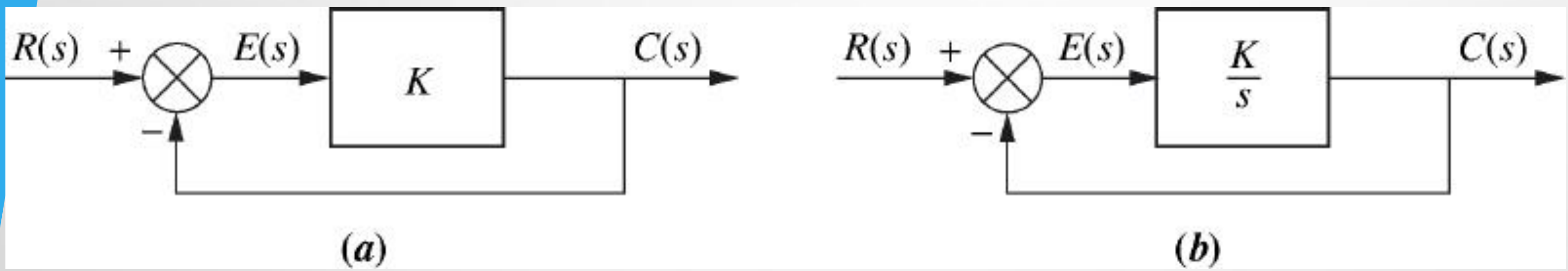
$y(t) = 0.9(1- e^{-10t})$



$y(\infty) = 0.9$

$E(s) = R(s) - Y(s)$

$e_{ss} = \lim_{s \to 0} s\,E(s) = 0.1$



$$E(s) = \frac{R(s)}{1+K}$$

$$E(s) = \frac{R(s)}{1+\dfrac{K}{s}}$$

$$e(t) = \frac{1}{1+K}$$

$$e(t) = e^{-K\,t}$$

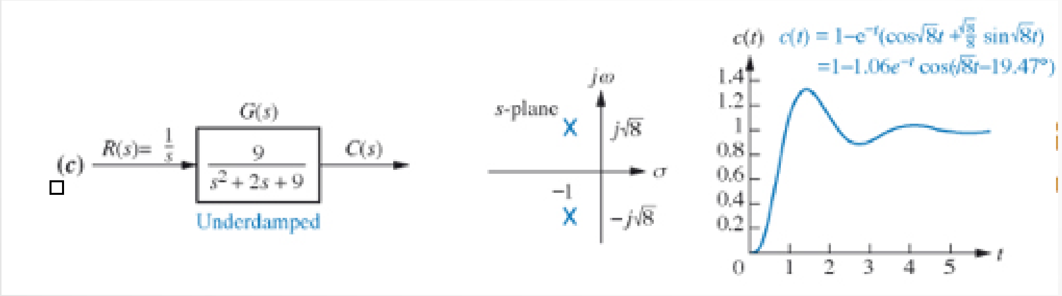
InverseLaplaceTransform[{$\frac{1}{s + a}$, $\frac{a}{s + a}$}, s, t]

{$E^{-(a\ t)}$, $\frac{a}{E^{a\ t}}$}

InverseLaplaceTransform[{$\frac{1}{s} * \frac{1}{s + a}$, $\frac{1}{s} * \frac{a}{s + a}$}, s, t]

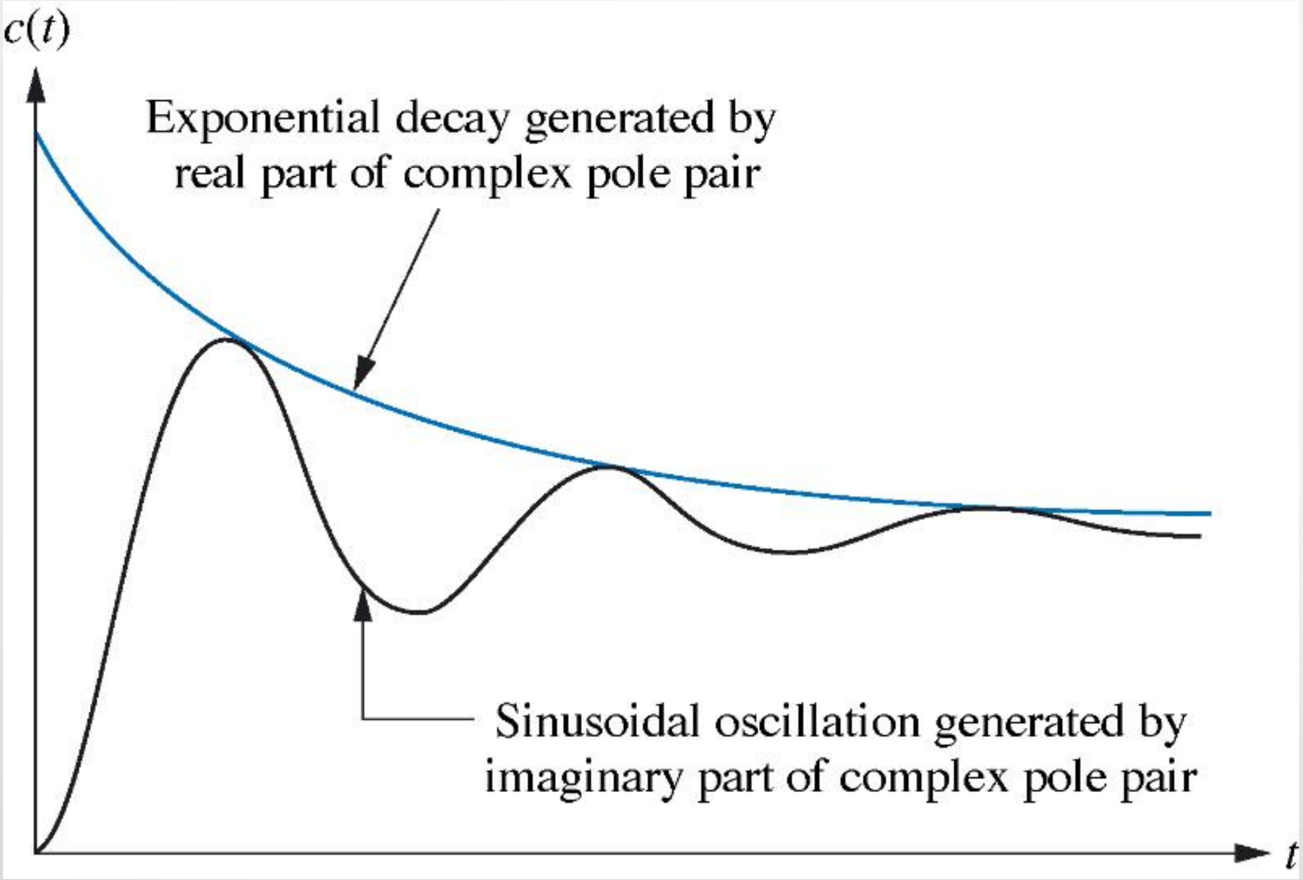
{$\frac{1 - E^{-(a\ t)}}{a}$, $1 - E^{-(a\ t)}$}

Underdamped



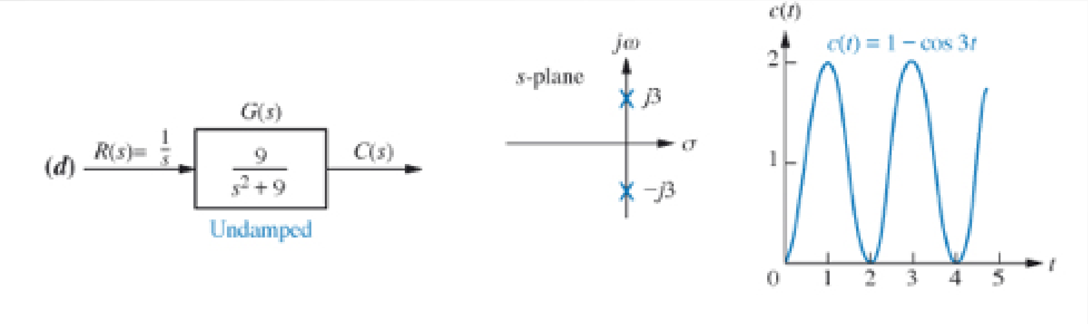
$$C(s)=\frac{9}{s(s^2+2s+9)}=\frac{9}{s(s+1-j\sqrt{8})(s+1+j\sqrt{8})}$$

$$c(t)=1-e^{-t}(\cos\sqrt{8}t+\frac{\sqrt{8}}{8}\sin\sqrt{8}t)$$



$$c(t) = 1 - e^{-t}(\cos\sqrt{8}t + \frac{\sqrt{8}}{8}\sin\sqrt{8}t)$$

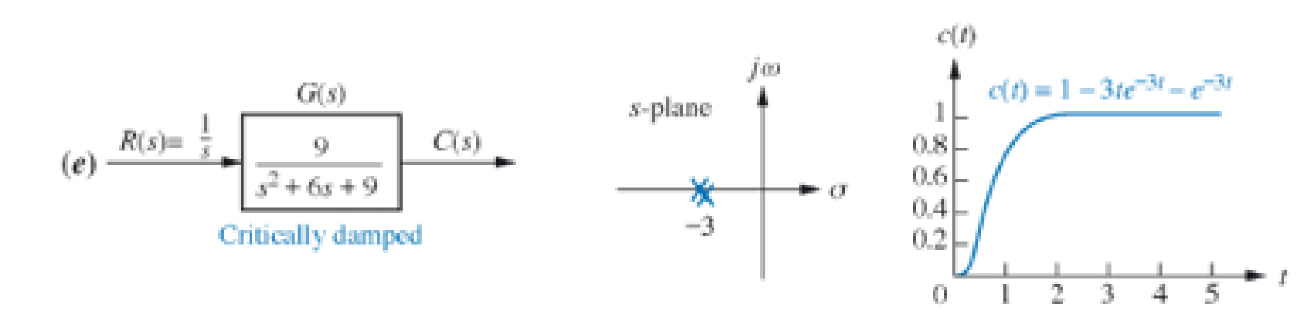
Undamped



$$C(s) = \frac{9}{s(s^2+9)}$$

$$c(t) = 1 - \cos 3t$$

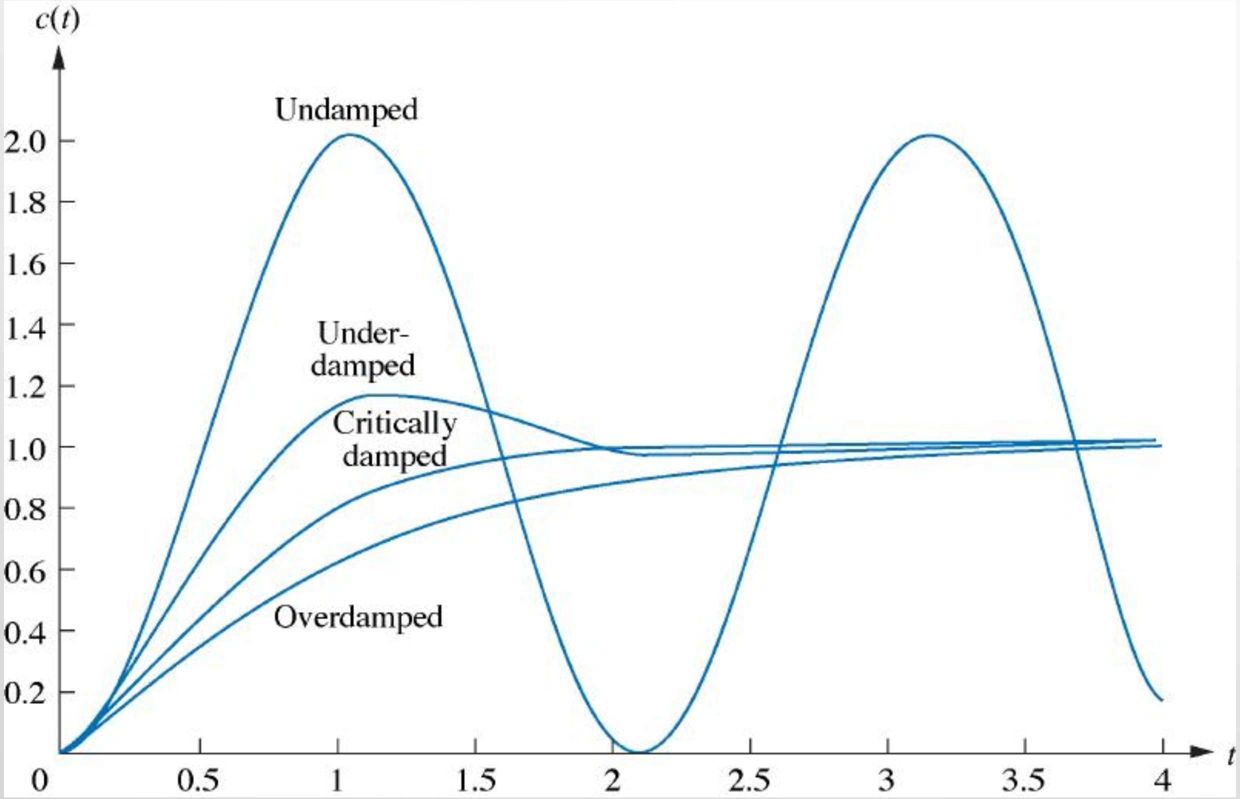
Critically damped



$$C(s) = \frac{9}{s(s^2+6s+9)} = \frac{9}{s(s+3)^2}$$

$$c(t) = 1 - 3te^{-3t} - e^{-3t}$$

Step response for second order system damping cases



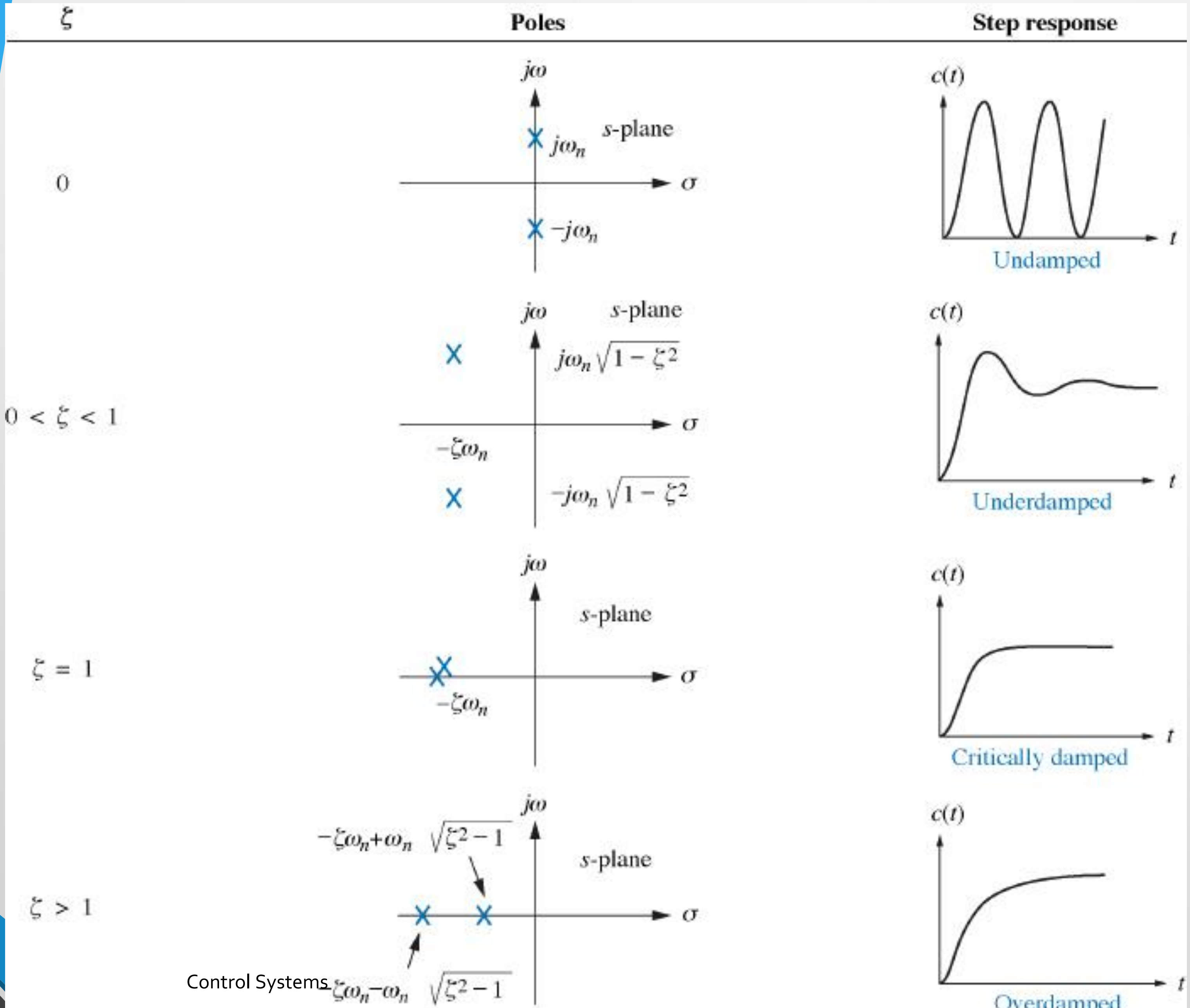

ζ
Poles
Step response
0
jω
s-plane
jω_n
−jω_n
σ
c(t)
t
Undamped
0 < ζ < 1
jω_n √(1 − ζ²)
−ζω_n
−jω_n √(1 − ζ²)
Underdamped
ζ = 1
Critically damped
ζ > 1
−ζω_n+ω_n √(ζ² − 1)
−ζω_n−ω_n √(ζ² − 1)
Overdamped

fig_04_11

Summary

- **Overdamped**

  Poles: Two real at $-\sigma_1, -\sigma_2$
- **Underdamped**

  Poles: Two complex at $-\sigma_d + j\omega_d, -\sigma_d - j\omega_d$
- **Undamped**

  Poles: Two imaginary at $+j\omega_1, -j\omega_1$
- **Critically damped**

  Poles: Two real at $-\sigma_1,$