# Control Systems

# Lecture: 2

# Poles & Zeros

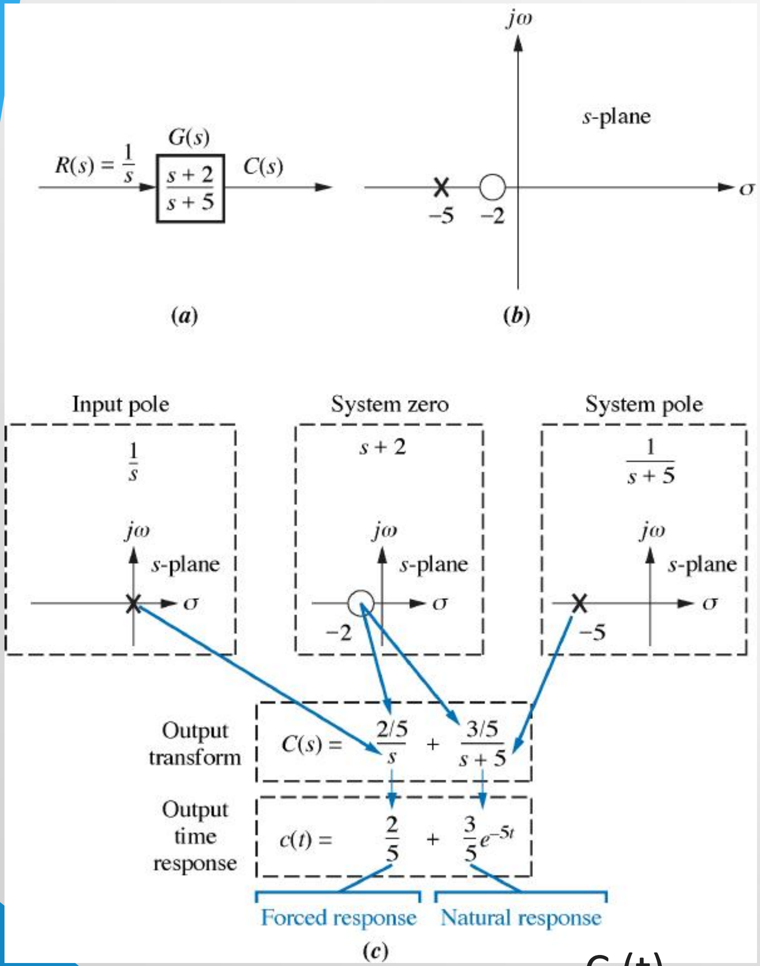
# Topics Covered

**Poles & Zeros for 1st order & 2nd order system**

# Poles and zeros

1. A pole of the input function generates the form of the forced response ( that is the pole at the origin generated a step function at the output).
2. A pole of the transfer function generate the form of the exponential response
3. The zeros and poles generate the amplitudes for both the transit and steady state responses ( see A, B in partial fraction extension)

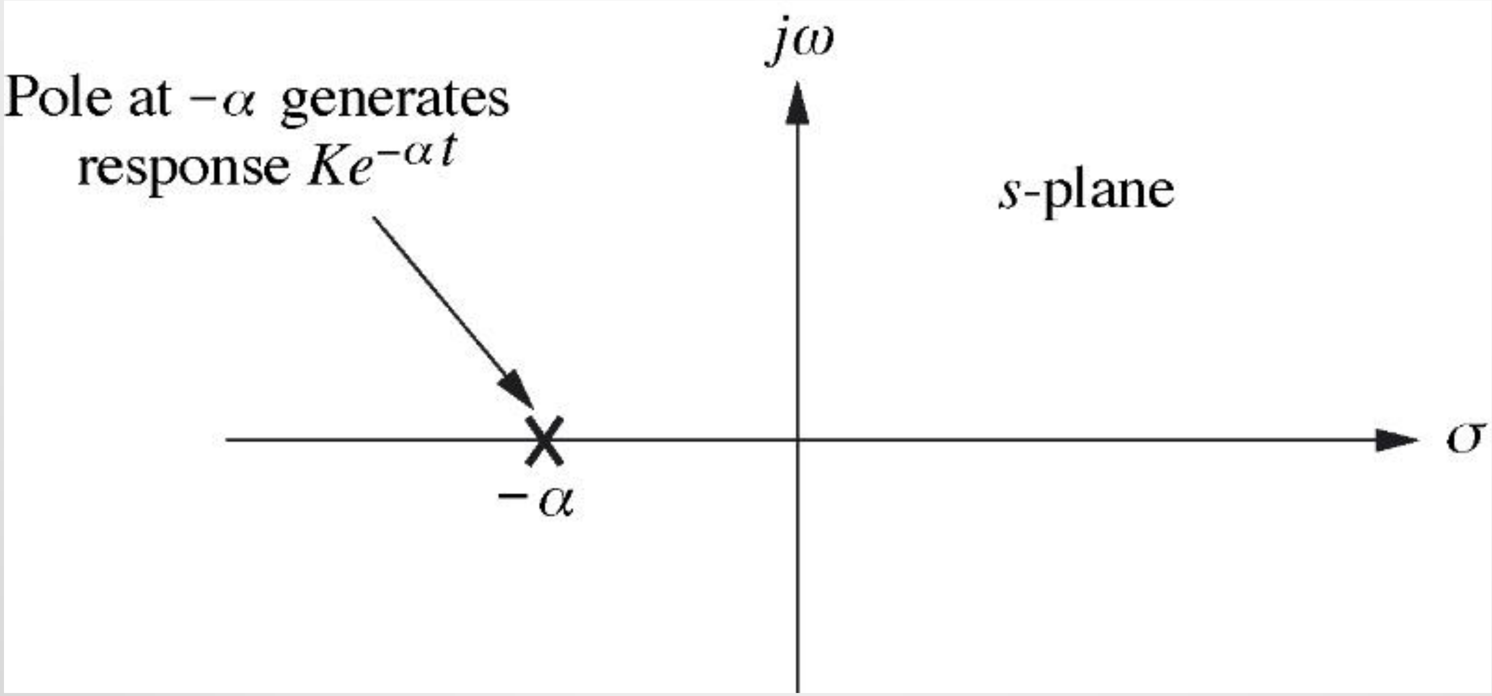
# Poles and zeros of a first order system




$C_t(t)$

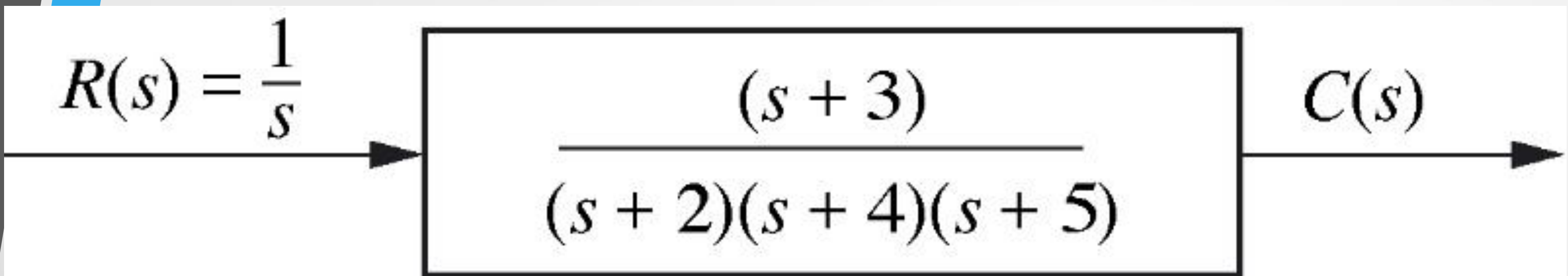
$C_{ss}(t)$

# Effect of a real-axis pole upon transient response




A pole on the real axis generate an exponential response of the form Exp[$-\alpha$t] where $-\alpha$ is the pole location on real axis. The farther to the left a pole is on the negative real axis, the faster the exponential transit response will decay to zero.

# Evaluating response using poles

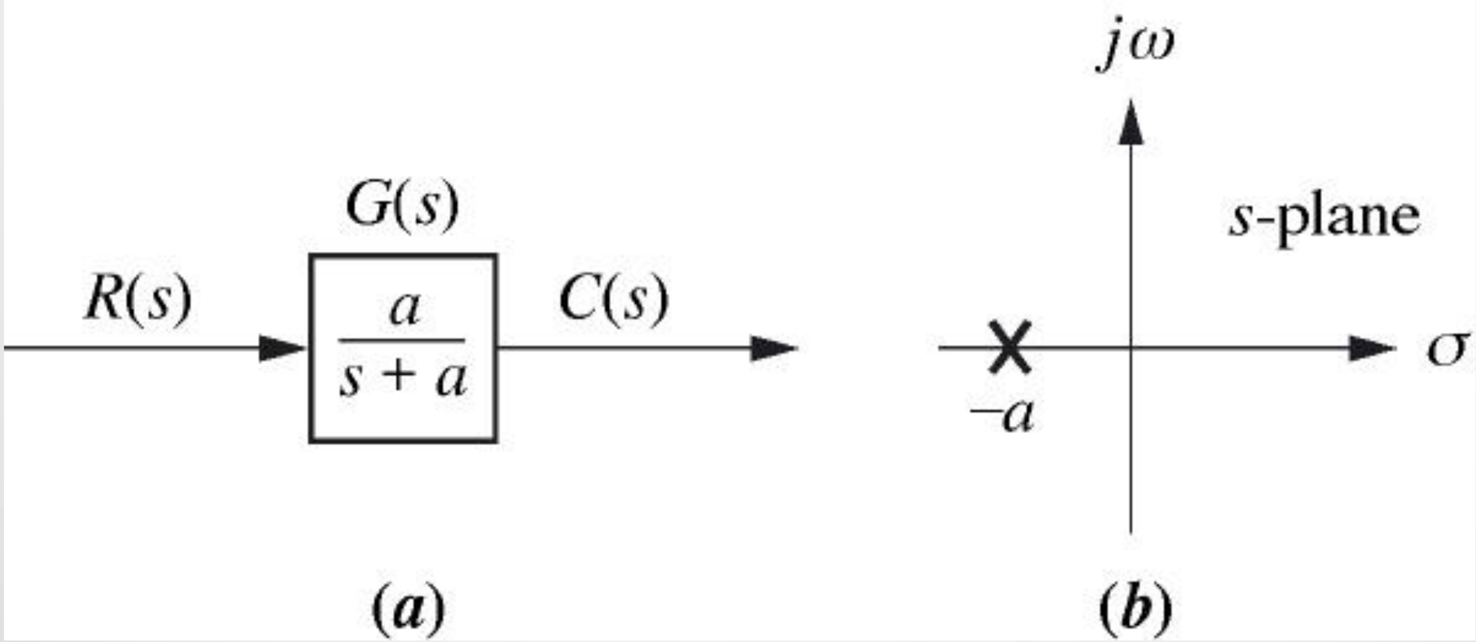



$$C(s) = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+4} + \frac{K_4}{s+5}$$

$C_{ss}(t)$ $C_t(t)$

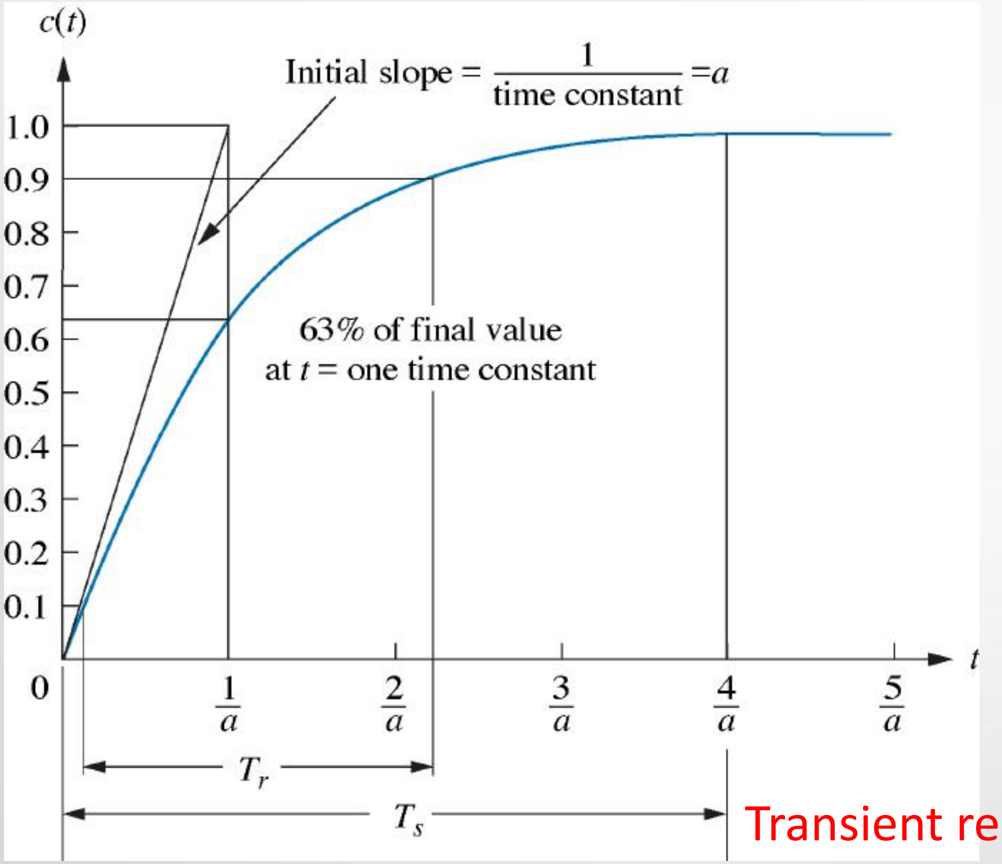
$$c(t) = K_1 + K_2 e^{-2t} + K_3 e^{-4t} + K_4 e^{-5t}$$

# First order system




$$C(s) = R(s)\, G(s) = \frac{a}{s(s+a)} \qquad c(t) = 1 - e^{-at}$$

# First-order system response to a unit step




Transient response specification:

1. Time-constant, 1/a
2. Rise time, $T_r$
3. Settling time, $T_s$

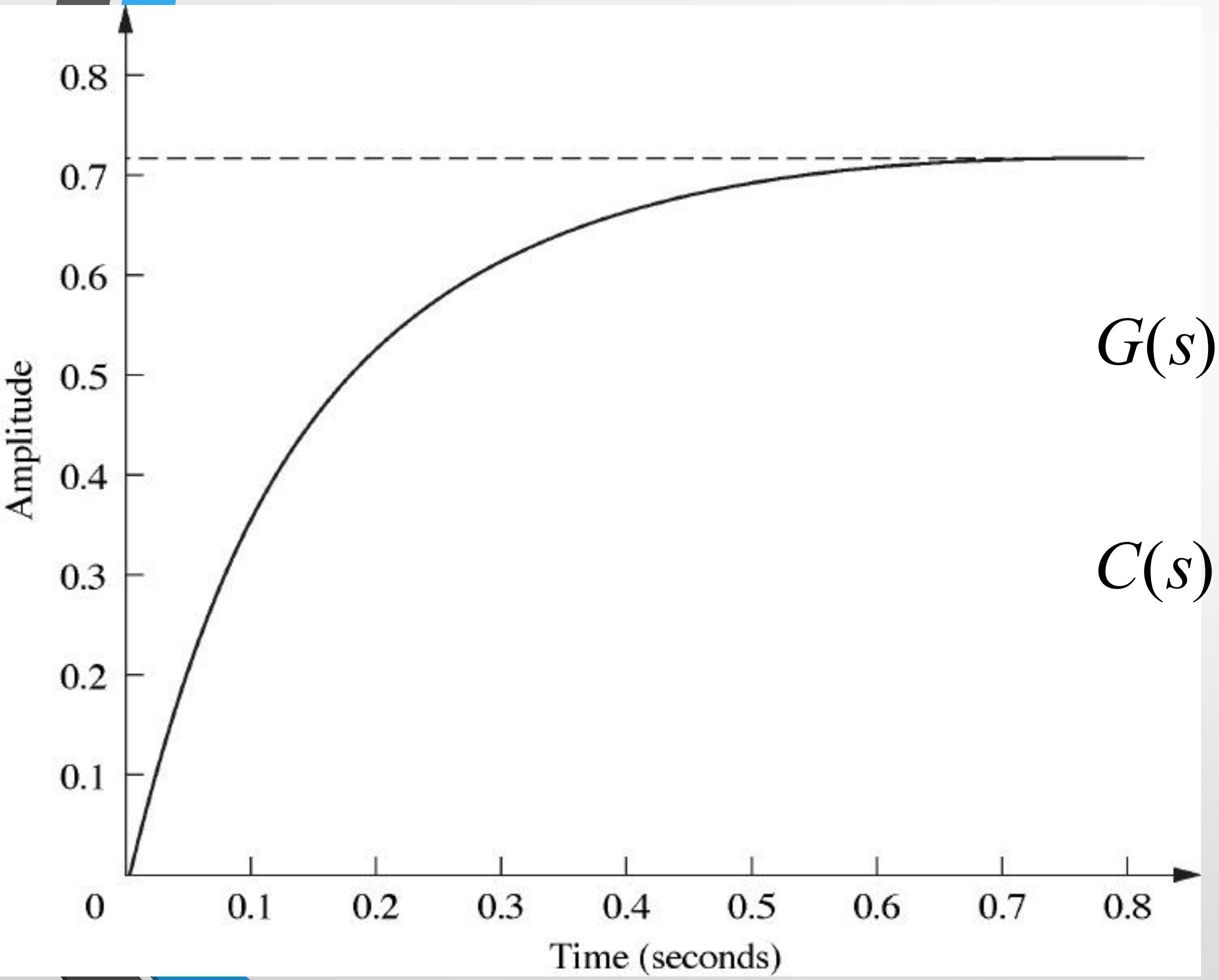
# Transient response specification for a first-order system

1. **Time-constant, 1/a**
   Can be described as the time for (1 - Exp[- a t])
   to rise to 63 % of initial value.

1. **Rise time, $T_r$ = 2.2/a**
   The time for the waveform to go from 0.1 to 0.9
   of its final value.

3. **Settling time, $T_s$ = 4/a**
   The time for response to reach, and stay within, 2% of its final value

# Transfer function via laboratory testing




$$G(s)=\frac{K}{(s+a)}$$

$$C(s)=\frac{K}{s(s+a)}=\frac{K/a}{s}-\frac{K/a}{(s+a)}$$

# Identify K and a from testing

**The time for amplitude to reach 63% of its final value:**
**63 x 0.72 = 0.45, or about 0.13 sec , a = 1/0.13 = 7.7**

**From equation, we see that the forced response reaches**
**a steady-state value of K/a =0.72 .**
**K= 0.72 x 7.7= 5.54**

**G(s) = 5.54/(s+7.7) .**