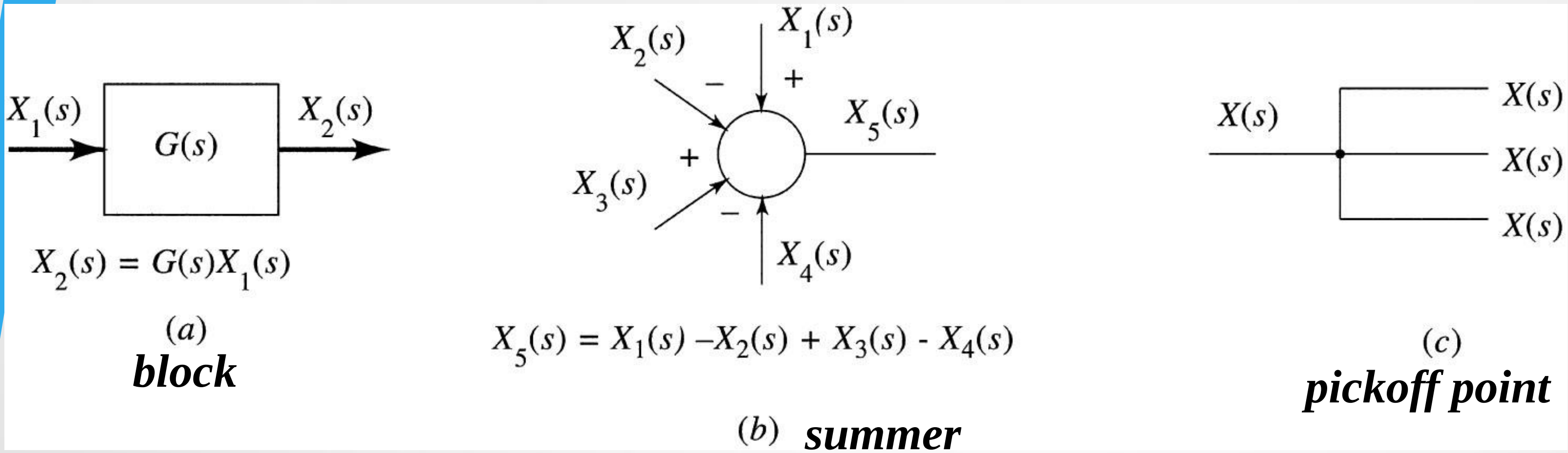
Control Systems

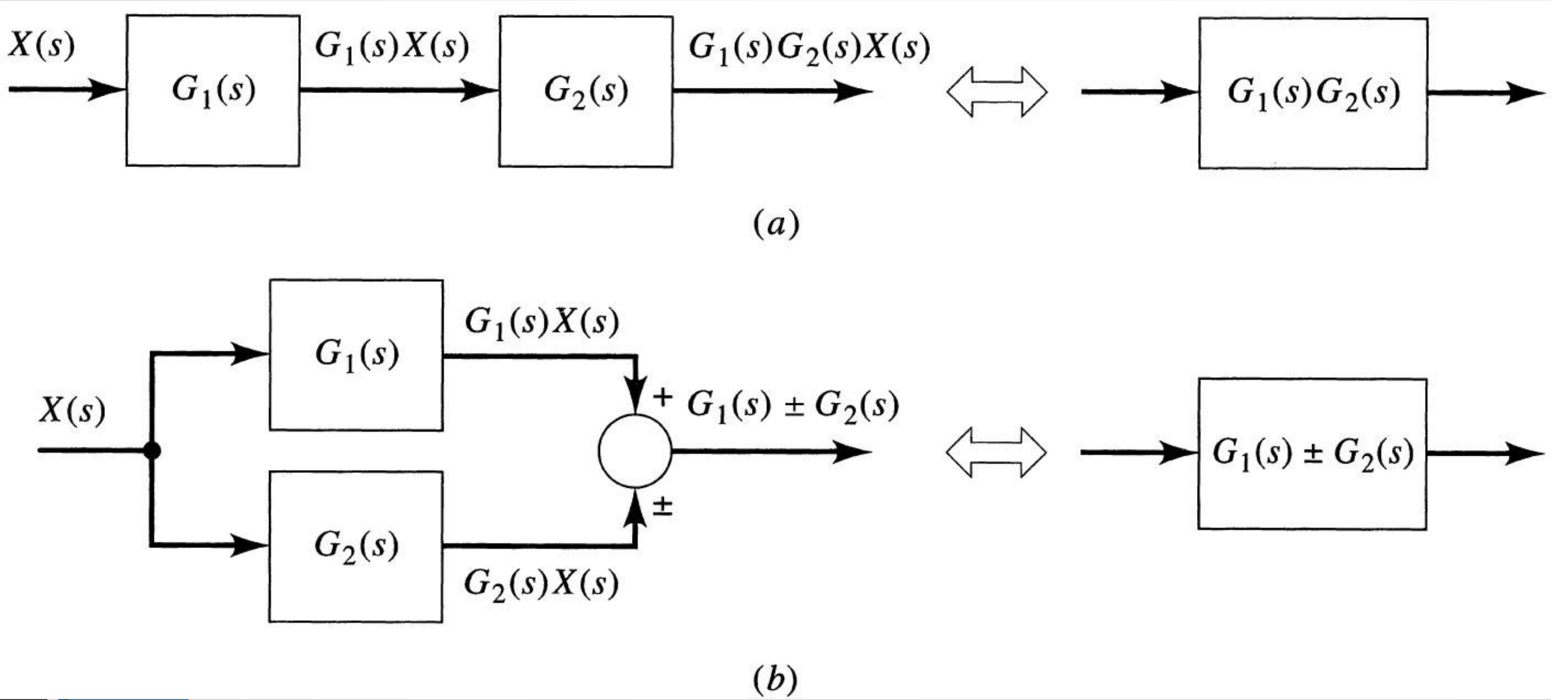
Lecture: 9

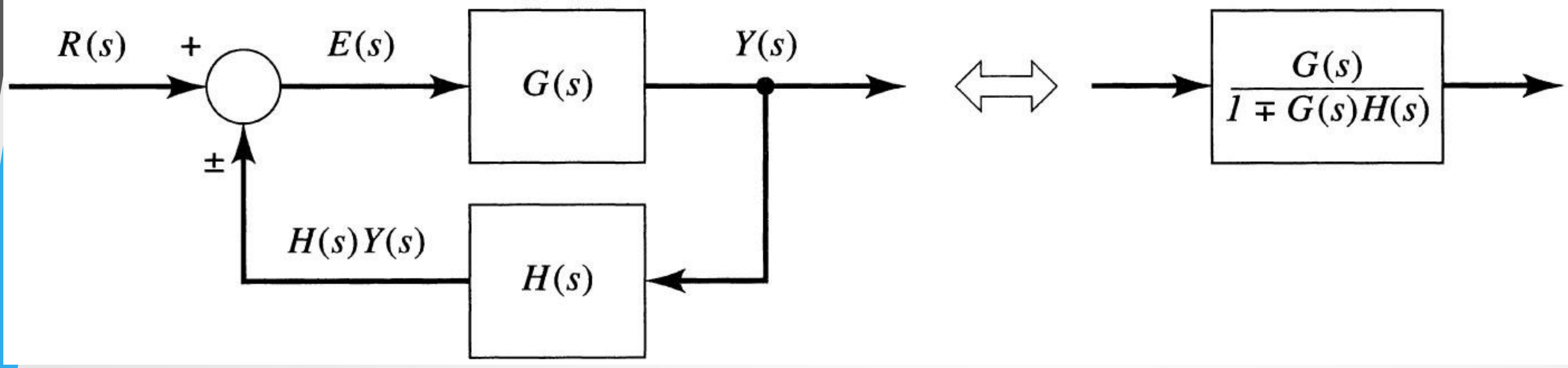
Block diagrams

&

Signal flow graphs


$X_1(s)$
$G(s)$
$X_2(s)$
$X_2(s) = G(s)X_1(s)$
(a)
block
$X_1(s)$
$X_2(s)$
$X_3(s)$
$X_4(s)$
$X_5(s)$
$X_5(s) = X_1(s) - X_2(s) + X_3(s) - X_4(s)$
(b) summer
$X(s)$
$X(s)$
$X(s)$
$X(s)$
(c)
pickoff point


$X(s)$
$G_1(s)$
$G_1(s)X(s)$
$G_2(s)$
$G_1(s)G_2(s)X(s)$
$G_1(s)G_2(s)$
(a)
$X(s)$
$G_1(s)$
$G_1(s)X(s)$
$G_2(s)$
$G_2(s)X(s)$
$G_1(s) \pm G_2(s)$
$G_1(s) \pm G_2(s)$
(b)

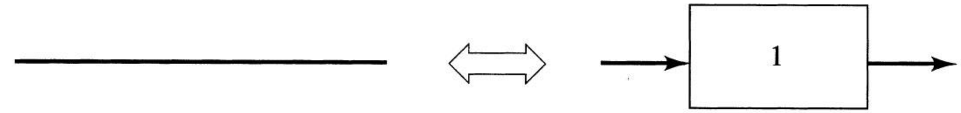


$$Y(s) = G(s)E(s)$$

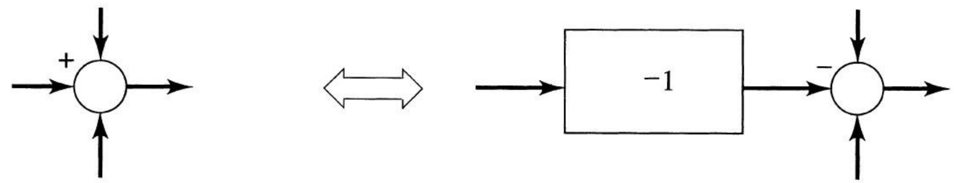
$$E(s) = R(s) \pm H(s)Y(s)$$

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)] = G(s)R(s) \pm G(s)H(s)Y(s)$$

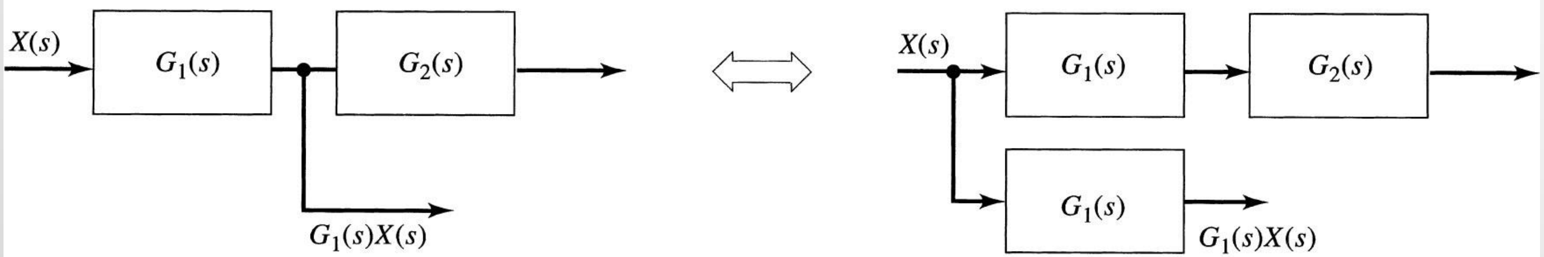
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$



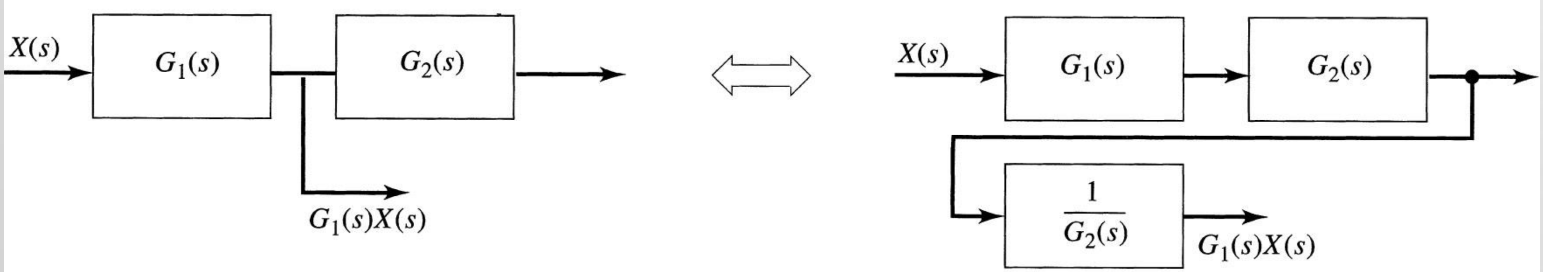
(*a*) Insertion or removal of unity gain



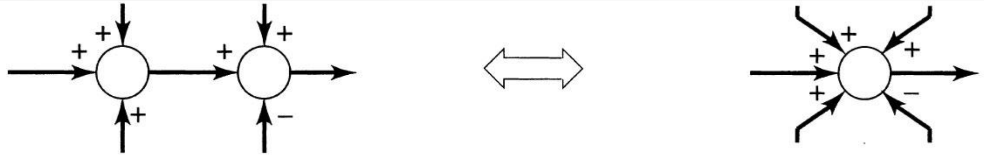
(*b*) Changing a summer sign



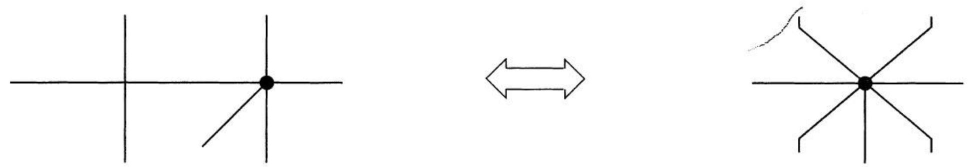
(*c*) Moving a pickoff point back



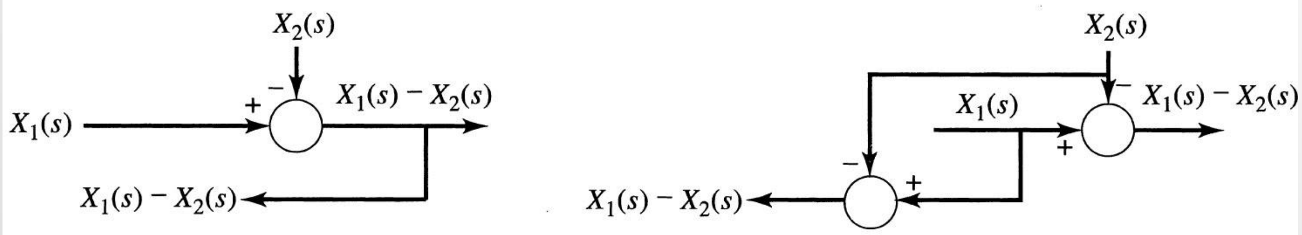
(*d*) Moving a pickoff point forward



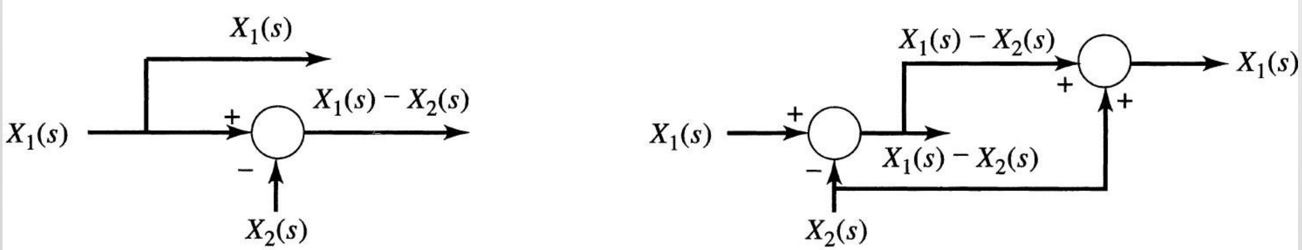
(*e*) Combining or expanding summations



(*f*) Combining or expanding junctions

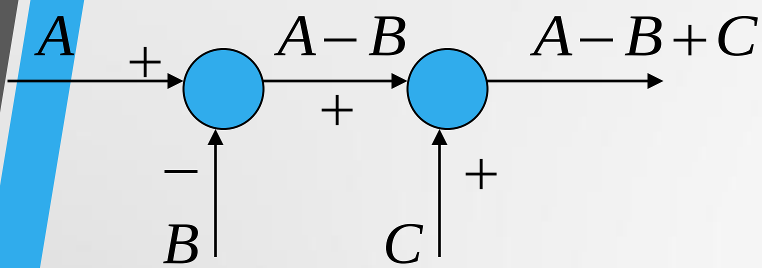


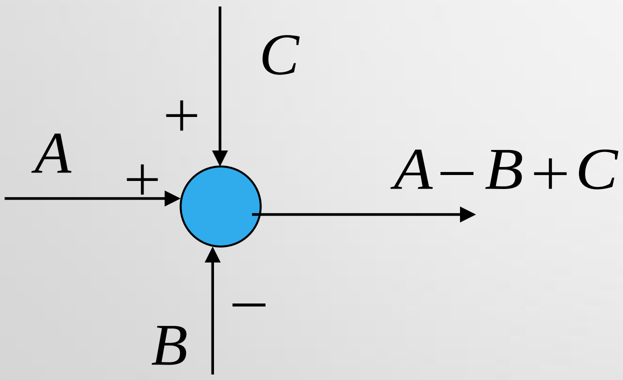
(*g*) Moving a pickoff point behind a summation



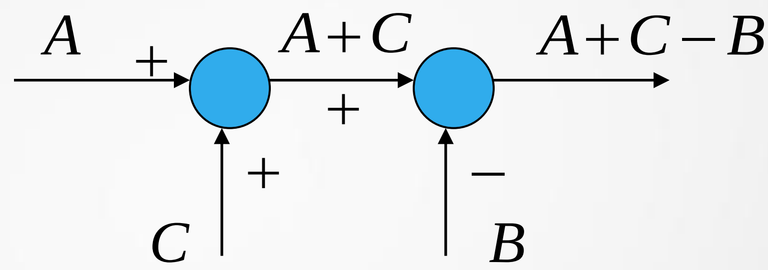
(*h*) Moving a pickoff point forward of a summation

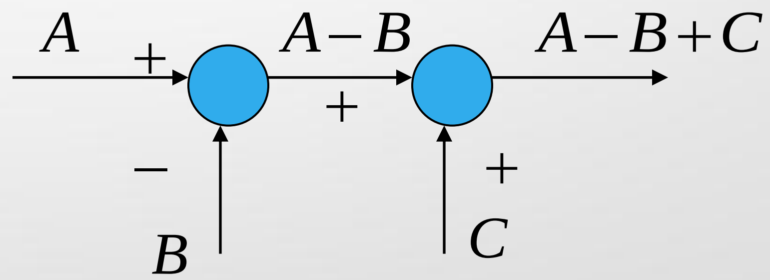
original



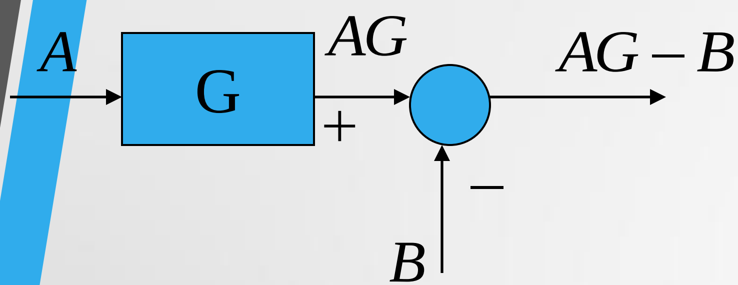


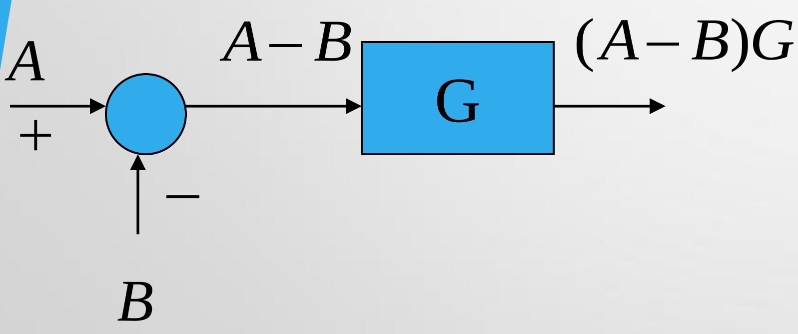
equivalent



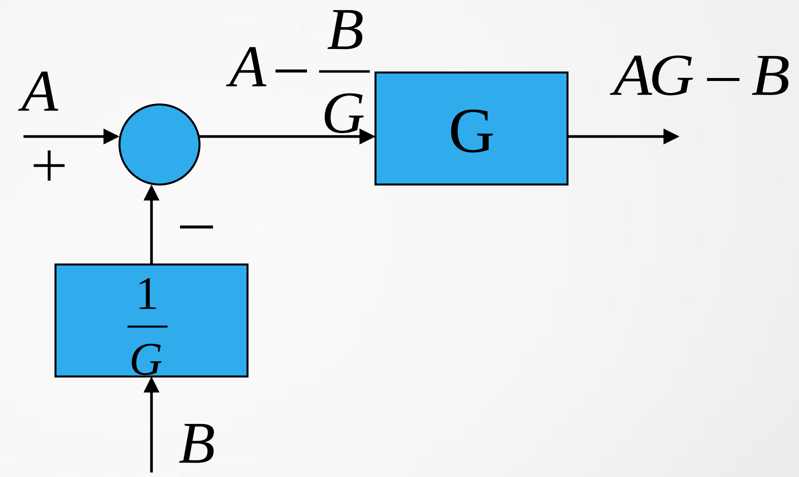


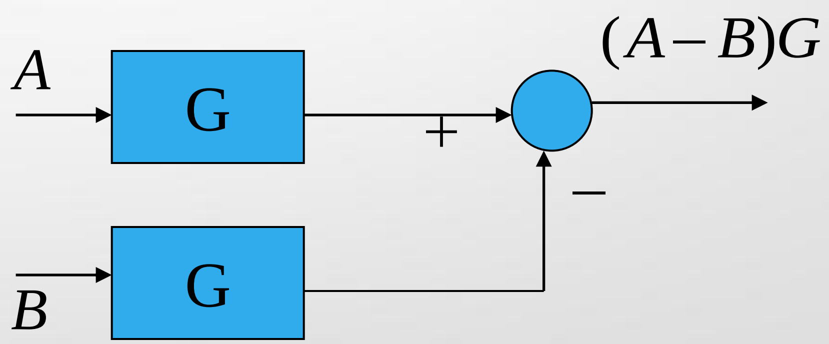
original



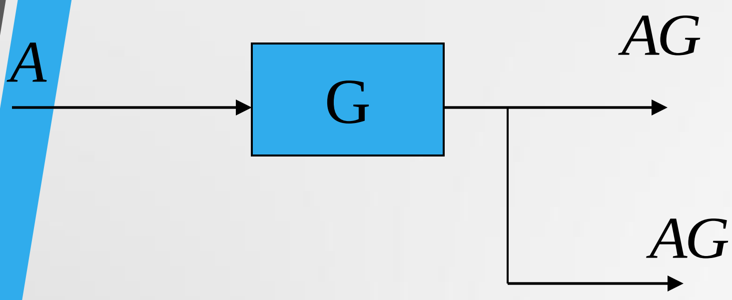


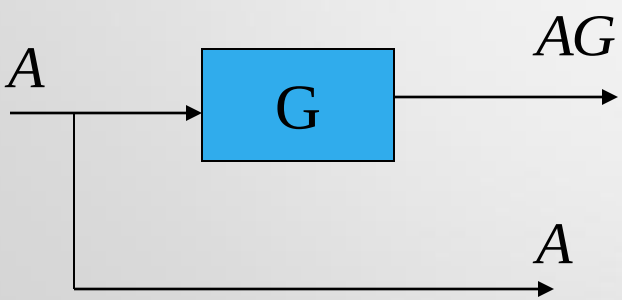
equivalent



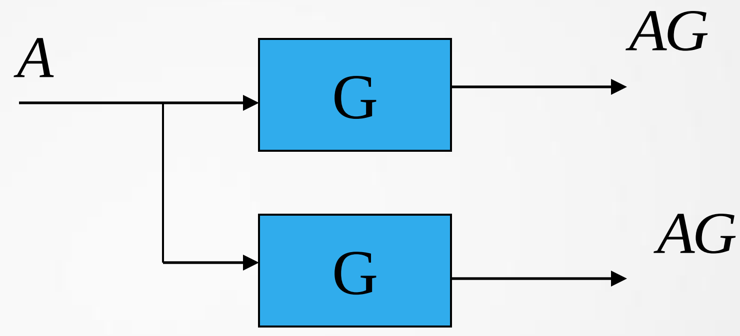


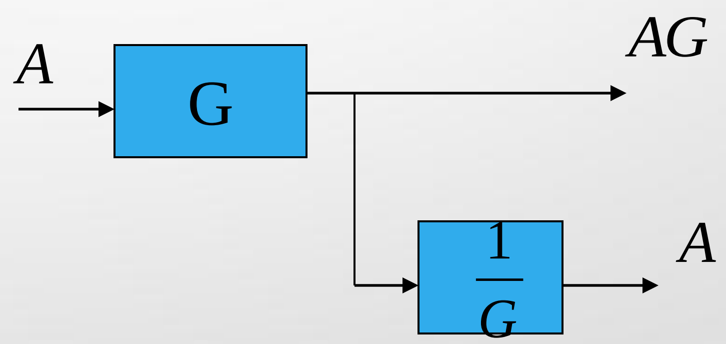
original

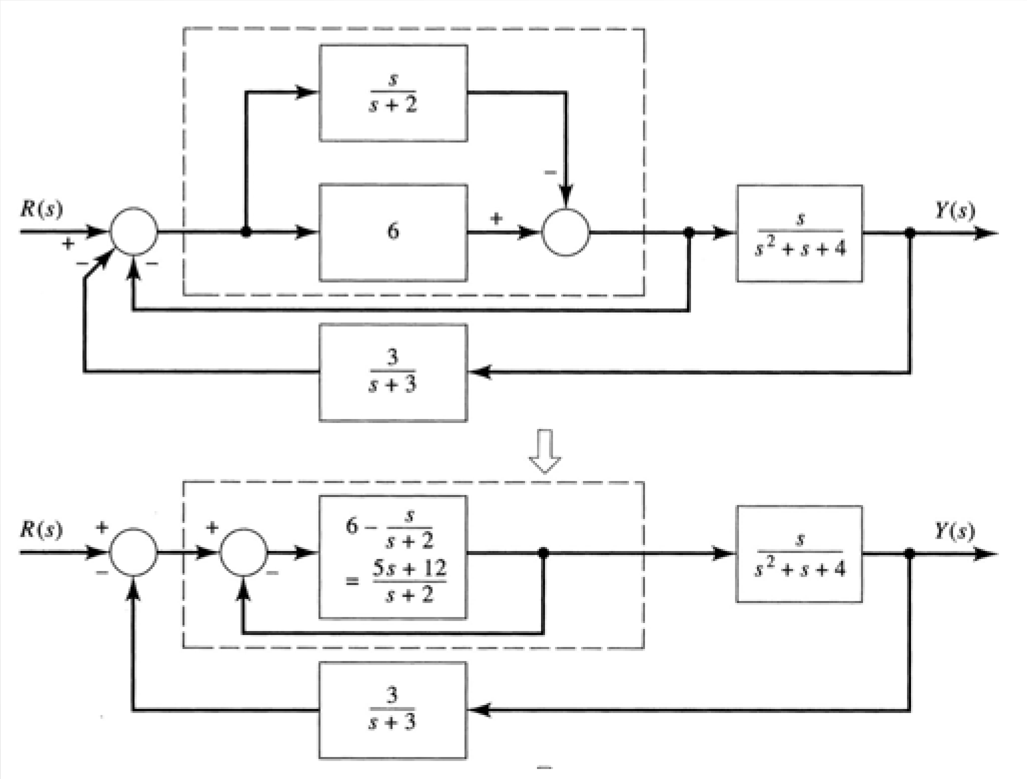




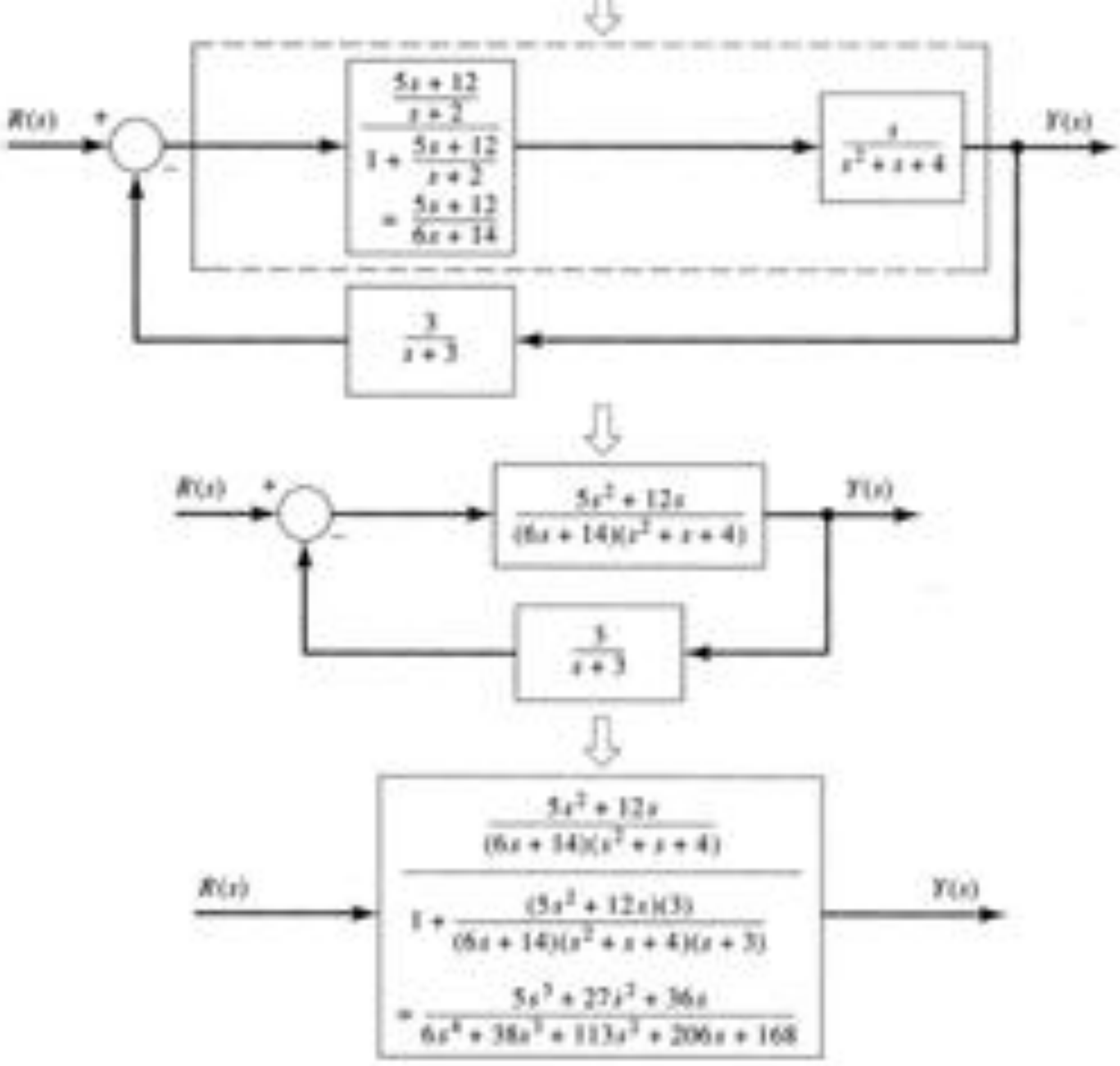
equivalent



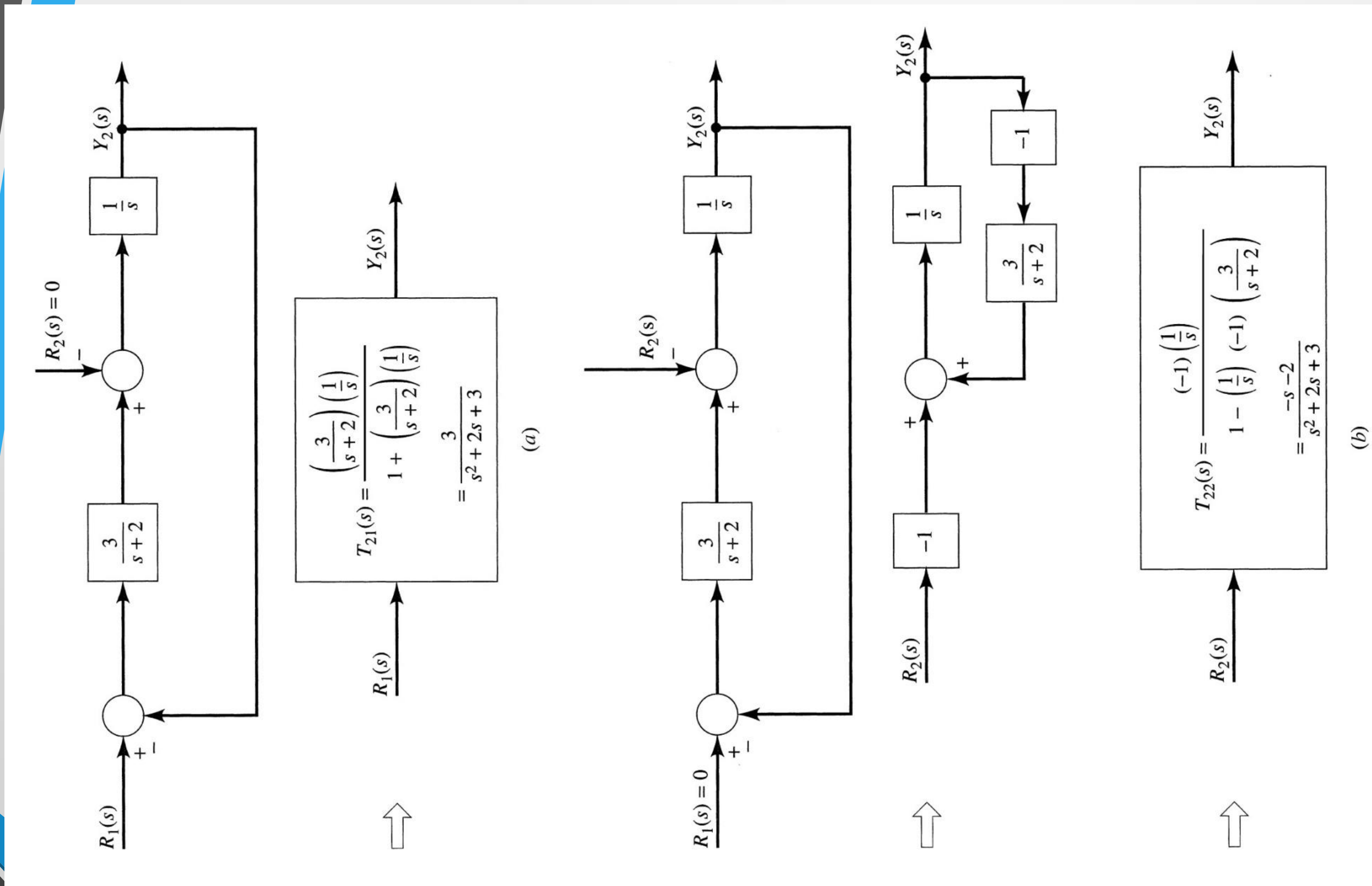




R(s)
+
−
−
$\frac{s}{s+2}$
6
−
+
$\frac{s}{s^2+s+4}$
Y(s)
$\frac{3}{s+3}$
R(s)
+
−
+
−
$6-\frac{s}{s+2}$
$=\frac{5s+12}{s+2}$
$\frac{s}{s^2+s+4}$
Y(s)
$\frac{3}{s+3}$

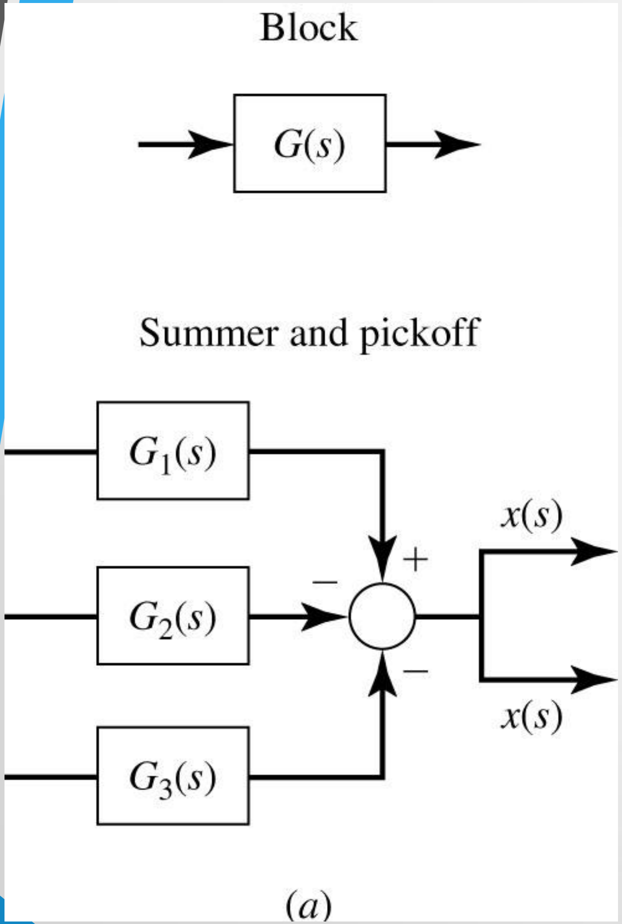

R(s)
+
−
$\frac{\frac{5s+12}{s+2}}{1+\frac{5s+12}{s+2}} = \frac{5s+12}{6s+14}$
$\frac{s}{s^2+s+4}$
Y(s)
$\frac{3}{s+3}$
R(s)
+
−
$\frac{5s^2+12s}{(6s+14)(s^2+s+4)}$
Y(s)
$\frac{3}{s+3}$
R(s)
$\frac{\frac{5s^2+12s}{(6s+14)(s^2+s+4)}}{1+\frac{(5s^2+12s)(3)}{(6s+14)(s^2+s+4)(s+3)}} = \frac{5s^3+27s^2+36s}{6s^4+38s^3+113s^2+206s+168}$
Y(s)

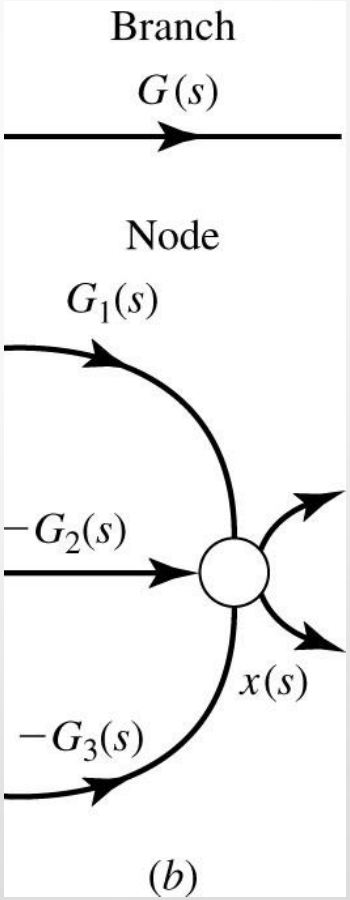
Example 2

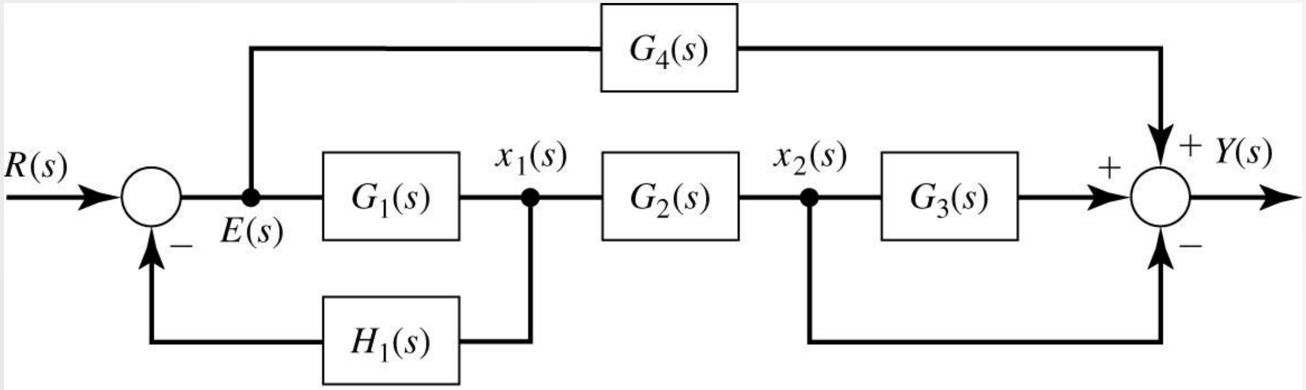


R1(s)
R2(s) = 0
Y2(s)
3/(s + 2)
1/s
T21(s) = (3/(s + 2))(1/s) / [1 + (3/(s + 2))(1/s)]
= 3/(s^2 + 2s + 3)
(a)
R1(s) = 0
R2(s)
−1
T22(s) = (−1)(1/s) / [1 − (1/s)(−1)(3/(s + 2))]
= (−s −2)/(s^2 + 2s + 3)
(b)

Signal flow graphs

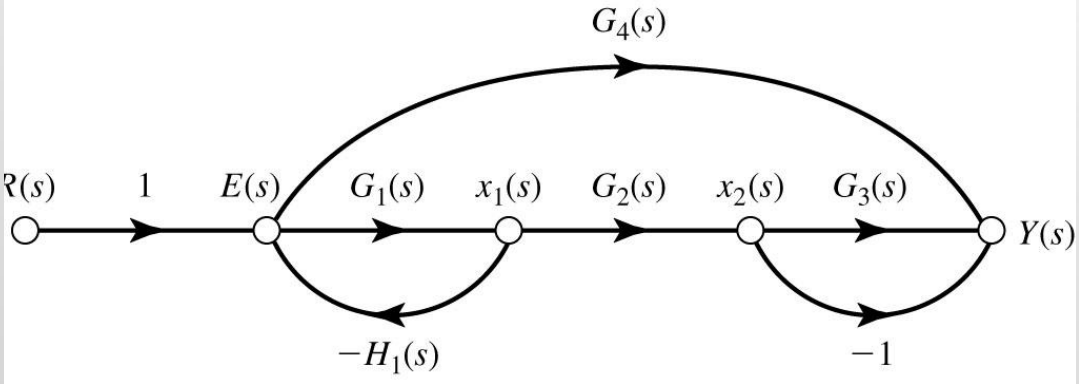






$G_4(s)$
$R(s)$
$x_1(s)$
$x_2(s)$
$+$ $Y(s)$
$+$
$G_1(s)$
$G_2(s)$
$G_3(s)$
$-$
$E(s)$
$-$
$H_1(s)$

(a)



$G_4(s)$
(s)
1
$E(s)$
$G_1(s)$
$x_1(s)$
$G_2(s)$
$x_2(s)$
$G_3(s)$
$Y(s)$
$-H_1(s)$
-1

(b)

Mason's Rule

Mason's gain rule is as follows: the transfer function of a system with signal-input, signal-output flow graphs is

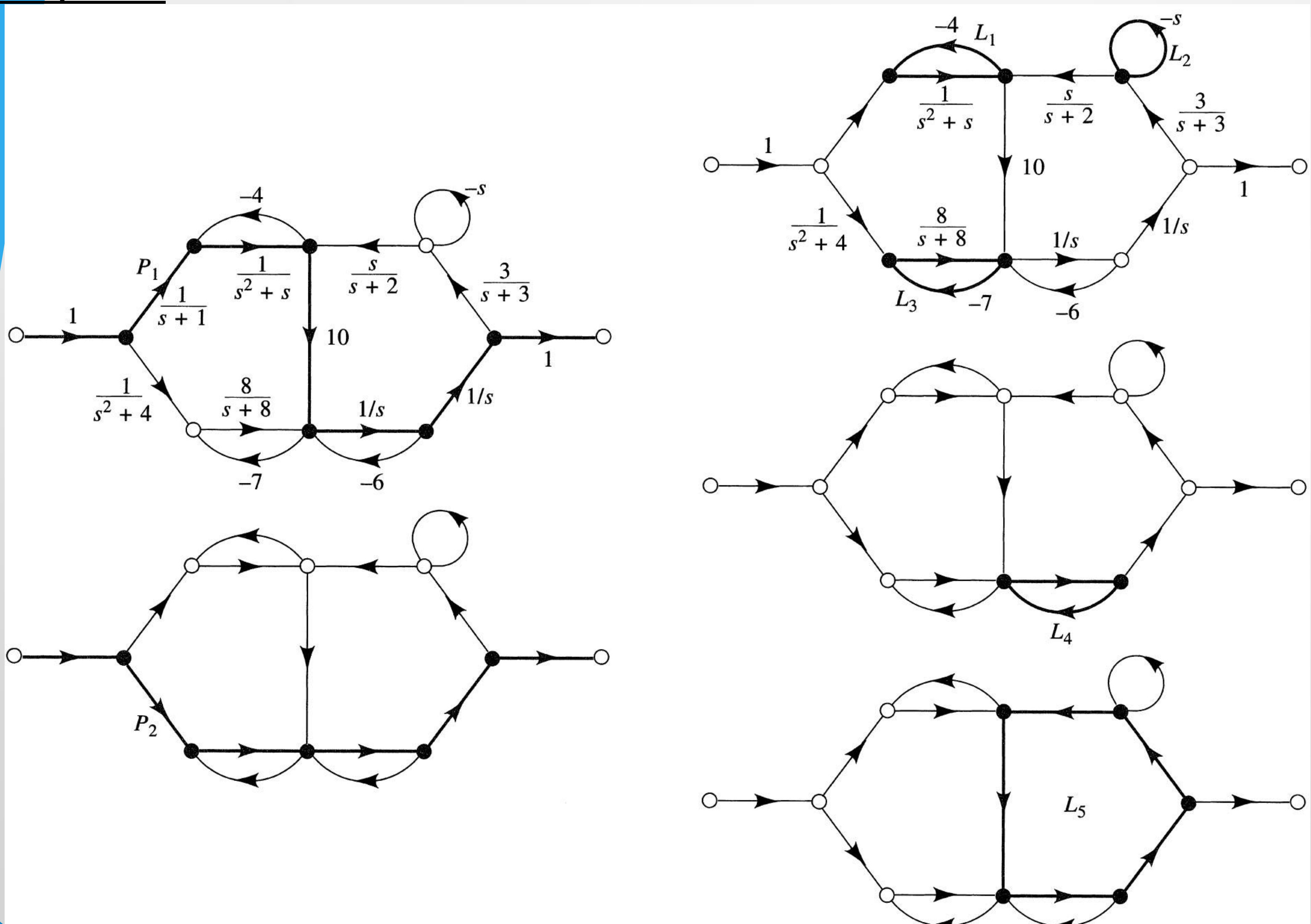
$$T(s) = \frac{p_1\Delta_1 + p_2\Delta_2 + p_3\Delta_3 + \cdots}{\Delta}$$

Δ=1-(sum of all loop gains)+(sum of products of gains of all combinations if 2 nontouching loops)- (sum of products of gains of all combinations if 3 nontouching loops)+…

*A **path** is any succession of branches, from input to output, in the direction of the arrows, that does not pass any node more than once.*

*A **loop** is any closed succession of branches in the direction of the arrows that does not pass any node more than once.*

Example 3



–4
–s
P_1
1
$\frac{1}{s+1}$
$\frac{1}{s^2+s}$
$\frac{s}{s+2}$
$\frac{3}{s+3}$
10
1
$\frac{1}{s^2+4}$
$\frac{8}{s+8}$
1/s
1/s
–7
–6
P_2
–4
L_1
–s
L_2
$\frac{1}{s^2+s}$
$\frac{s}{s+2}$
$\frac{3}{s+3}$
1
10
1
$\frac{1}{s^2+4}$
$\frac{8}{s+8}$
1/s
1/s
L_3
–7
–6
L_4
L_5

Example 4

find $\dfrac{y_5}{y_3}$