Control Systems

Topics Covered

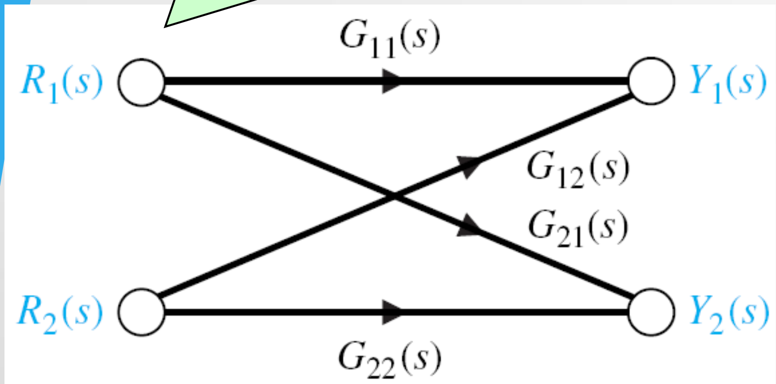
Signal- flow graph

- Alternative method to block diagram representation, developed by S.J.Mason.
- Advantage: the availability of a flow graph gain formula, also called Mason’s gain formula.
- A signal-flow graph consists of a network in which nodes are connected by directed branches.
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals. Note that the signal flows in only one direction.

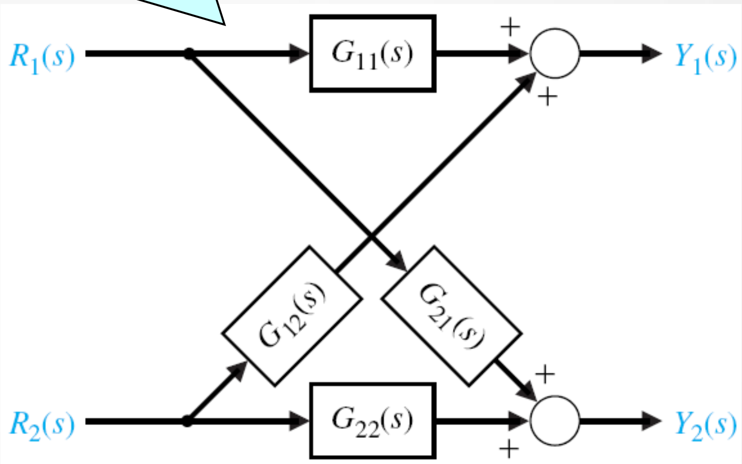
Definitions

- **Node - a point representing a signal or variable.**
- **Branch – unidirectional line segment joining two nodes.**
- **Path – a branch or a continuous sequence of branches that can be traversed from one node to another node.**
- **Loop – a closed path that originates and terminates on the same node and along the path no node is met twice.**
- **Nontouching loops – two loops are said to be nontouching if they do not have a common node.**

Signal-flow graph of interconnected system

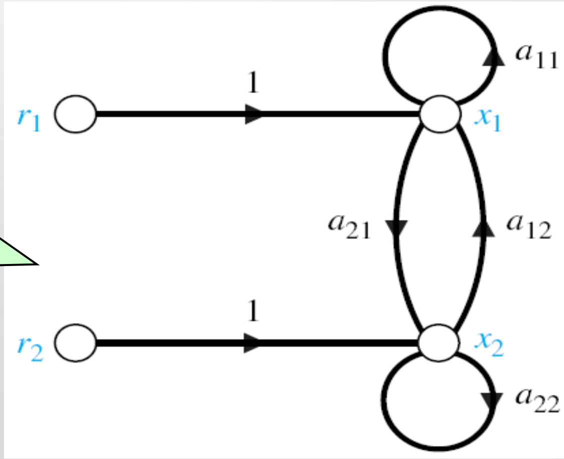


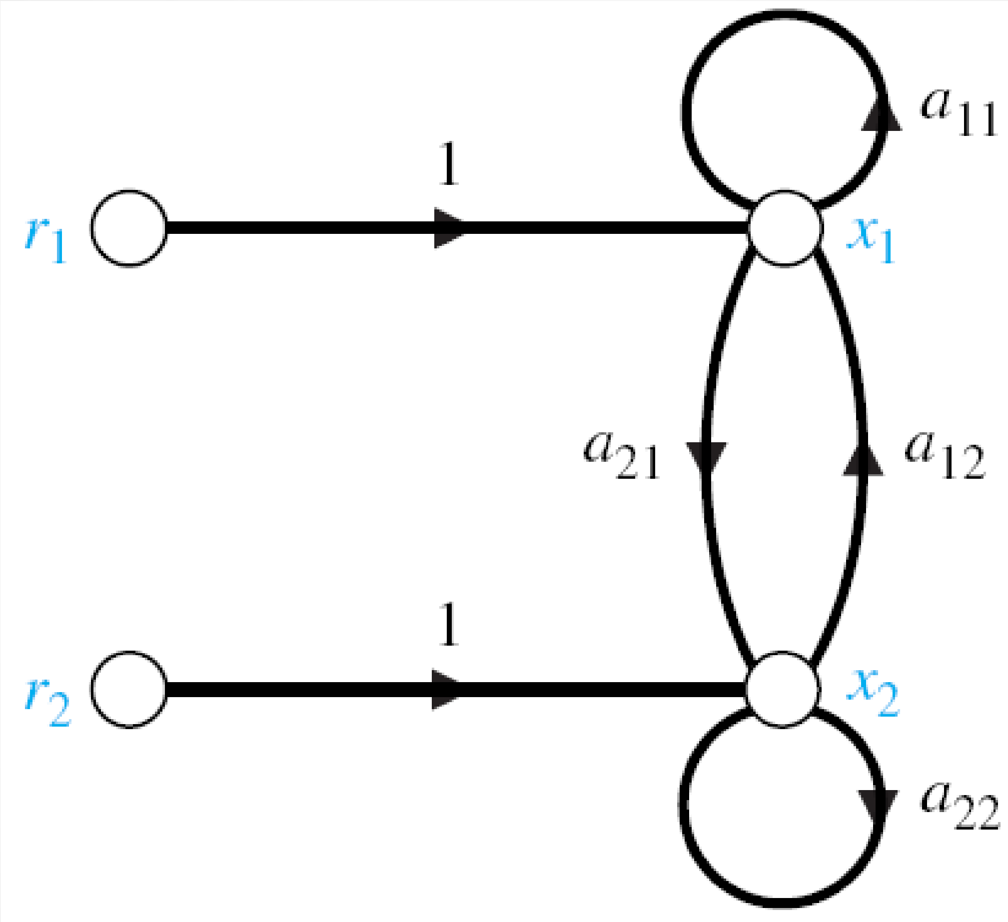
Corresponding block diagram



$$a_{11}x_1 + a_{12}x_2 + r_1 = x_1$$
$$a_{21}x_1 + a_{22}x_2 + r_2 = x_2$$

Signal-flow graph of two algebraic equations





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Mason's gain formula

The linear dependence T_{ij} between the independent variable x_i (also called the input variable) and a dependent variable x_j is

$$T_{ij} = \frac{\Sigma_k P_{ijk} \Delta_{ijk}}{\Delta},$$

P_{ijk} = kth path from variable x_i to variable x_j,

Δ = determinant of the graph,

Δ_{ijk} = cofactor of the path P_{ijk},

and the summation is taken over all possible k paths from x_i to x_j. The cofactor Δ_{ijk} is the determinant with the loops touching the kth path removed. The determinant Δ is

$$\Delta = 1 - \sum_{n=1}^{N} L_n + \sum_{m=1,q=1}^{M,Q} L_m L_q - \sum L_r L_s L_t + \cdots,$$

where L_q equals the value of the qth loop transmittance. Therefore the rule for evaluating Δ in terms of loops $L_1, L_2, L_3, \ldots, L_N$ is

$\Delta = 1$ − (sum of all different loop gains)

+ (sum of the gain products of all combinations of two nontouching loops)

− (sum of the gain products of all combinations of three nontouching loops)

$+ \cdots$.

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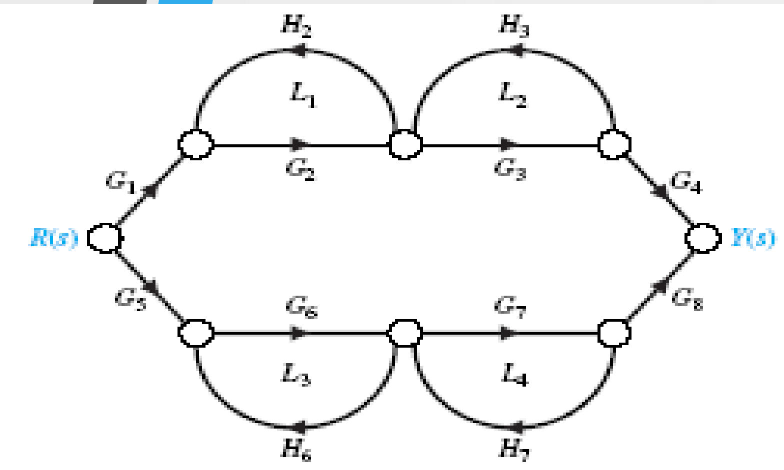
Write the gain formula in a simplified form:

$$T = \frac{\sum_k P_k \Delta_k}{\Delta}$$

Systematic approach:

1. Calculate forward path transfer function P_k for each forward path k
2. Calculate all loop transfer functions
3. Consider nontouching loops 2 at a time
4. Consider nontouching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_k as portion of Δ not touching forward path k



1.Calculate forward path transfer function *Pk* for each forward path *k.*

$$P_1 = G_1G_2G_3G_4 \text{ (path1)} \quad \text{and} \quad P_2 = G_5G_6G_7G_8 \text{ (path2)}$$

2.Calculate all loop TF's.

$$L_1 = G_2H_2, \quad L_2 = H_3G_3, \quad L_3 = G_6H_6, \quad L_4 = G_7H_7$$

3.Consider nontouching loops 2 at a time.

Loops L1 and L2 do not touch Loops L3 and L4

4. Consider nontouching loops 3 at a time.
   None.
5. Calculate Δ from steps 2,3,4.

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4)$$

6. Calculate Δ_k as portion of Δ not touching forward path *k.*

$$\Delta_1 = 1 - (L_3 + L_4) \quad \text{and} \quad \Delta_2 = 1 - (L_1 + L_2)$$

The TF of the system is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$