Control Systems

Topics Covered

Control Principles

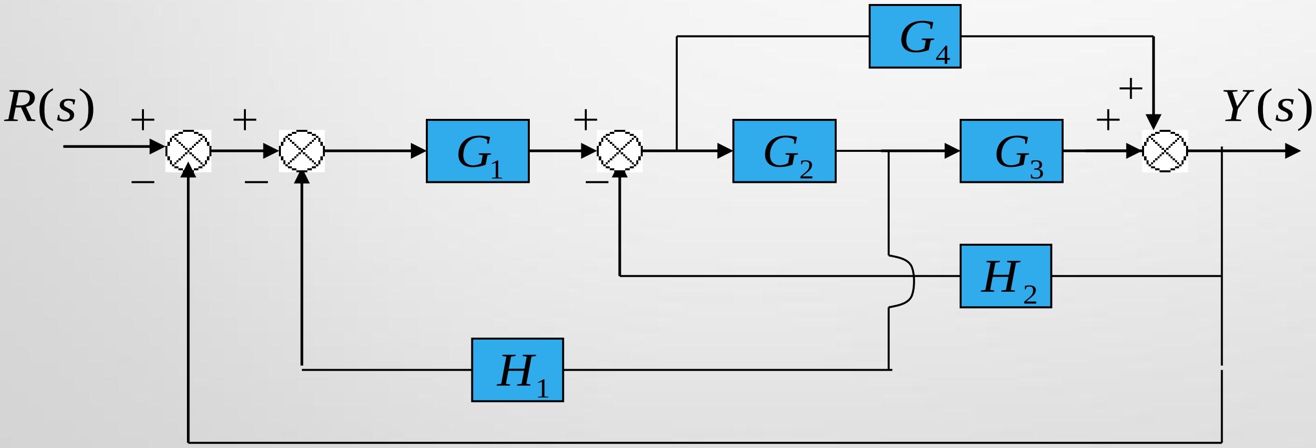
Block diagram practice

Block diagram

Transfer Function

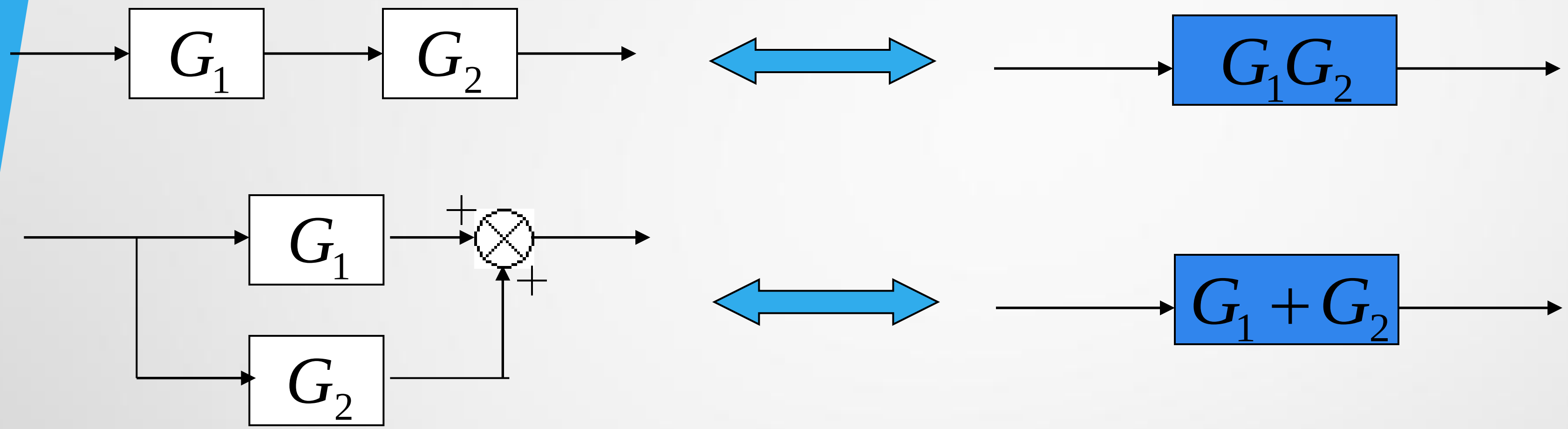
Consists of Blocks

Can be reduced

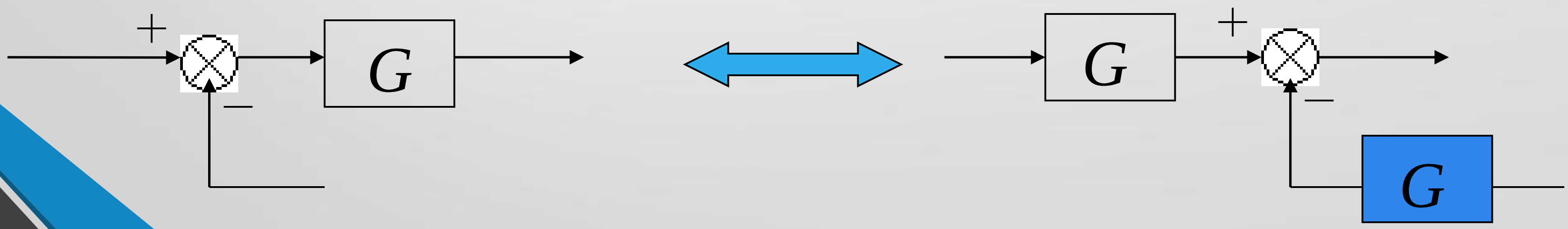


Reduction techniques

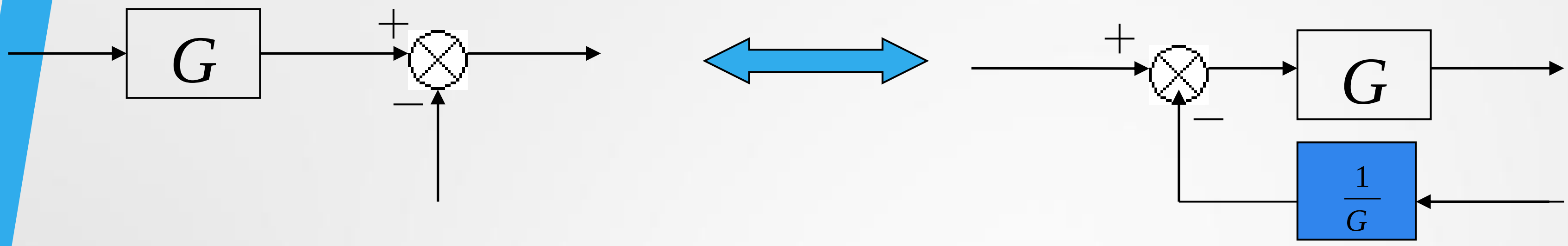
1. Combining blocks in cascade or in parallel



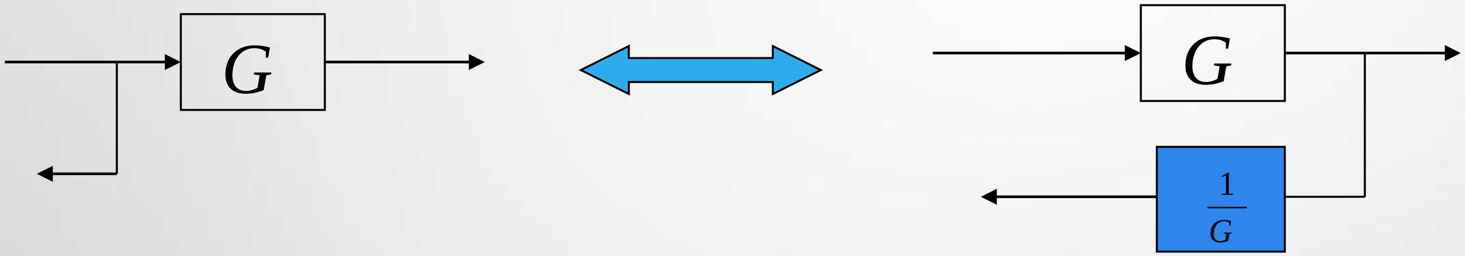
2. Moving a summing point behind a block



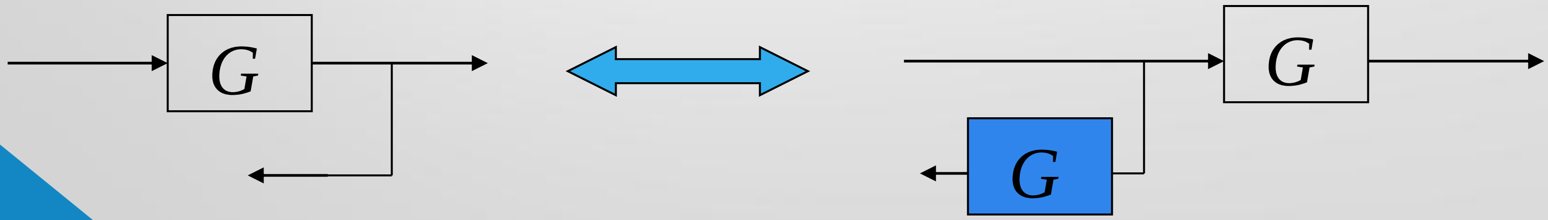
3. Moving a summing point ahead of a block



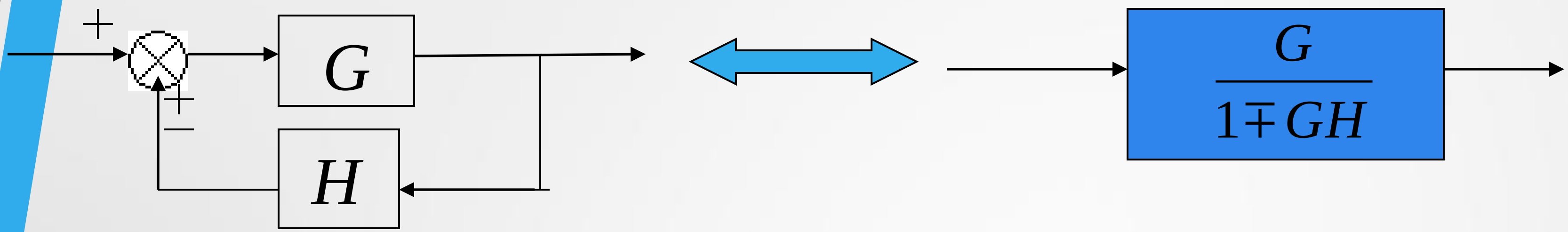
4. Moving a pickoff point behind a block

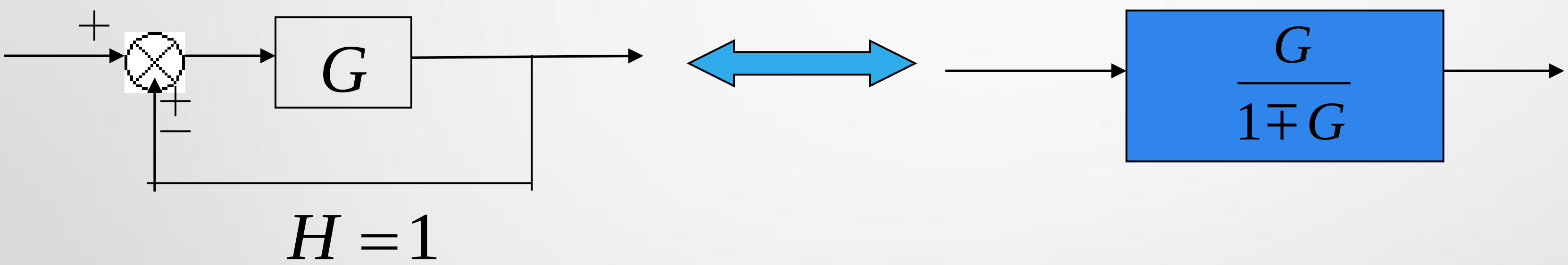


5. Moving a pickoff point ahead of a block

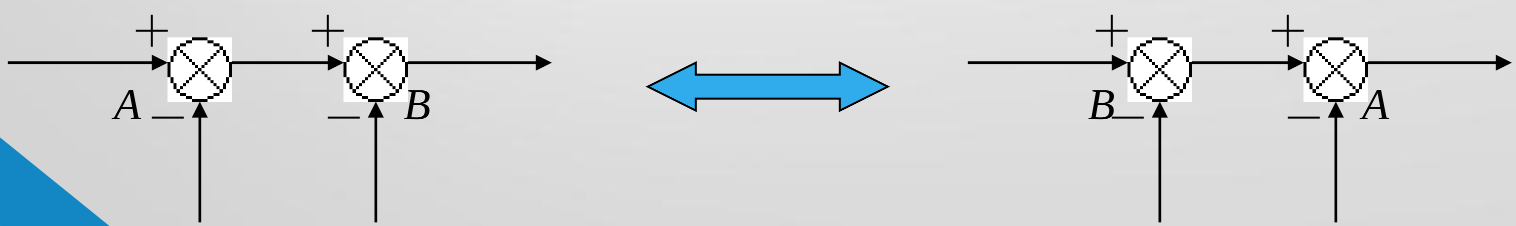


6. Eliminating a feedback loop





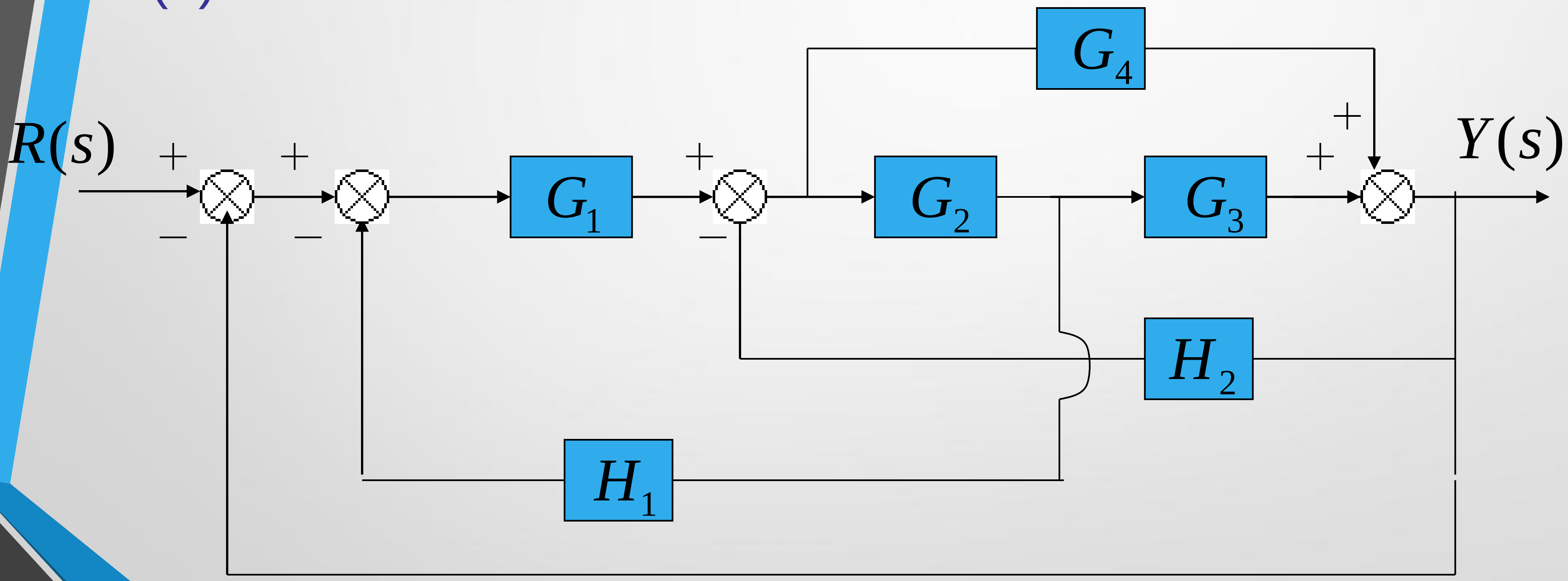
7. Swap with two neighboring summing points

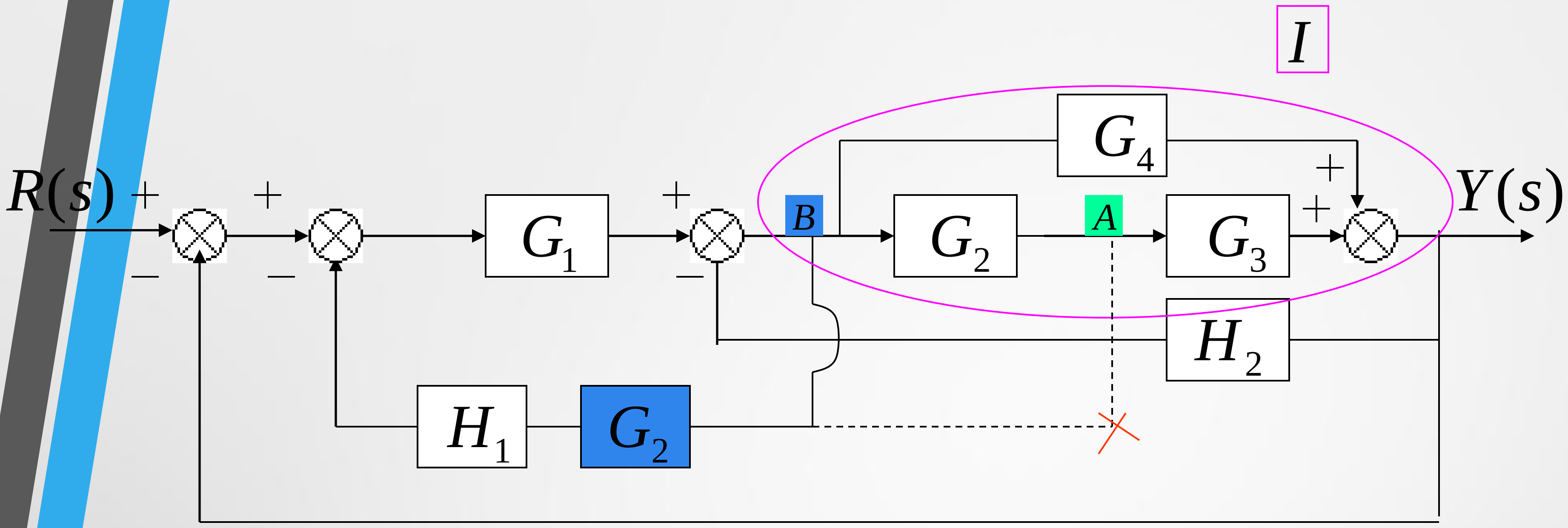


Example 1

Find the transfer function of the following block diagrams

(a)

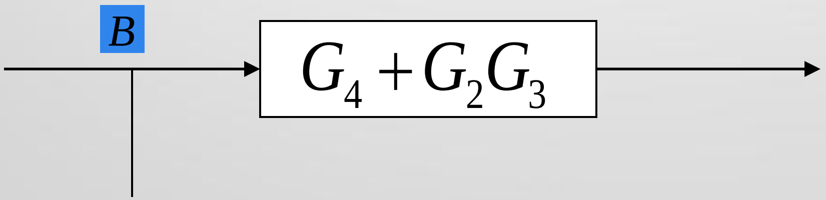


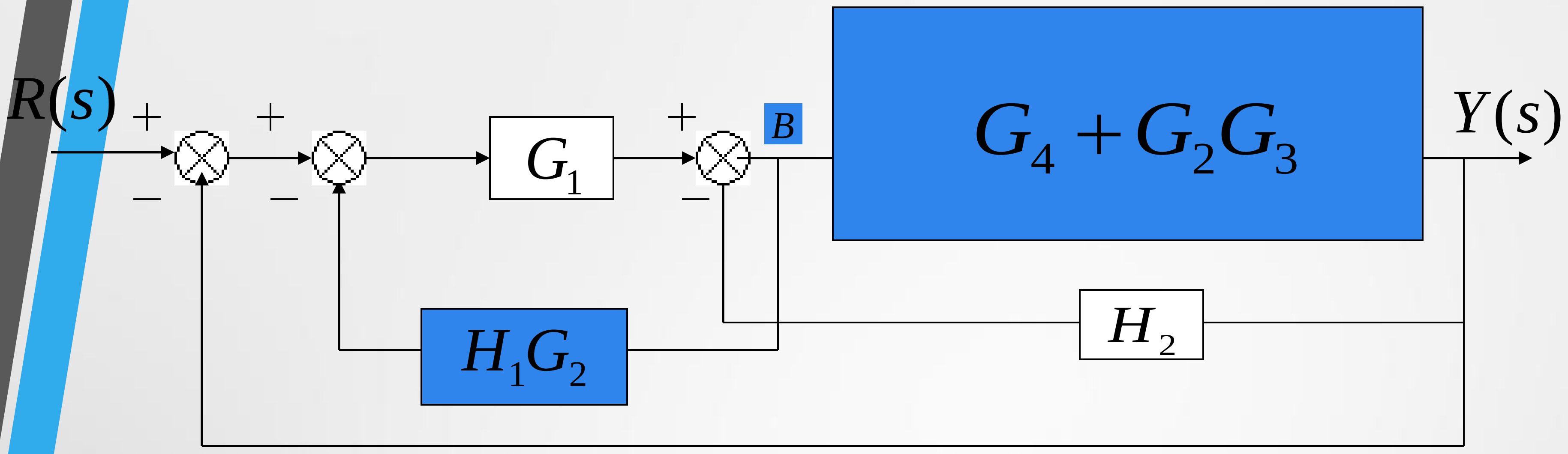


Solution:

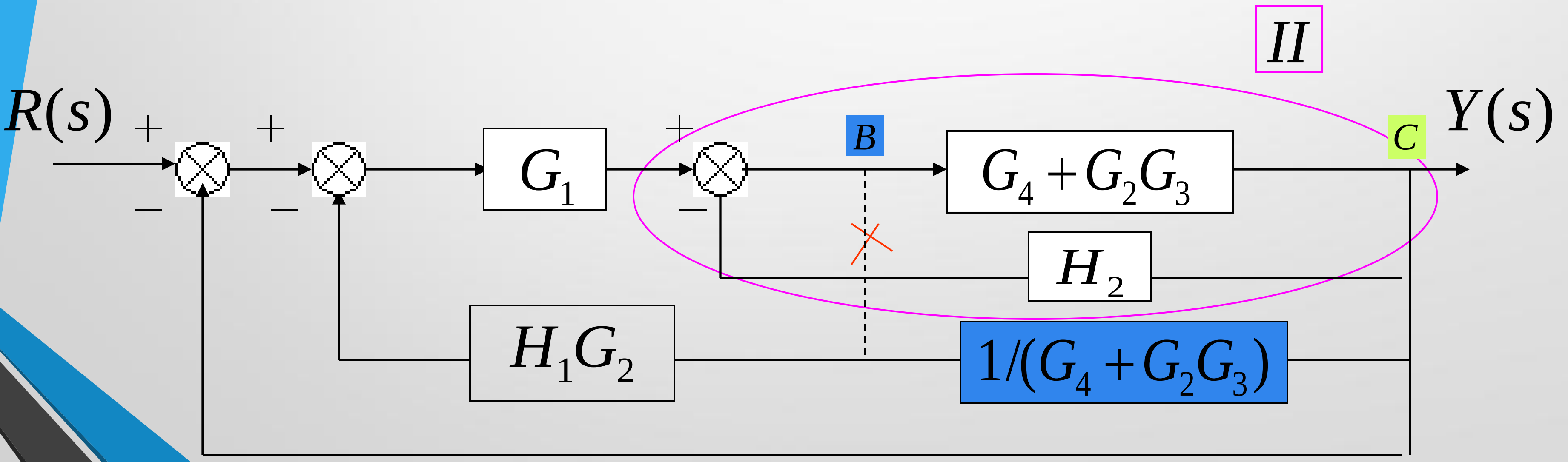
1. Moving pickoff point A ahead of block G_2

2. Eliminate loop I & simplify

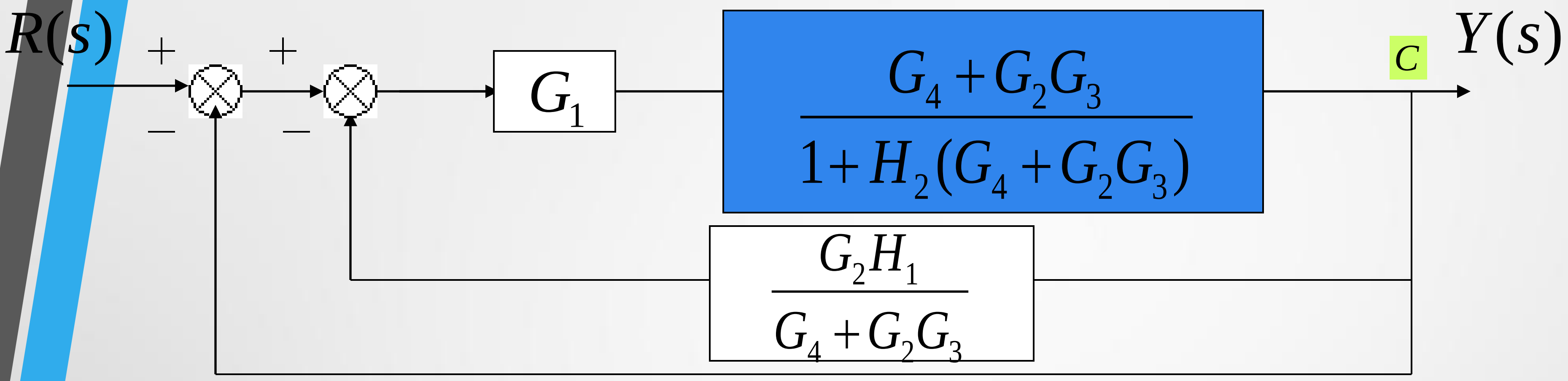




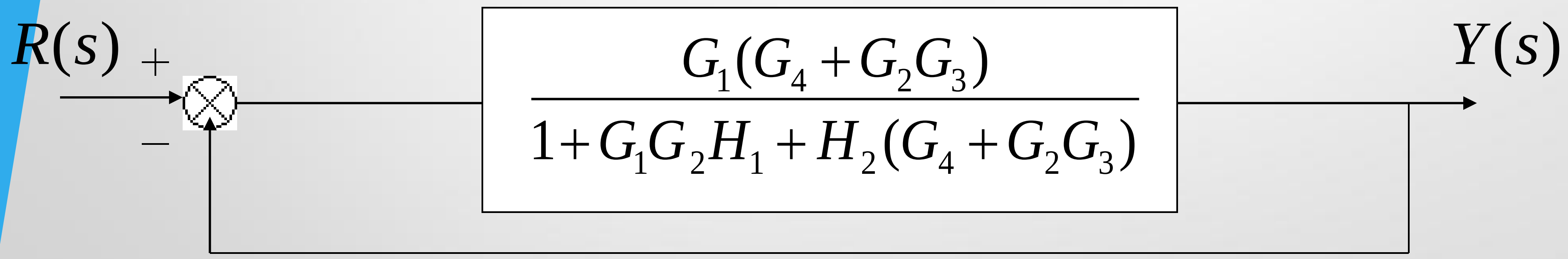
3. Moving pickoff point B behind block $G_4 + G_2G_3$



4. Eliminate loop III

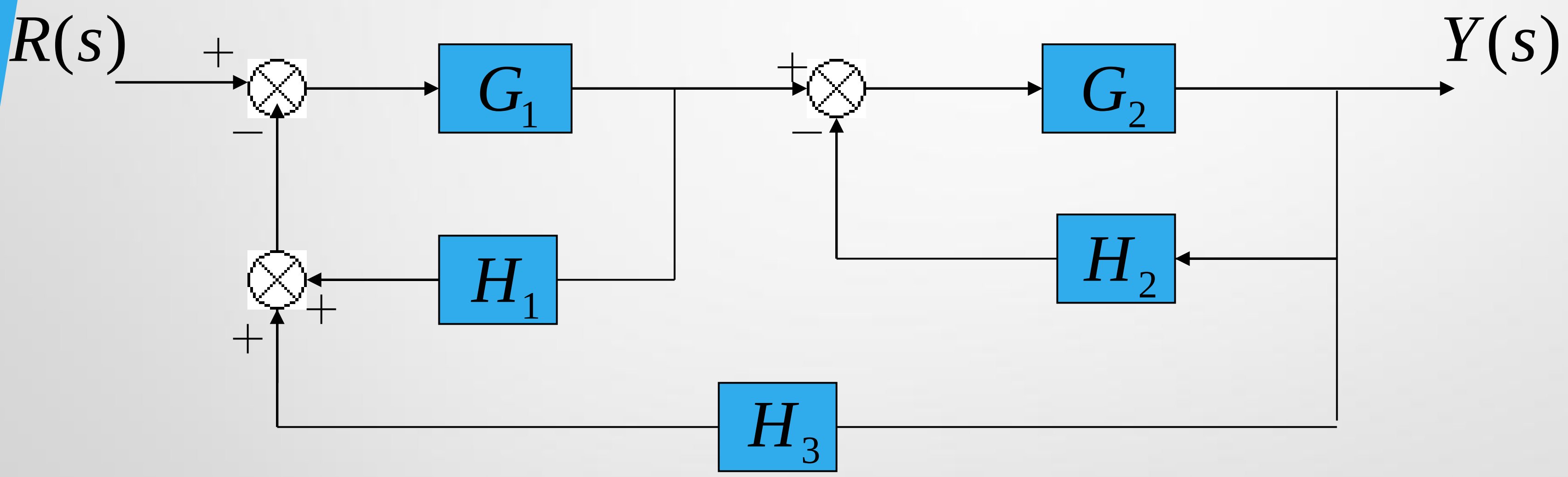


Using rule 6



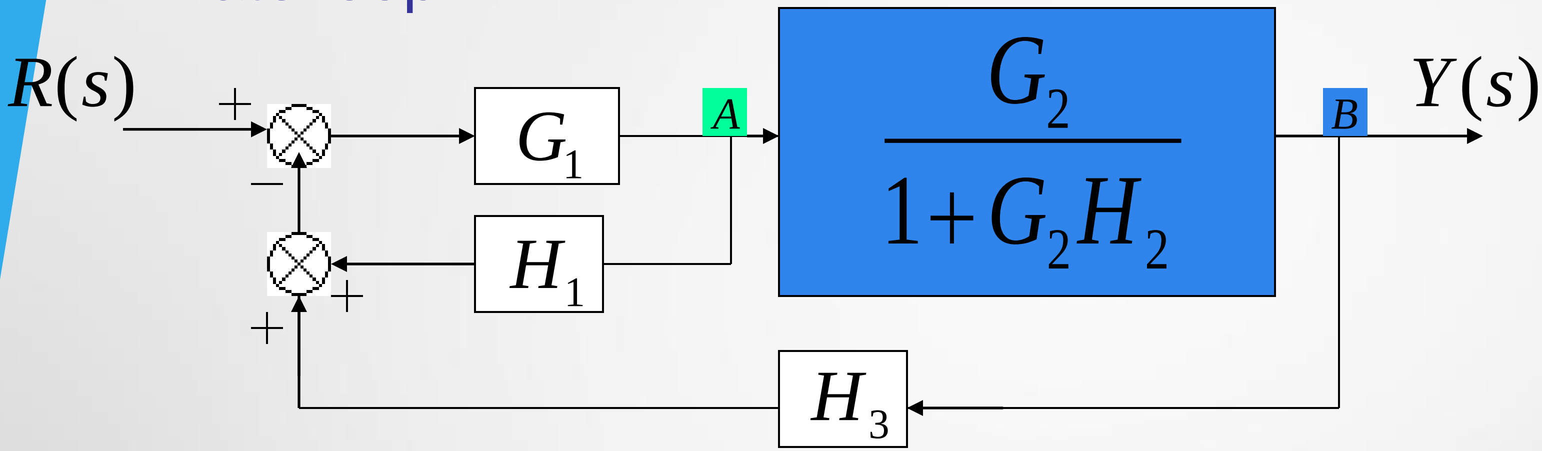
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1+H_2(G_4+G_2G_3)+G_1(G_4+G_2G_3)}$$

(b)

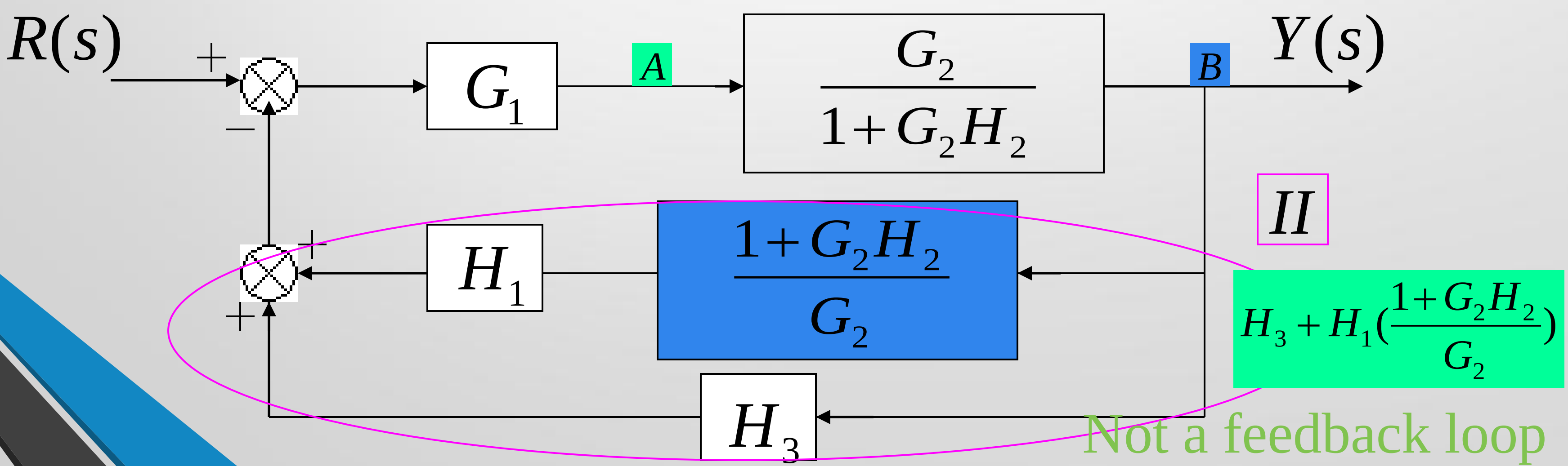


Solution:

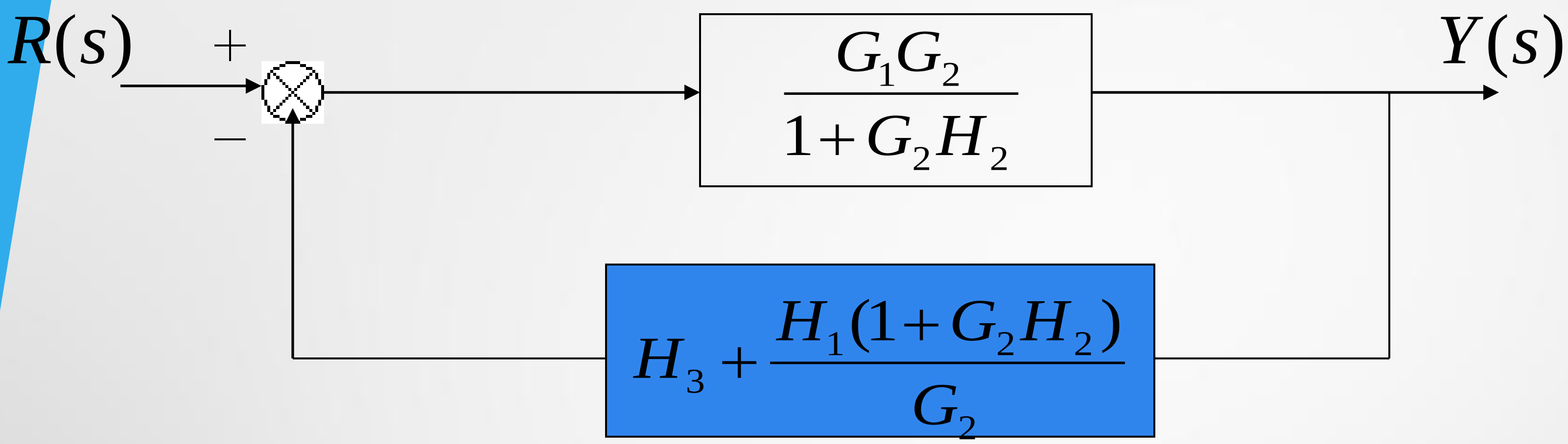
1. Eliminate loop I



2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



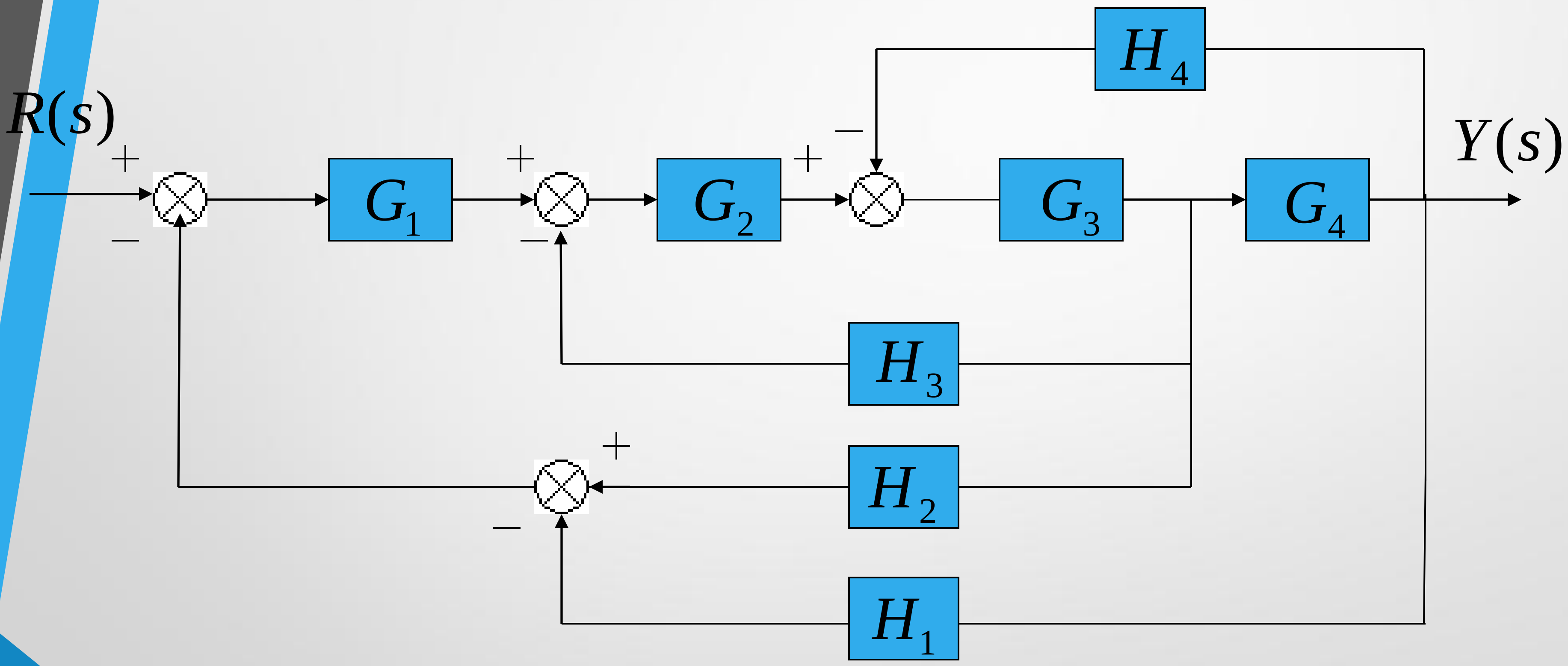
3. Eliminate loop II



Using rule 6

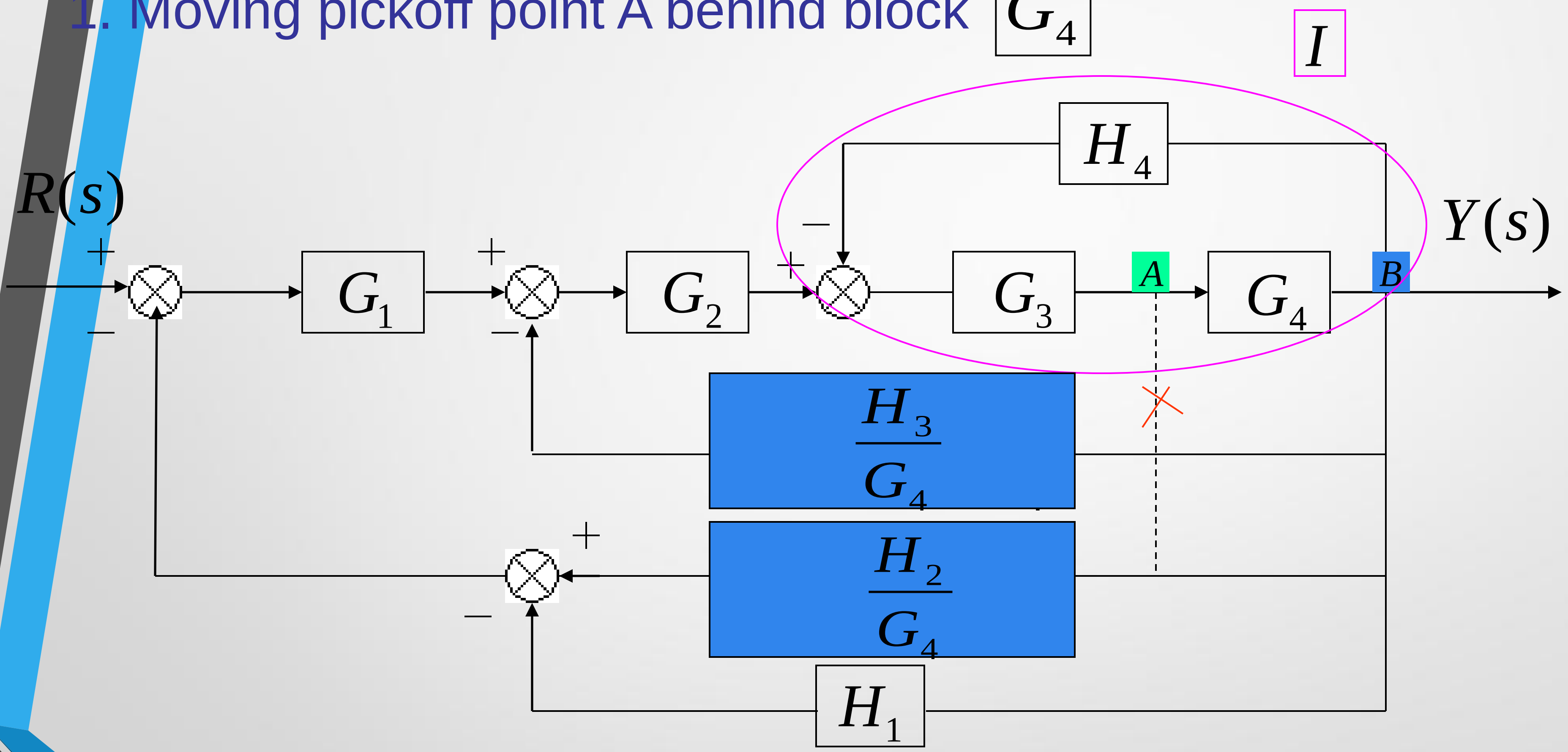
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_3+G_1H_1+G_1G_2H_1H_2}$$

(c)

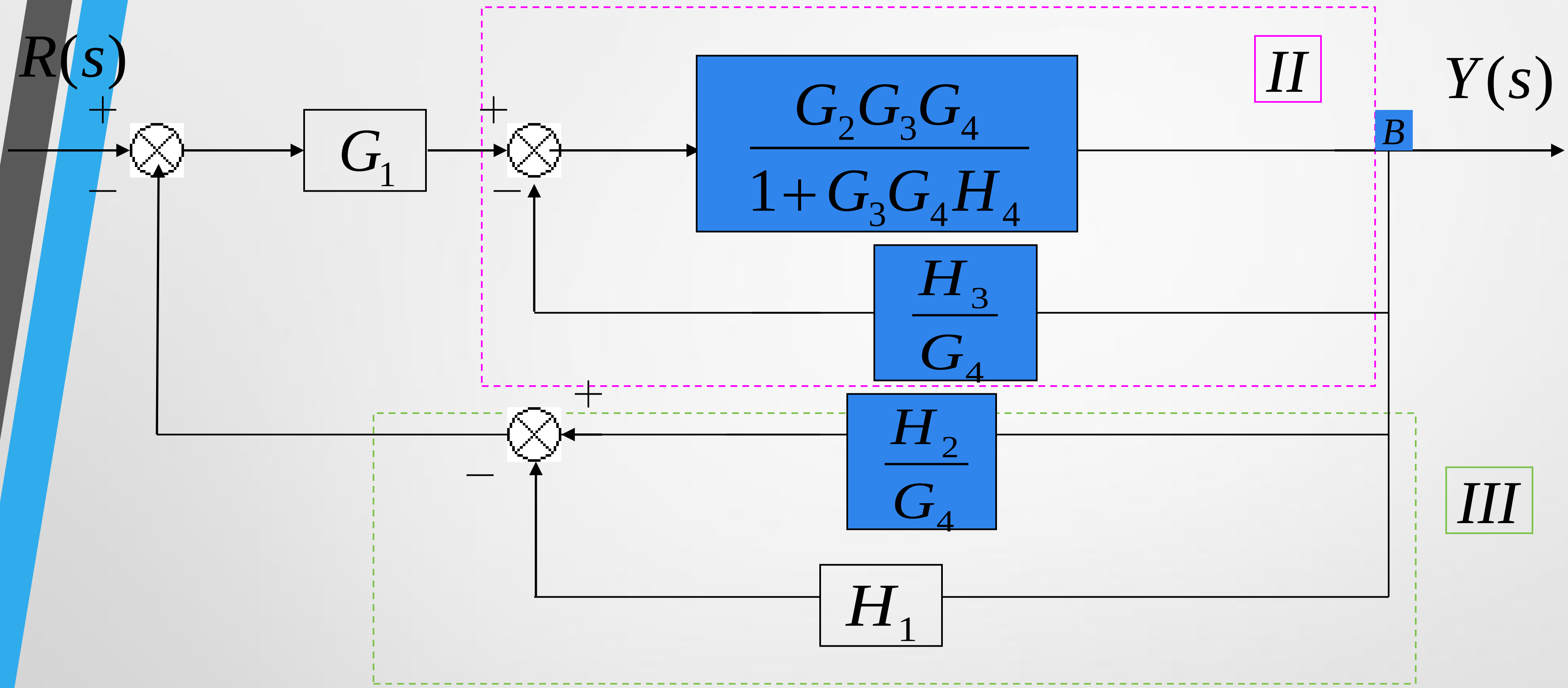


Solution:

1. Moving pickoff point A behind block G_4



2. Eliminate loop I and Simplify



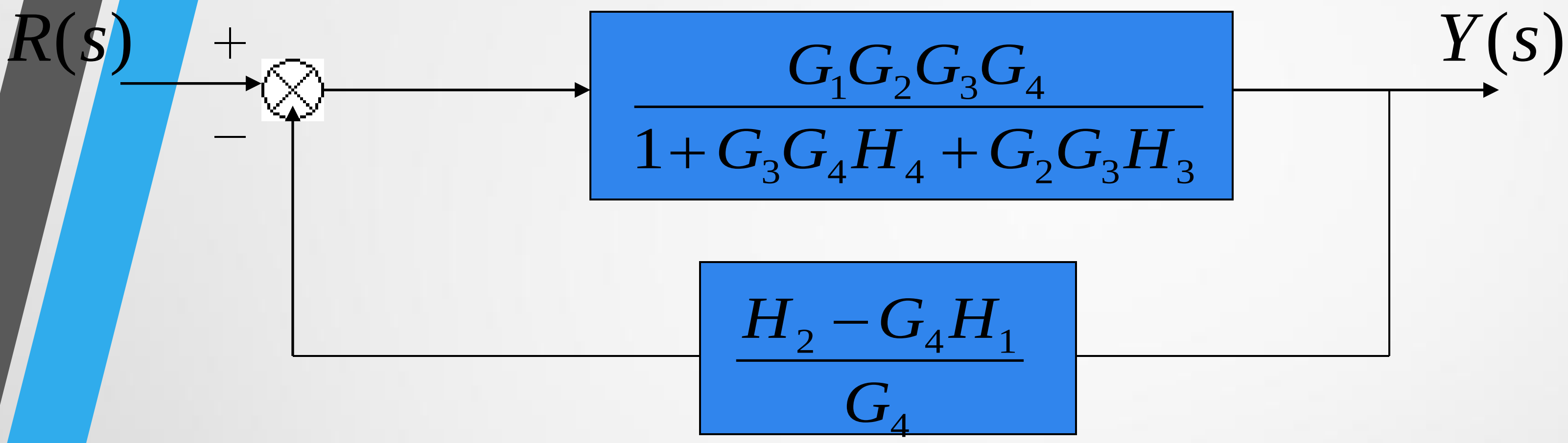
II ⟶ feedback

$$\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III ⟶ Not feedback

$$\frac{H_2-G_4H_1}{G_4}$$

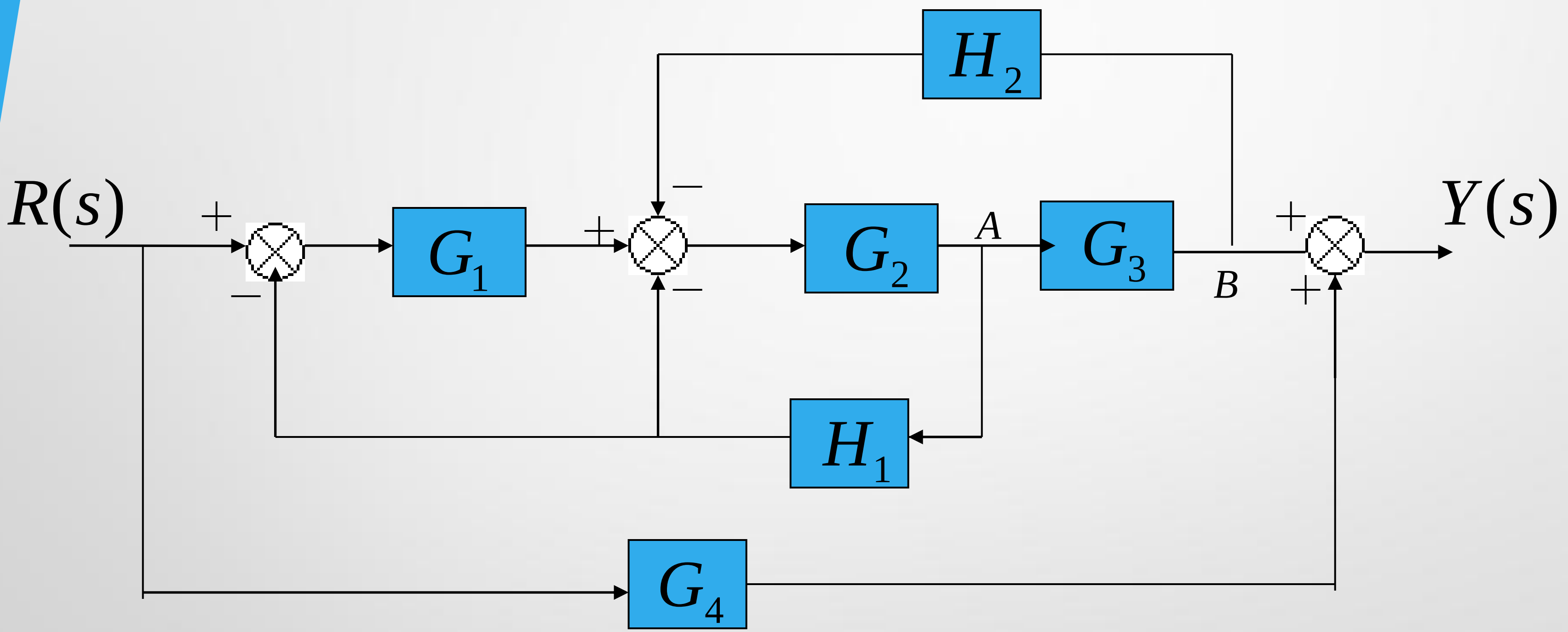
3. Eliminate loop II & IIII



Using rule 6

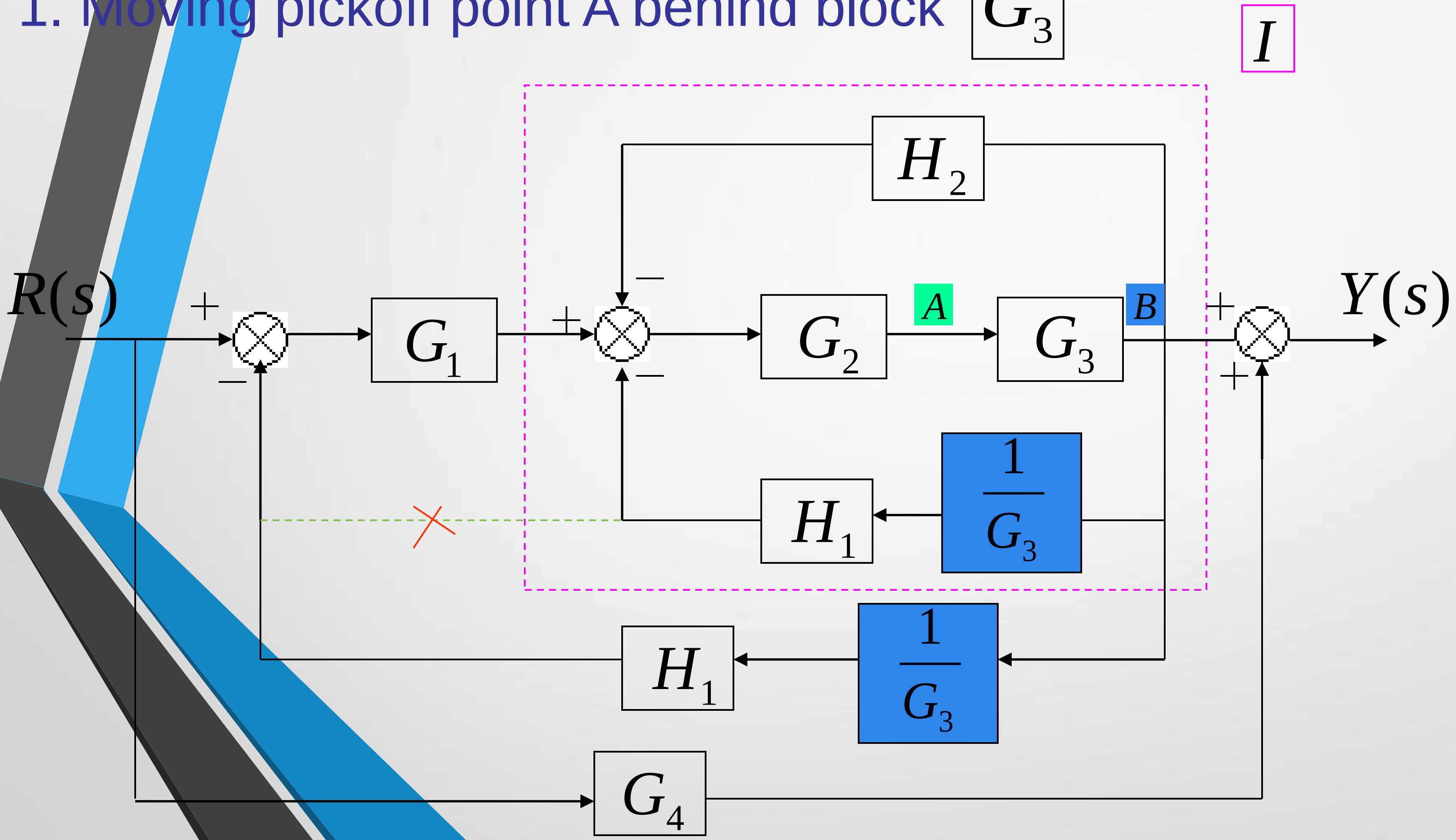
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

(d)

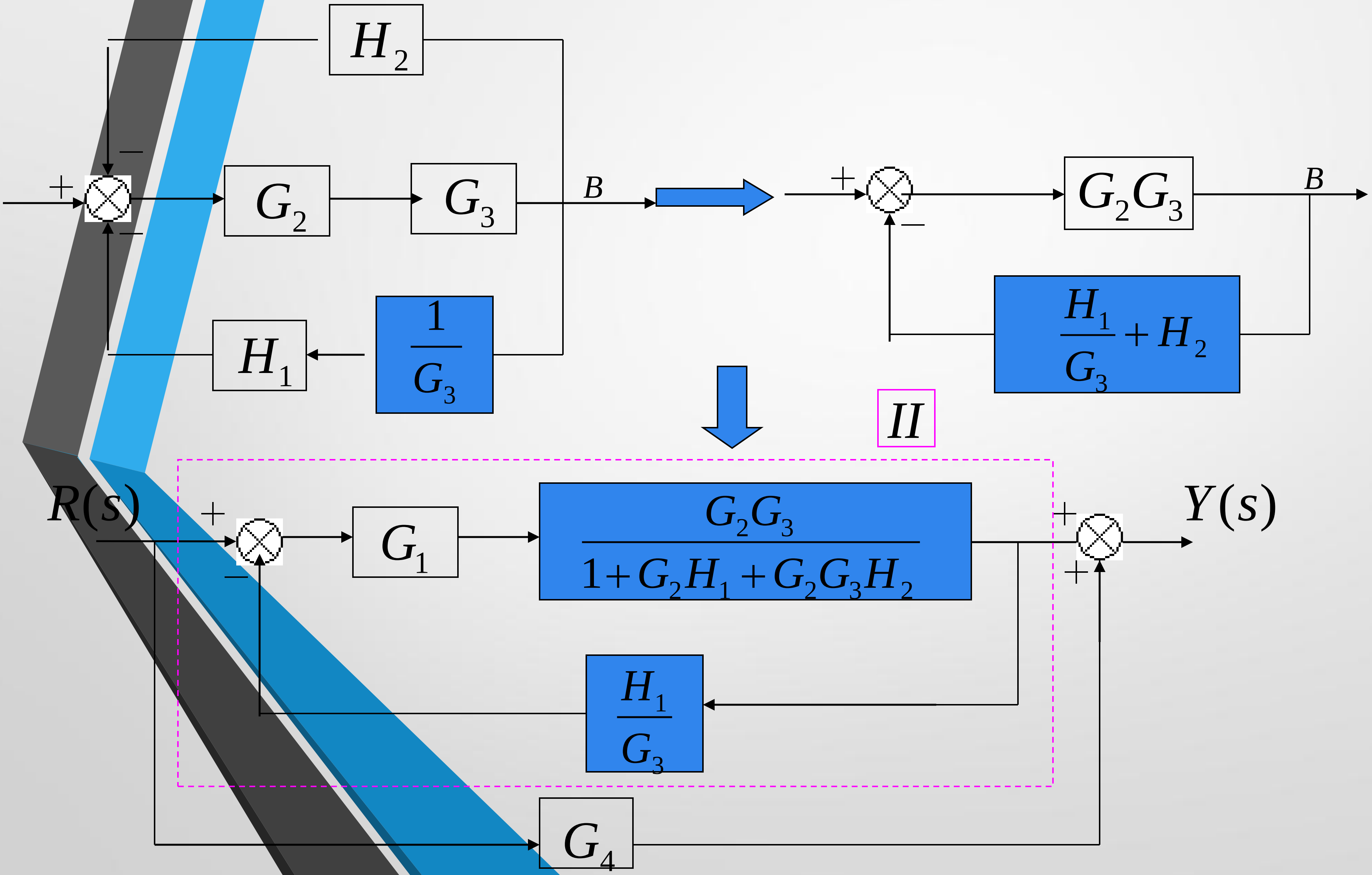


Solution:

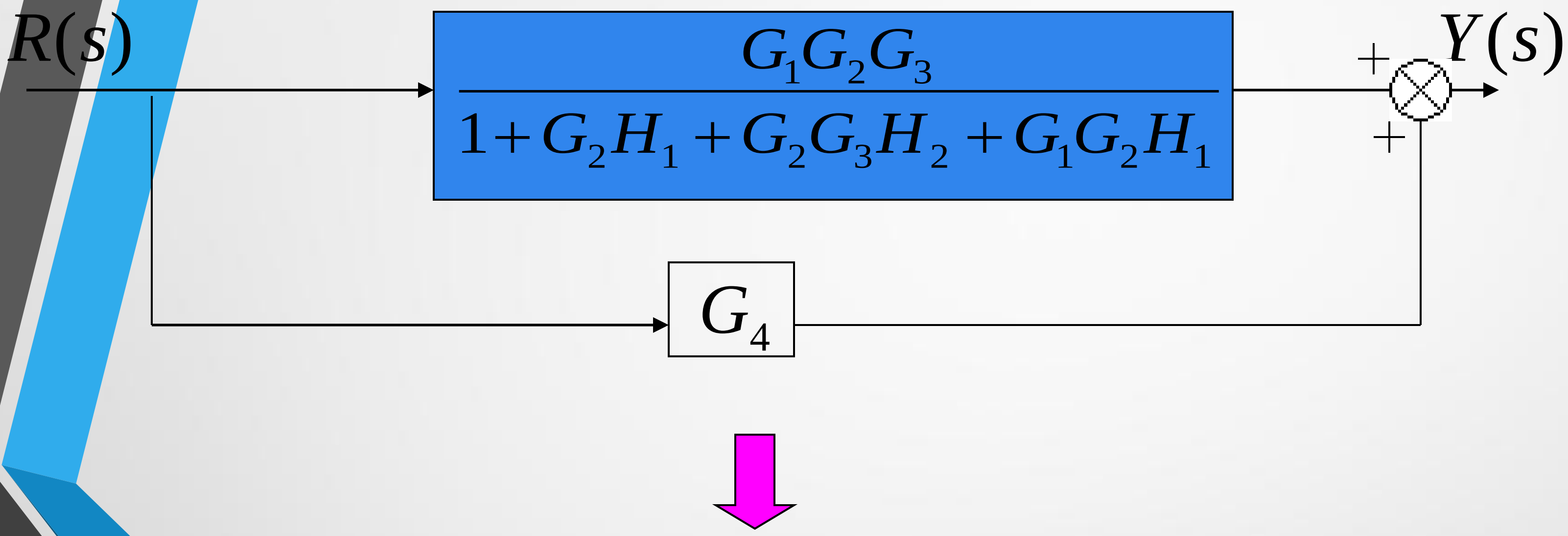
1. Moving pickoff point A behind block G_3



2. Eliminate loop I & Simplify



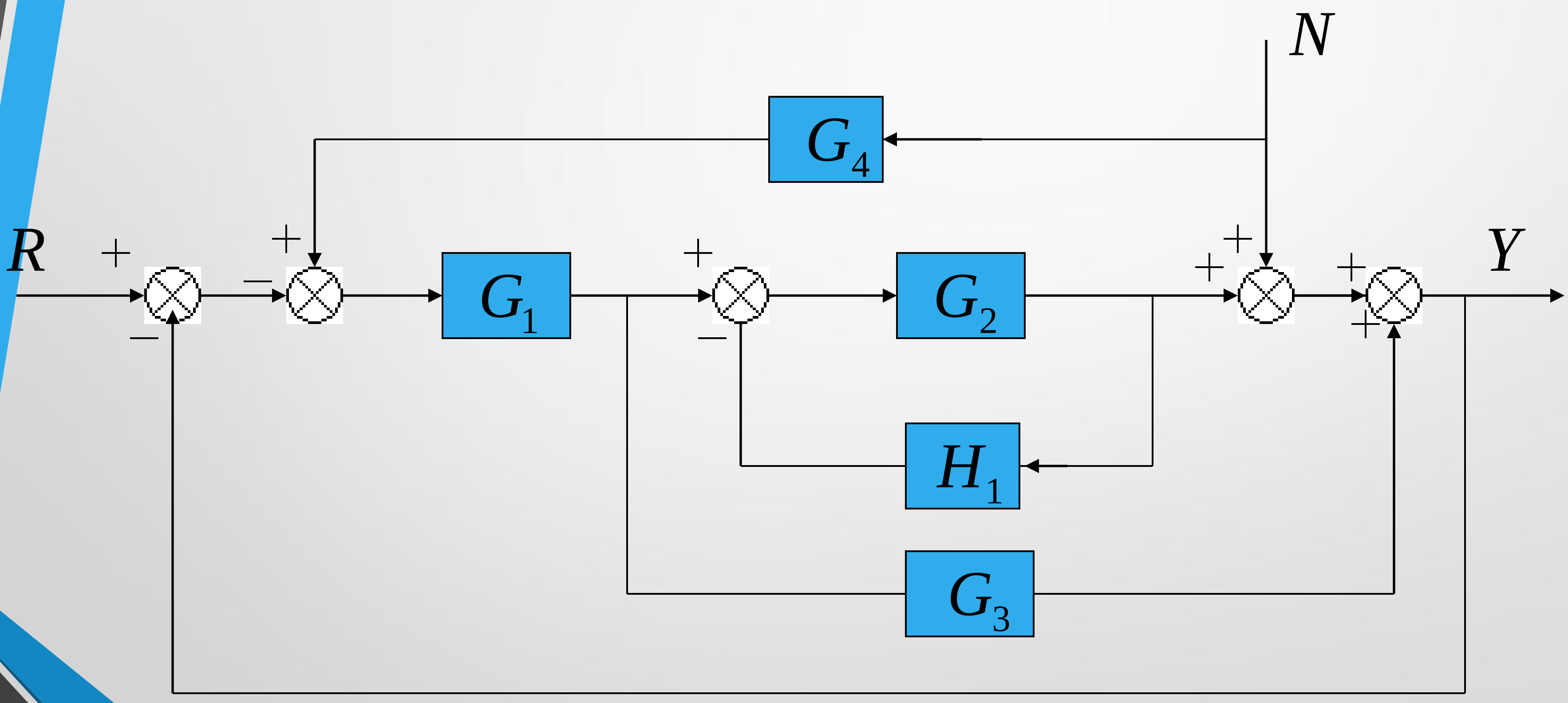
3. Eliminate loop II



$$T(s) = \frac{Y(s)}{R(s)} = G_4 + \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 H_1}$$

Example 2

Determine the effect of R and N on Y in the following diagram



In this linear system, the output Y contains two parts, one part is related to R and the other is caused by N:

$$Y = Y_1 + Y_2 = T_1 R + T_2 N$$

If we set N=0, then we can get Y1:

$$Y_1 = Y_{N=0} = T_1 R$$

The same, we set R=0 and Y2 is also obtained:

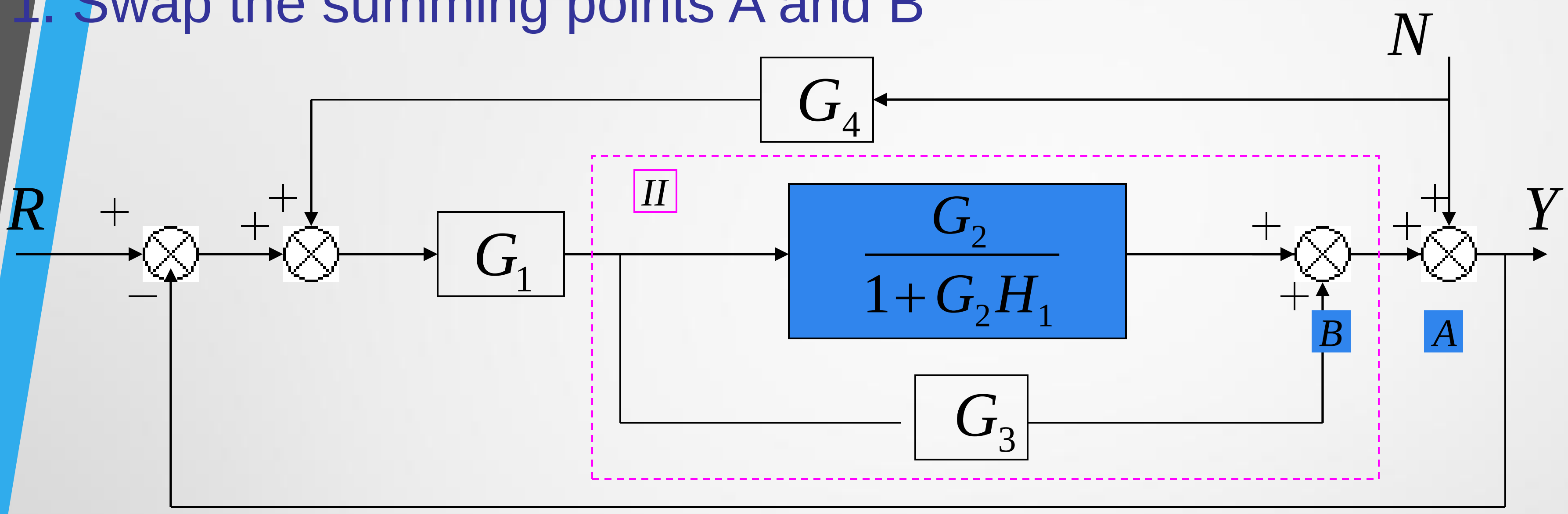
$$Y_2 = Y_{R=0} = T_2 N$$

Thus, the output Y is given as follows:

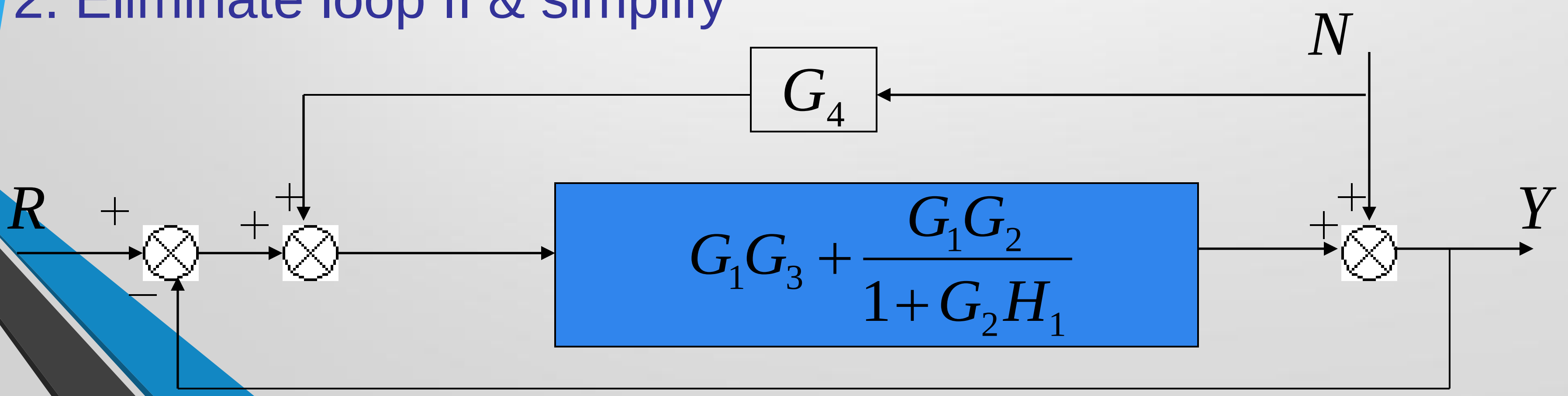
$$Y = Y_1 + Y_2 = Y_{N=0} + Y_{R=0}$$

Solution:

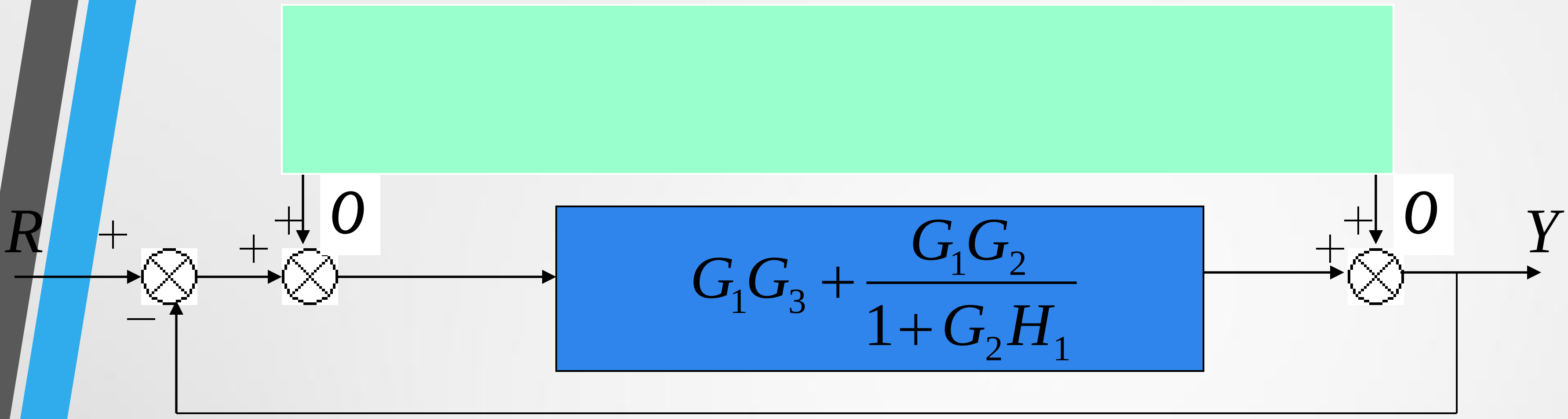
1. Swap the summing points A and B



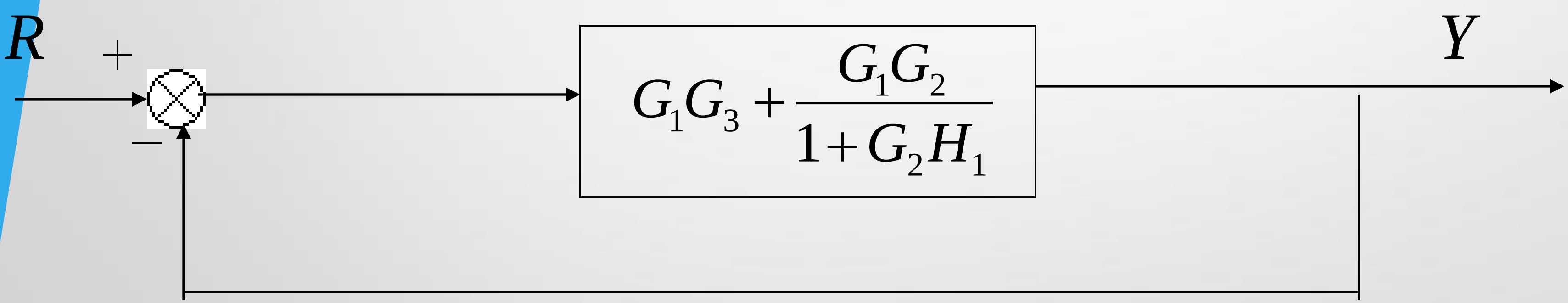
2. Eliminate loop II & simplify



Rewrite the diagram:



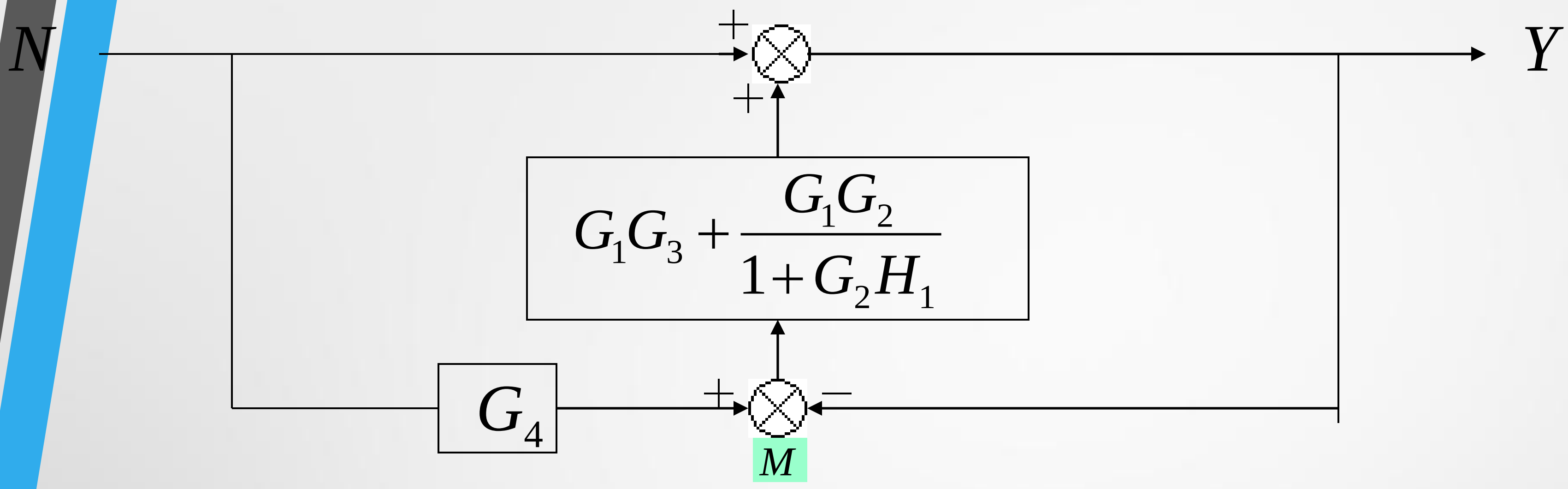
3. Let N=0



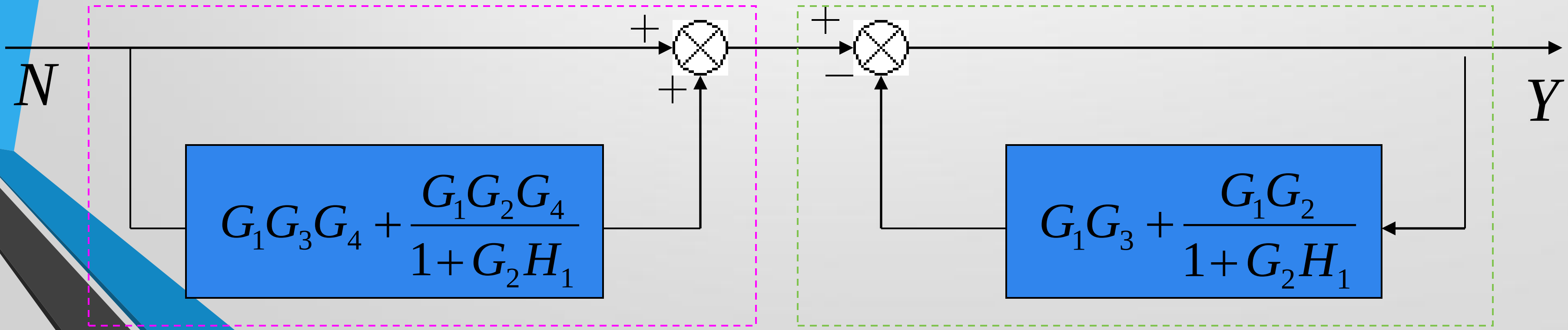
We can easily get Y_1

$$Y_1 = \frac{G_1G_2 + G_1G_3 + G_1G_2G_3H_1}{1 + G_2H_1 + G_1G_2 + G_1G_3 + G_1G_2G_3H_1} R$$

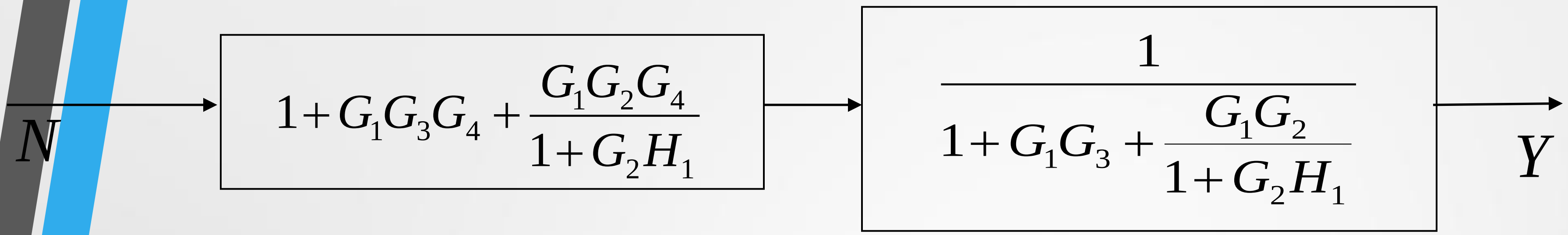
4. Let R=0, we can get:



5. Break down the summing point M:



6. Eliminate above loops:



$$Y_2=\frac{1+G_2H_1+G_1G_2G_4+G_1G_3G_4+G_1G_2G_3G_4H_1}{1+G_2H_1+G_1G_2+G_1G_3+G_1G_2G_3H_1}N$$

7. According to the principle of superposition, Y_1 and Y_2 can be combined together, So:

$$Y=Y_1+Y_2$$

$$=\frac{1}{1+G_2H_1+G_1G_2+G_1G_3+G_1G_2G_3H_1}[(G_1G_2+G_1G_3+G_1G_2G_3H_1)R$$

$$+(1+G_2H_1+G_1G_2G_4+G_1G_3G_4+G_1G_2G_3G_4H_1)N]$$

End