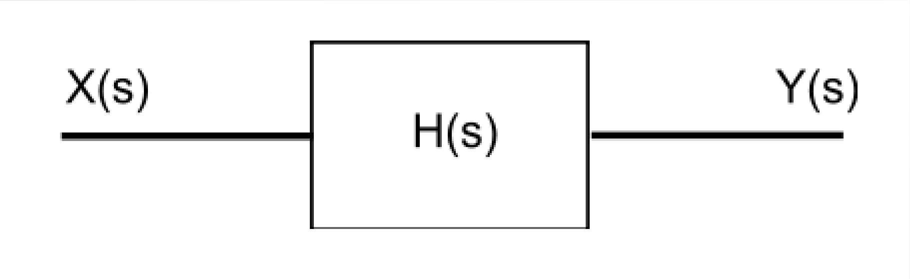
Control Systems

Lecture: 2

Mathematical Modelling

Definition of Transfer Function

- **Transfer Function reveals how the circuit modifies the input amplitude in creating output amplitude.**
- **Therefore, transfer function describes how the circuit processes the input to produce output.**



$$H(s) = \frac{Y(s)}{X(s)}$$

Transfer Function is the ratio of the output of a system to the input of a system, in the Laplace domain considering its initial conditions to be zero.

3

Frequency Domain

Resistor	$V(s) = RI(s)$	$V = RI$	R
Inductor	$V(s) = sLI(s)$	$V = sLI$	Ls
Capacitor	$V(s) = \frac{1}{sC}I(s)$	$V = \frac{1}{sC}I$	$\frac{1}{sC}$

$$v_R(t) = Ri(t) \qquad V_R(s) = RI(s)$$

$$v_L(t) = L\frac{di}{dt} \qquad V_L(s) = sLI(s)$$

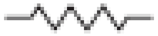
$$v_C(t) = \frac{1}{C}\int idt \qquad V_C(s) = \frac{1}{sC}I(s)$$

Time constants:

$$T_{RC} = RC = \frac{V}{A}\frac{Q}{V} = \frac{VQ}{\frac{Q}{\sec}V} = \sec$$

$$T_{RL} = \frac{L}{R} = \frac{\frac{\frac{V}{A}}{\sec}}{\frac{V}{A}} = \sec$$

Frequency Domain

Resistor	$V(s) = RI(s)$	$V = RI$	R 
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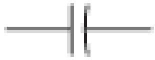
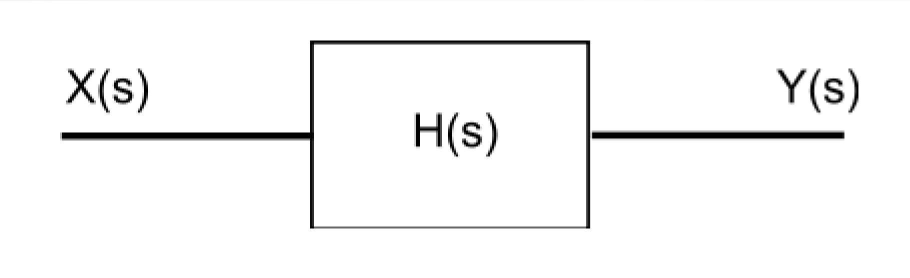
Capacitor	$V(s) = \frac{1}{sC}I(s)$	$V = \frac{1}{sC}I$	$\frac{1}{sC}$ 

Definition of Transfer Function

Transfer Function is the ratio of the output of a system to the input of a system, in the Laplace domain considering its initial conditions to be zero.



$$H(s) = \frac{Y(s)}{X(s)}$$

Frequency Domain

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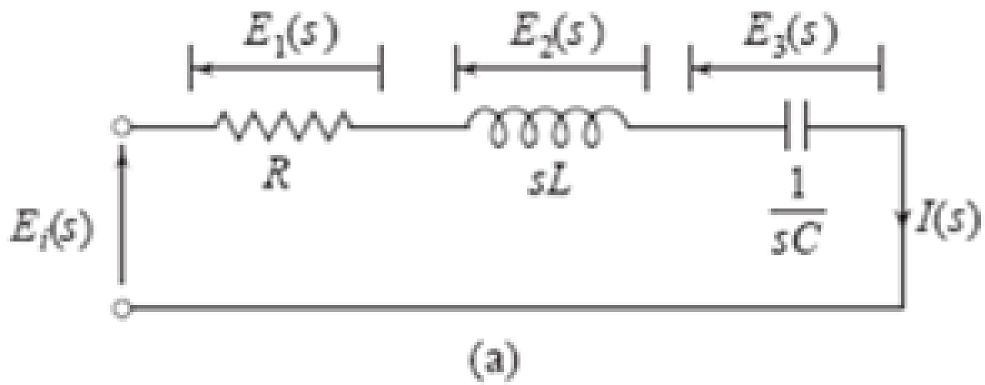
$$\frac{V(s)}{I(s)} = R$$

$$\frac{V(s)}{I(s)} = sL$$

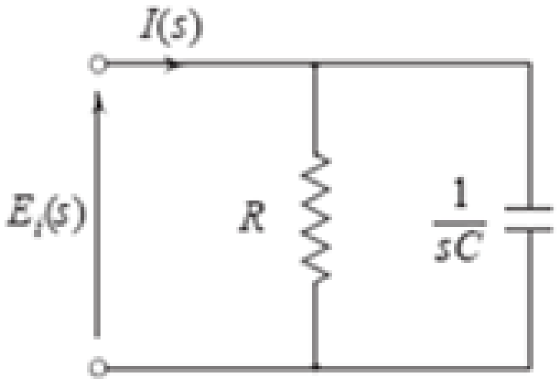
$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

Impedances in series



$$Z(s)_{eq} = \frac{Ei(s)}{I(s)} = R + \frac{1}{sC} + Ls$$

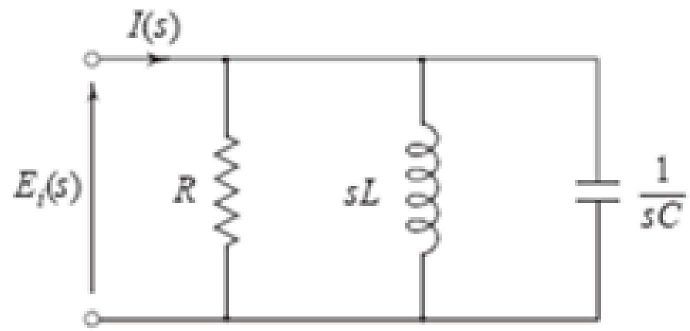
Impedances in parallel



(b)

$$Z(s)_{eq} = \frac{Ei(s)}{I(s)} = \frac{R}{C\left(R + \frac{1}{Cs}\right)s}$$

Impedance Approach



(c)

$$\frac{1}{Z(s)_{eq}} = Y(s)_{eq} = \frac{I(s)}{Ei(s)} = \frac{1}{R} + Cs + \frac{1}{Ls} \qquad \text{or}$$

$$Z(s)_{eq} = \frac{Ei(s)}{I(s)} = \frac{LR}{C\left(R + \frac{1}{Cs}\right)\left(\frac{R}{C\left(R + \frac{1}{Cs}\right)s} + Ls\right)}$$

Derivation of Transfer Function

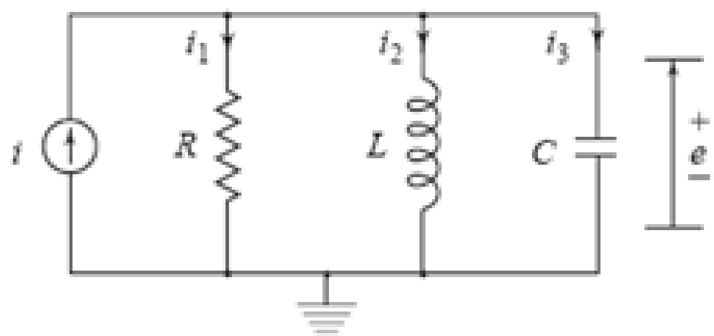
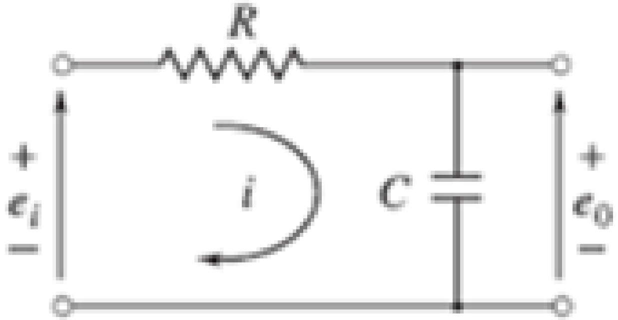


Fig. 2.28 *An RLC circuit*

$$H(s) = \frac{\dfrac{L}{C\left(\dfrac{1}{Cs} + Ls\right)}}{R + \dfrac{L}{C\left(\dfrac{1}{Cs} + Ls\right)}}$$

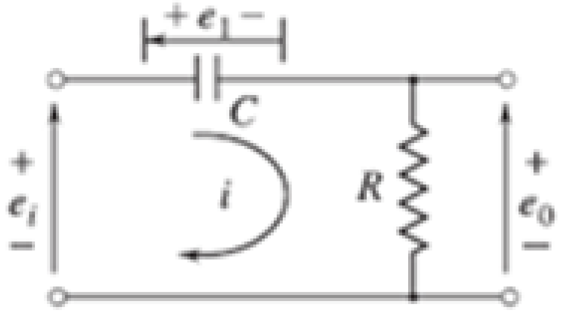
Derivation of Transfer Function



(a)

$$H(s) = \frac{1}{C\left(R + \frac{1}{Cs}\right)s}$$

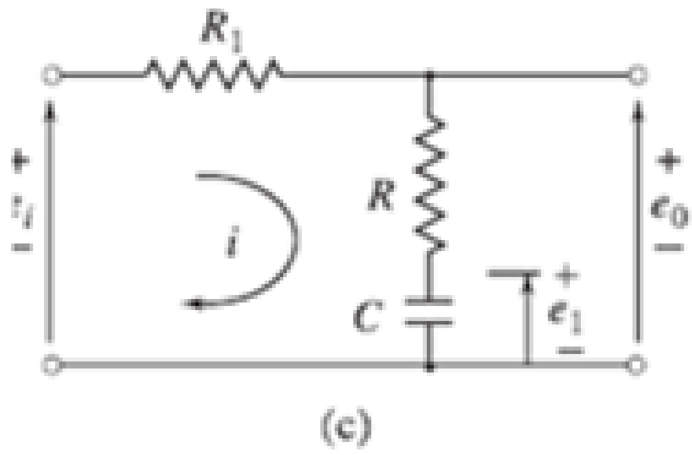
Derivation of Transfer Function



(b)

$$H(s) = \frac{R}{R + \frac{1}{Cs}}$$

Derivation of Transfer Function



$$H(s) = \frac{R + \frac{1}{Cs}}{R + R1 + \frac{1}{Cs}}$$