SYSTEM SIMULATION AND MODELLING

LECTURE 3

Section D

TOPIC COVERED: Tests for Random Numbers, Chi – Square Test

TESTS OF HYPOTHESIS (HYPOTHESIS TESTING)

Hypothesis testing or **significance testing** is a method for testing a claim or hypothesis about a parameter in a population, using data measured in a sample.

There are following steps of hypothesis testing:

Step 1: State the hypothesis. A researcher states a null hypothesis about a value in the population (H_0) and an alternative hypothesis that contradicts the null hypothesis.

Step 2: Set the criteria for a decision. A criterion is set upon which a researcher will decide whether to retain or reject the value stated in the null hypothesis.

A sample is selected from the population and a sample mean is measured

Step 3: Compute the test statistic. This will produce a value that can be compared to the criterion that was set before the sample was selected.

Step 4: Make a decision. If the probability of obtaining a sample mean is less than 5% when the null is true, then reject the null hypothesis.

If the probability of obtaining a sample mean is greater than 5% when the null is true, then retain the null hypothesis.

GOODNESS OF FIT TESTS

The goodness of fit of a statistical model describes how well it fits a set of observations. Measures of goodness of fit typically summarize the discrepancy between observed values and the value expected under the model in question.

A goodness of fit test is a statistical hypothesis test which provides helpful guidance for evaluating the suitability of a potential input model.

This is used to assess formally whether the observations $X_1, X_2, \ldots, X_n$ are an independent sample from a particular distribution with distribution function $\hat{F}$.

The null and the alternate hypothesis may be set as follows:

H_0 : The observation follow a specific statistical distribution, for example, Normal (mean = 10, standard deviation = 1)

H_1 : The observation do not follow the distribution specified in H_0.

A hypothesis test sufferers from two types of error as given in Table 4.3.

Table 4.3

S.No.	*Decision*	*H_0 is true*	*H_0 is false*
1.	Fail to reject H_0	No error	Make type II error
2.	Reject H_0	Make type I error	No error

Before proceeding with a discussion of several specific goodness of fit tests, we should understand the formal structure and properties of these tests.

NOTE:

Failure to reject H_0 should not be interpreted as "accepting H_0 as being true."

Goodness of fit tests are not very useful for small to moderate sample sizes. These tests are unlikely to reject any candidate distribution for small sample size. On the other hand, if sample size is very then these tests will almost always reject H_0.

NOTE:

The sensitivity of the test also depends on the sample size.

Chi-square Test

The oldest goodness of fit hypothesis test is the chi-square test. This test is valid for large sample sizes and for both discrete and continuous distributional assumptions when parameters are estimated by maximum likelihood. This test may be viewed as a formal comparison of the histogram of the data to the shape of the candidate density or probability mass function. There are following steps of chi-square test.

Step 1: We must first divide the entire range of the observed data into k adjacent intervals $[a_0, a_1]$, $[a_1, a_2]$, $[a_2, a_3]$, ... $[a_{k-1}, a_k]$.

Step 2: Calculate the number of observations in each intervals. O_i is the number of observations in the i_{th} interval $[a_{i-1}, a_i]$.

Step 3: Calculate the expected number of observations in the i_{th} interval (E_i). The expected number (expected frequency) for each class interval is calculated as $E_i = np_i$ where p_i is the theoretical hypothesized probability associated with the i_{th} class interval.

- For a continuous distribution

$$p_i = \int_{a_{i-1}}^{a_i} f(x)\,dx \qquad \ldots(4.22)$$

- For a discrete distribution

$$p_i = \sum_{a_{i-1} \le x_j < a_i} \hat{p}(x_j) \qquad \text{...(4.23)}$$

where $\hat{p}$ is the mass function of the fitted distribution.

Step 4: Finally, the test statistic is given by

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \qquad \text{...(4.24)}$$

The test statistic follows chi-square distribution with $k - s - 1$ degrees of freedom, where k = number of intervals used and s = number of parameters estimated from the data.

Note:

A large value of χ^2 indicates a poor fit. Therefore, we reject H_0 if χ^2 is too large. We would expect χ^2 to be small if the fit were good.