# SYSTEM SIMULATION AND MODELLING

# LECTURE 5

Section C

TOPIC COVERED: Steady State Behavior of finite Population Models.

# M/M/1/GD/∞/∞ System

Corresponding birth and death coefficients are as follows:

$$\lambda_k = \lambda \quad k = 0, 1, 2, \ldots \text{ average inter-arrival time} = \frac{1}{\lambda}$$

$$\mu_k = \mu \quad k = 0, 1, 2, \ldots \text{ average service time} = \frac{1}{\mu}.$$

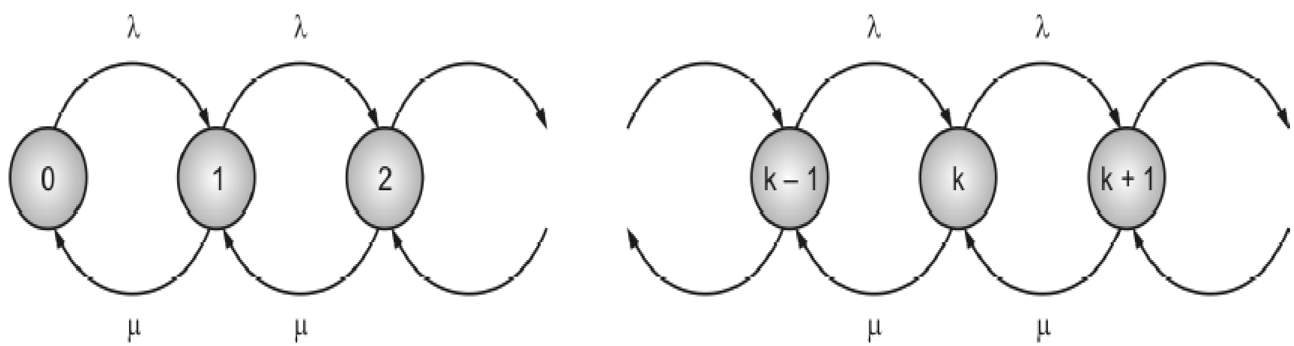


**Fig. 5.10** State-transition diagram for M/M/1 system.

The following relationships hold in steady state:

$$p_0 \lambda = p_1 \mu \quad \text{or} \; - \rho p_0 + p_1 = 0,$$

Server utilization

$$\rho = \left(\frac{\lambda}{\mu}\right) \quad \text{for} \quad k = 0_s$$

$$-p_k(\lambda + \mu) + p_{k+1}\,\mu + p_{k-1}\,\lambda = 0, \text{ for } k > 0.$$

# Multiple Parallel Servers with Finite System Capacity M/M/c/GD/N/∞ System

$c$ = Number-of-servers

$N$ = System capacity, $N > c$

$p_k = 0; \quad k \geq N + 1.$

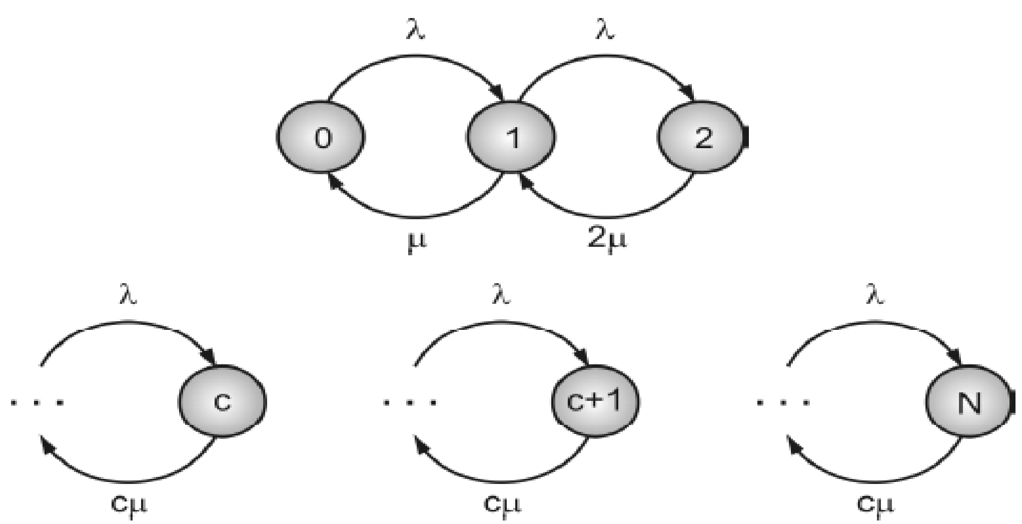


**Fig. 5.14** State transition diagram

The following relationships hold:

$$\lambda_k = 1 \qquad 0 \le k \le N$$
$$\lambda_k = 0 \qquad k \ge N$$
$$\mu_k = k\mu \qquad 0 \le k \le c$$
$$\mu_k = c\text{m} \qquad c \le k \le N$$

Using $p = \dfrac{\lambda}{\mu}$

In steady state,

$$p_k = \left(\frac{\rho^k}{k!}\right) p_0, \qquad 0 \le k \le c$$

$$p_k = \left(\frac{\rho^k}{c^{k-c}\, c!}\right) p_0, \qquad c \le k \le N$$

$$p_0 = \left[\sum_{k=0}^{c-1} \frac{\rho^k}{k!} + \frac{\rho^c\left[1-\left(\frac{\rho}{c}\right)^{N-c+1}\right]}{c!\left(1-\frac{\rho}{c}\right)}\right]^{-1}, \qquad \frac{\rho}{c} \ne 1$$

$$p_0 = \left[\sum_{k=0}^{c-1} \frac{\rho^k}{k!} + \frac{\rho^c}{c!}(N-c+1)\right]^{-1}, \qquad \frac{\rho}{c} = 1.$$

Utilization factor $\rho/c$ need not be $< 1$.