SYSTEM SIMULATION AND MODELLING

LECTURE 4

Section C

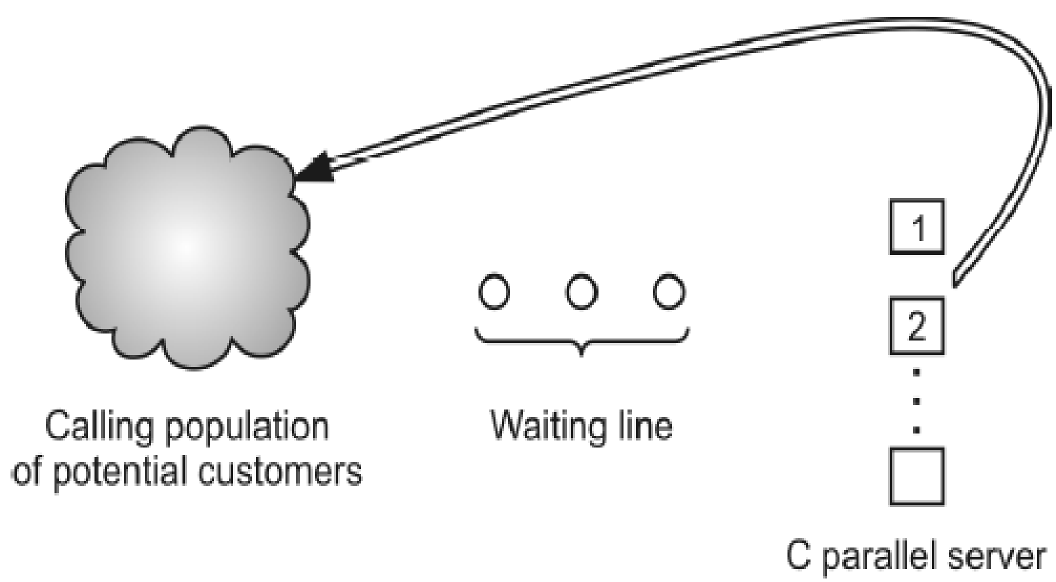
TOPIC COVERED: Steady State Behavior of infinite Population Markovian Models,

Queuing notation

S.No.	Queueing notation	
1.	P_n	Steady-state probability of having n customers in system
2.	$P_n(t)$	Probability of n customers in system at time t
3.	λ	Arrival rate
4.	λ_e	Effective arrival rate
5.	μ	Service rate of one server
6.	ρ	Server utilization
7.	A_n	Interarrival time between customers $n-1$ and n
8.	S_n	Service time of the nth arriving customer
9.	W_n	Total time spent in system by the nth arriving customer
10.	W_n^Q	Total time spent in the waiting line by customer n
11.	$L(t)$	The number of customers in system at time t
12.	$L_Q(t)$	The number of customers in queue at time t
13.	L	Long-run time-average number of customers in system
14.	L_Q	Long-run time-average number of customers in queue
15.	w	Long-run average time spent in system per customer
16.	w_Q	Long-run average time spent in queue per customer

MULTIPLE SERVER QUEUEING SYSTEM

- Let us assume that there are *C* channels operating in parallel. Each of these channel has an independent and identical exponential service time distribution with mean .
- In queuing theory, a discipline within the mathematic theory of probability, the *M*/*M*/*C* queue(or Erlong-C model) is a multiple-server queuing model.


1
2
Calling population
of potential customers
Waiting line
C parallel server

- The multiserver queueing system is shown in Fig. 5.8. If the number in the system is $n < c$, an arrival will enter an available channel. A queue is built if arrival occurs when $n \geq c$.
- The offered load is defined by λ/μ. If $\lambda \geq C\mu$, the arrival rate is greater than or equal to the maximum service rate of the system; thus, the system cannot handle the load put upon it, and therefore it has no statistical equilibrium.
- If $\lambda > C\mu$, the waiting line grows in length at the rate $(\lambda - C\mu)$ customers per unit time unit, on the average.
- Customers are entering the system at a rate λ per time unit but are leaving the system at a maximum rate of $C\mu$ per time unit.
- The offered load must satisfy $\lambda/\mu < C$, in which case $\lambda/(C\mu) = \rho$, the server utilization, for the *M*/*M*/*C* queue to have statistical equilibrium.

MARKOVIAN QUEUING SYSTEMS

- The basic queuing systems discussed in this section are characterized by birth-death process where the system state can change to an adjacent state only in the next transition. The steady-state solutions for birth-death systems can be derived by changing the birth and the death rate coefficients in the “Product Form” solution for a specific system. The Product Form of solution is described as follows:

- State variable k represents the number in system. For a birth-death system, transition to states $(k - 1)$ and $(k + 1)$ is permitted only from the current state k in the next transition. A birth takes the state to $(k + 1)$ while a death takes it to $(k - 1)$. The state transition diagram is shown in Fig. 5.9.

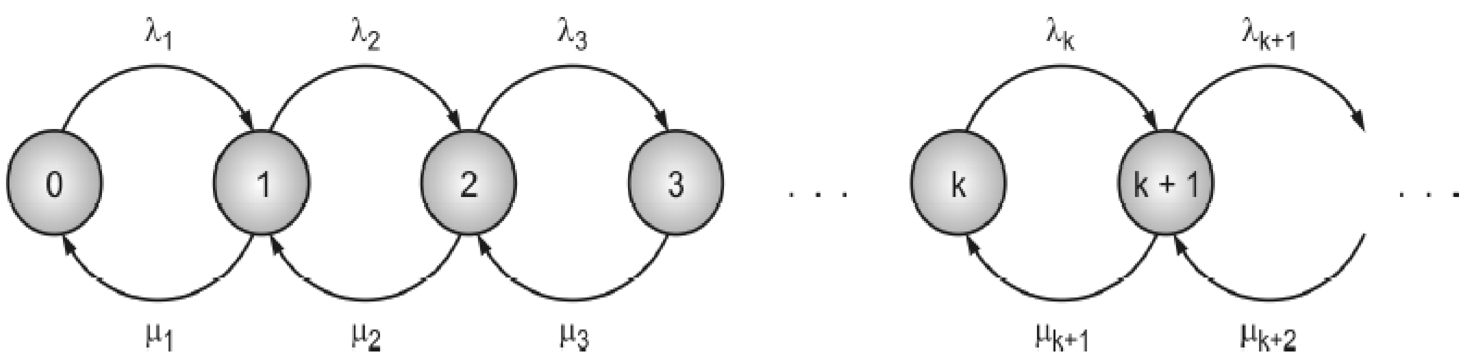


Fig. 5.9 State transion diagram.