Course Name: Analysis and Design of Algorithms

Topics to be covered

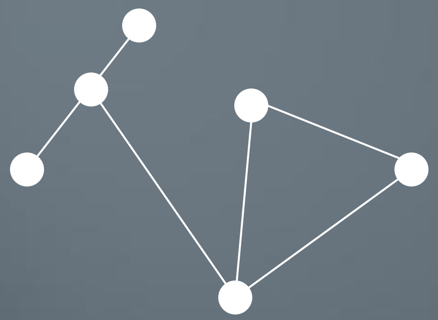
- Review of Graph Theory
- Basic Concepts
- Euler Circuits
- Hamilton Circuits and Algorithms
- Trees and Minimum Spanning Trees

Basic Concepts

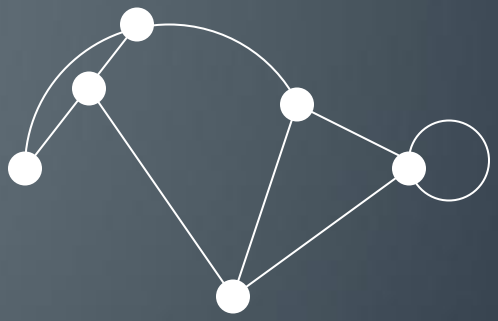
- Graphs
- Walks, Paths, and Circuits
- Complete Graphs and Subgraphs
- Graph Coloring

Simple Graphs

Graphs in which there is no more than one edge between any two vertices and in which no edge goes from vertex to the same vertex are called **simple**.



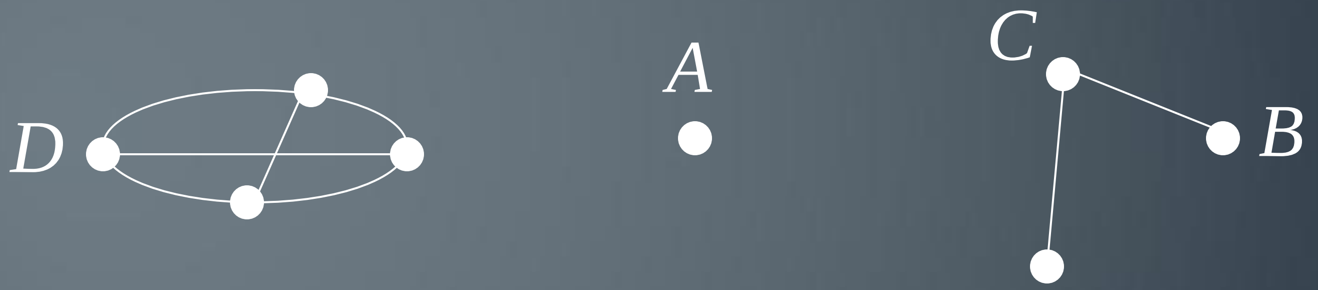
Simple



Not simple

Degree of the Vertex

The number of edges joined to a vertex is called the **degree of the vertex**.



A has degree 0, *B* has degree 1, *C* has degree 2 and *D* has degree 3.

Isomorphic Graphs

Two graphs are **isomorphic** if there is a one-to-one matching between vertices of the two graphs with the property that whenever there is an edge between two vertices of either one of the graphs, there is an edge between the corresponding vertices of the other graph.

Connected and Disconnected Graphs

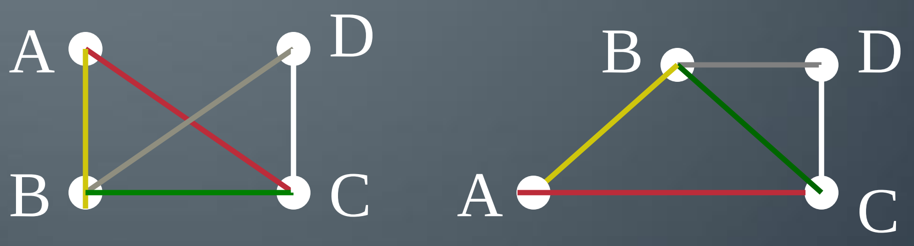
A graph is **connected** if one can move from each vertex of the graph to every other vertex of the graph *along edges of the graph*. If not, the graph is **disconnected**. The connected pieces of a graph are called the **components** of the graph.

Example: Isomorphic Graphs

Are the two graphs isomorphic?

Solution

Yes, and corresponding vertices are labeled below.



Sum of Degrees Theorem

In any graph, the sum of the degrees of the vertices equals twice the number of edges.

Example: Sum of Degrees

A graph has exactly four vertices, each of degree 3. How many edges does this graph have?

Solution

The sum of degrees of the vertices is 12. By the theorem, this number is twice the number of edges, so the number of edges is $12/2 = 6$.

Walk

A **walk** in a graph is a sequence of vertices, each linked to the next vertex by a specified edge of the graph.

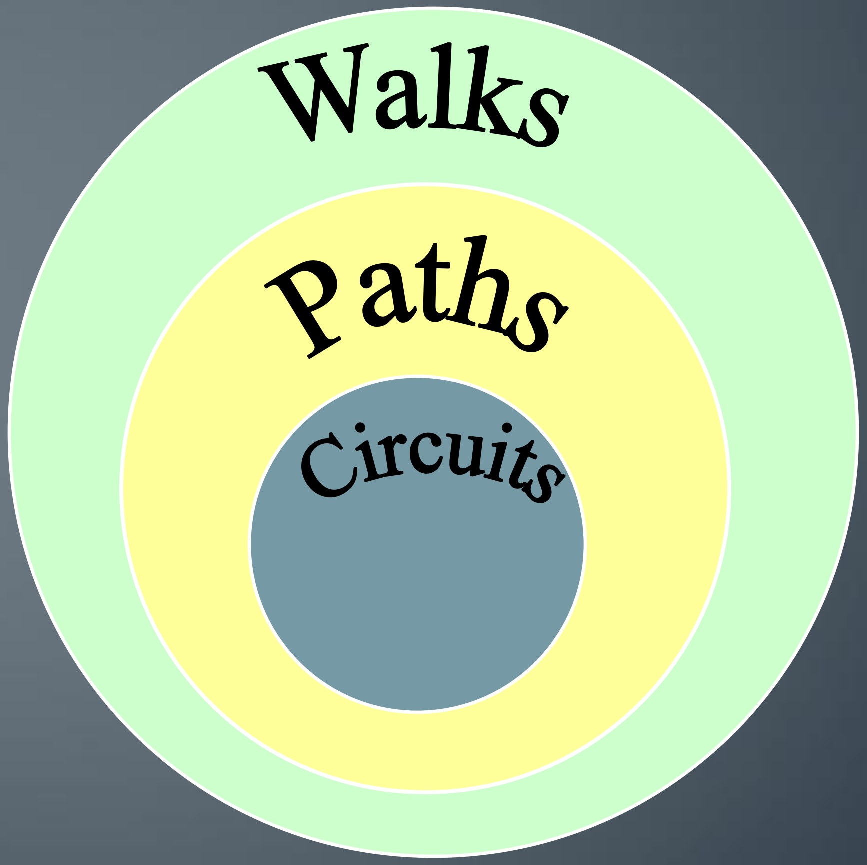
Path

A **path** in a graph is a walk that uses no edge more than once.

Circuit

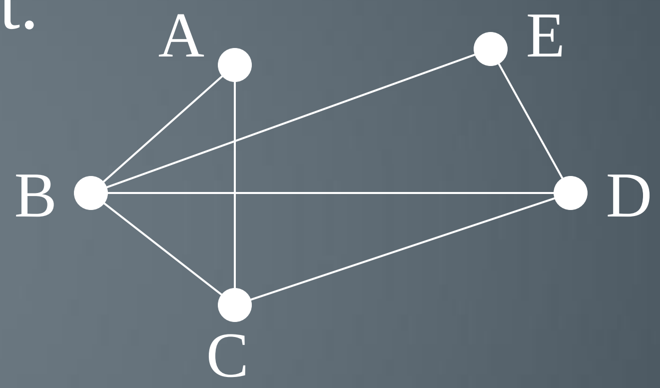
A **circuit** in a graph is a path that begins and ends at the same vertex.

Venn Diagram of Walks, Paths, and Circuits



Example: Classifying Walks

Using the graph, classify each sequence as a walk, a path or a circuit.



a) $E \rightarrow C \rightarrow D \rightarrow E$

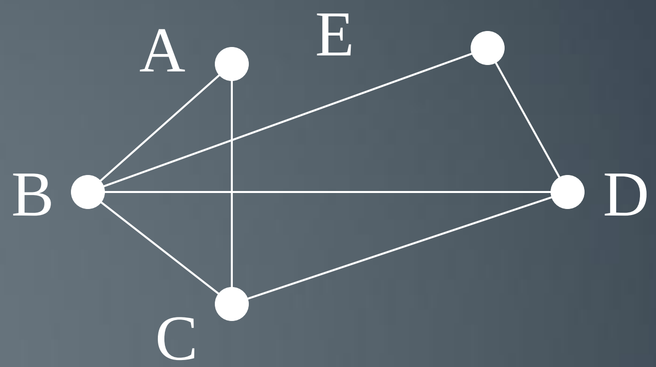
b) $A \rightarrow C \rightarrow D \rightarrow E \rightarrow B \rightarrow A$

c) $B \rightarrow D \rightarrow E \rightarrow B \rightarrow C$

d) $A \rightarrow B \rightarrow C \rightarrow D \rightarrow B \rightarrow A$

Example: Classifying Walks

Solution



Walk	Path	Circuit
a) No	No	No
b) Yes	Yes	Yes
c) Yes	No	No
d) Yes	No	No

Complete Graph

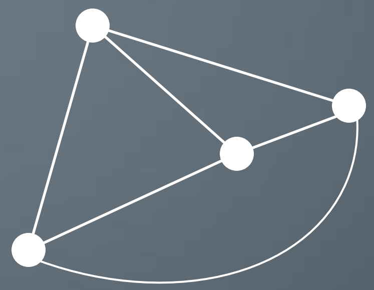
A **complete graph** is a graph in which there is exactly one edge going from each vertex to each other vertex in the graph.

Example: Complete Graph

Draw a complete graph with four vertices.

Solution

Answers may vary, but one solution is shown below.



Subgraph

A graph consisting of some of the vertices of the original graph and some the original edges between those vertices is called a **subgraph**.

Coloring and Chromatic Number

A **coloring** for a graph is a coloring of the vertices in such a way that the vertices joined by an edge have different colors. The **chromatic number** of a graph is the least number of colors needed to make a coloring.

Coloring a Graph

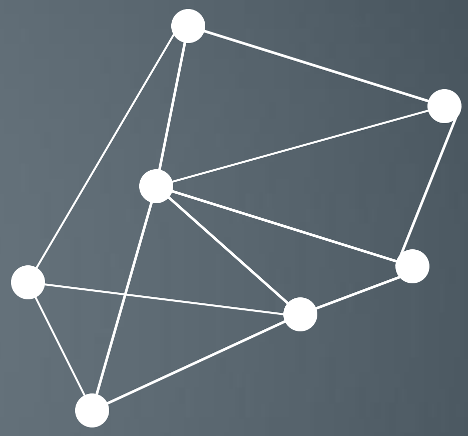
Step 1: Choose a vertex with highest degree, and color it. Use the same color to color as many vertices as you can without coloring vertices joined by an edge of the same color.

Step 2: Choose a new color, and repeat what you did in Step 1 for vertices not already colored.

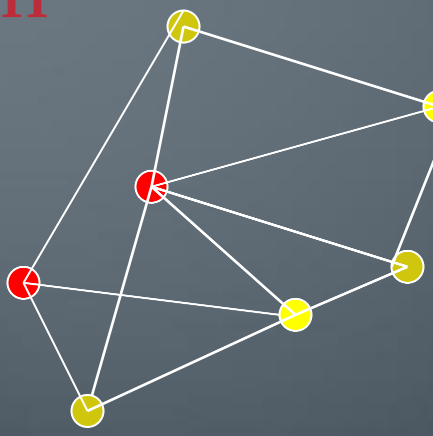
Step 3: Repeat Step 1 until all vertices are colored.

Example: Coloring a Graph

Color the graph below and give its chromatic number .



Solution



Its chromatic number is 3.