Internet Fundamentals

Lecture-34

Digital Signature

Chapter 13

Objectives

- To define a digital signature
- To define security services provided by a digital signature
- To define attacks on digital signatures
- To discuss some digital signature schemes, including RSA, ElGamal,
- Schnorr, DSS, and elliptic curve
- To describe some applications of digital signatures

13-1 COMPARISON

Let us begin by looking at the differences between conventional signatures and digital signatures.

<u>Topics discussed in this section:</u>

13.1.1 Inclusion

A conventional signature is included in the document; it is part of the document. But when we sign a document digitally, we send the signature as a separate document.

13.1.2 Verification Method

For a conventional signature, when the recipient receives a document, she compares the signature on the document with the signature on file. For a digital signature, the recipient receives the message and the signature. The recipient needs to apply a verification technique to the combination of the message and the signature to verify the authenticity.

13.1.3 Relationship

For a conventional signature, there is normally a one-to-many relationship between a signature and documents. For a digital signature, there is a one-to-one relationship between a signature and a message.

13.1.4 Duplicity

In conventional signature, a copy of the signed document can be distinguished from the original one on file. In digital signature, there is no such distinction unless there is a factor of time on the document.

13-2 PROCESS

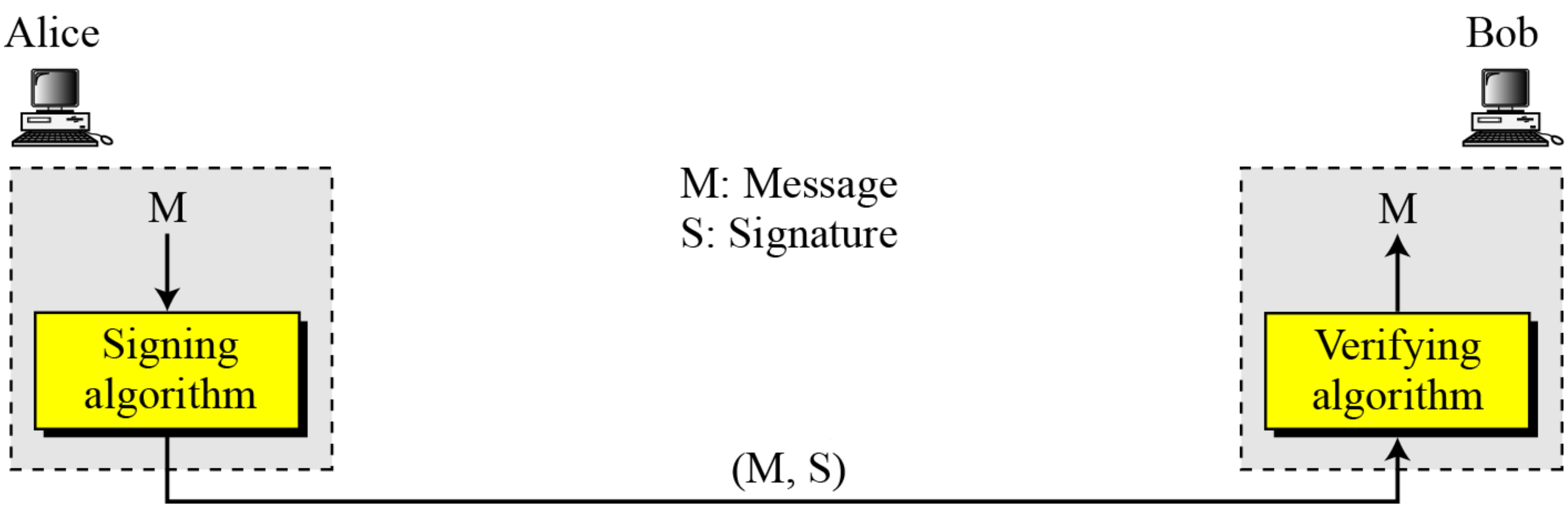
Figure 13.1 shows the digital signature process. The sender uses a signing algorithm to sign the message. The message and the signature are sent to the receiver. The receiver receives the message and the signature and applies the verifying algorithm to the combination. If the result is true, the message is accepted; otherwise, it is rejected.

Topics discussed in this section:

13.2.1 Need for Keys
13.2.2 Signing the Digest

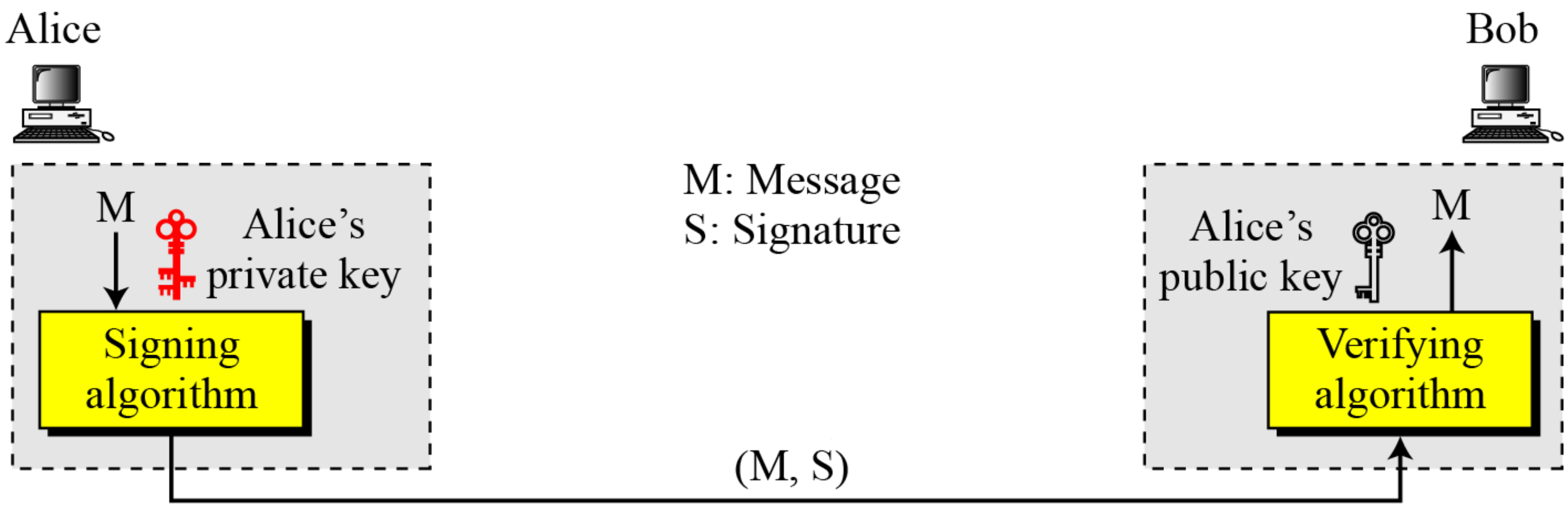
13-2 Continued

Figure 13.1 ***Digital signature process***



13.2.1 Need for Keys

Figure 13.2 *Adding key to the digital signature process*



Note

A digital signature needs a public-key system. The signer signs with her private key; the verifier verifies with the signer's public key.

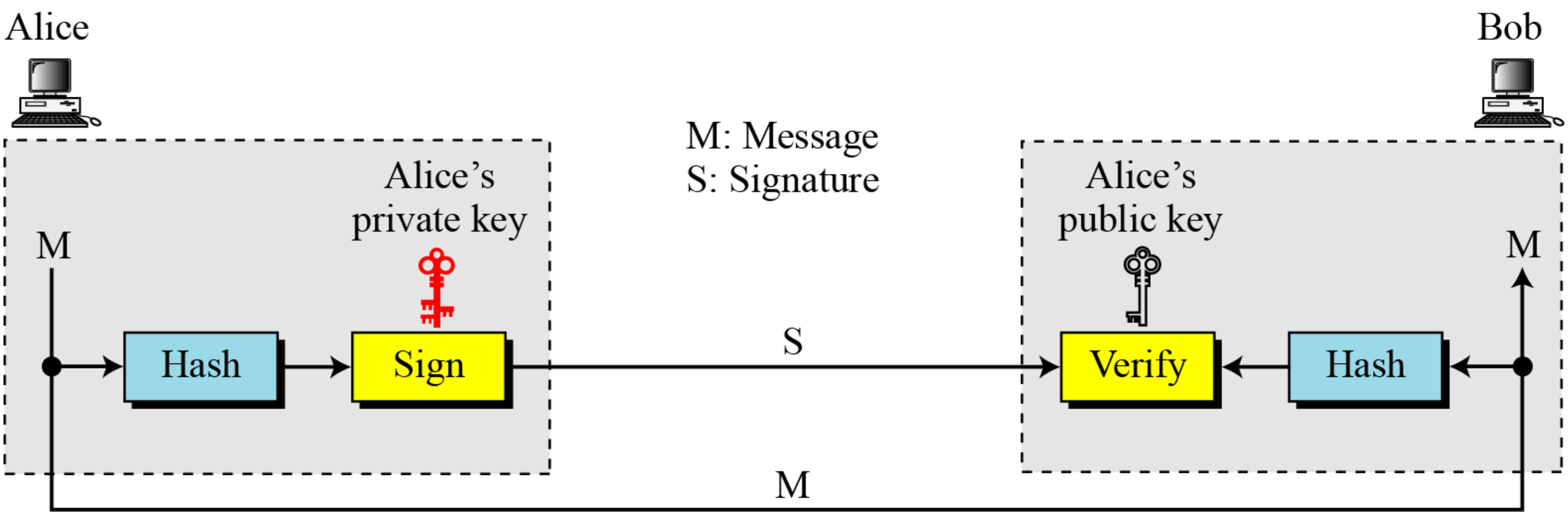
13.2.1 Continued

Note

A cryptosystem uses the private and public keys of the receiver: a digital signature uses the private and public keys of the sender.

13.2.2 Signing the Digest

Figure 13.3 ***Signing the digest***



13-3 SERVICES

We discussed several security services in Chapter 1 including message confidentiality, message authentication, message integrity, and nonrepudiation. A digital signature can directly provide the last three; for message confidentiality we still need encryption/decryption.

Topics discussed in this section:

13.3.1 Message Authentication
13.3.2 Message Integrity
13.3.3 Nonrepudiation
13.3.4 Confidentiality

13.3.1 Message Authentication

A secure digital signature scheme, like a secure conventional signature can provide message authentication.

Note

A digital signature provides message authentication.

13.3.2 Message Integrity

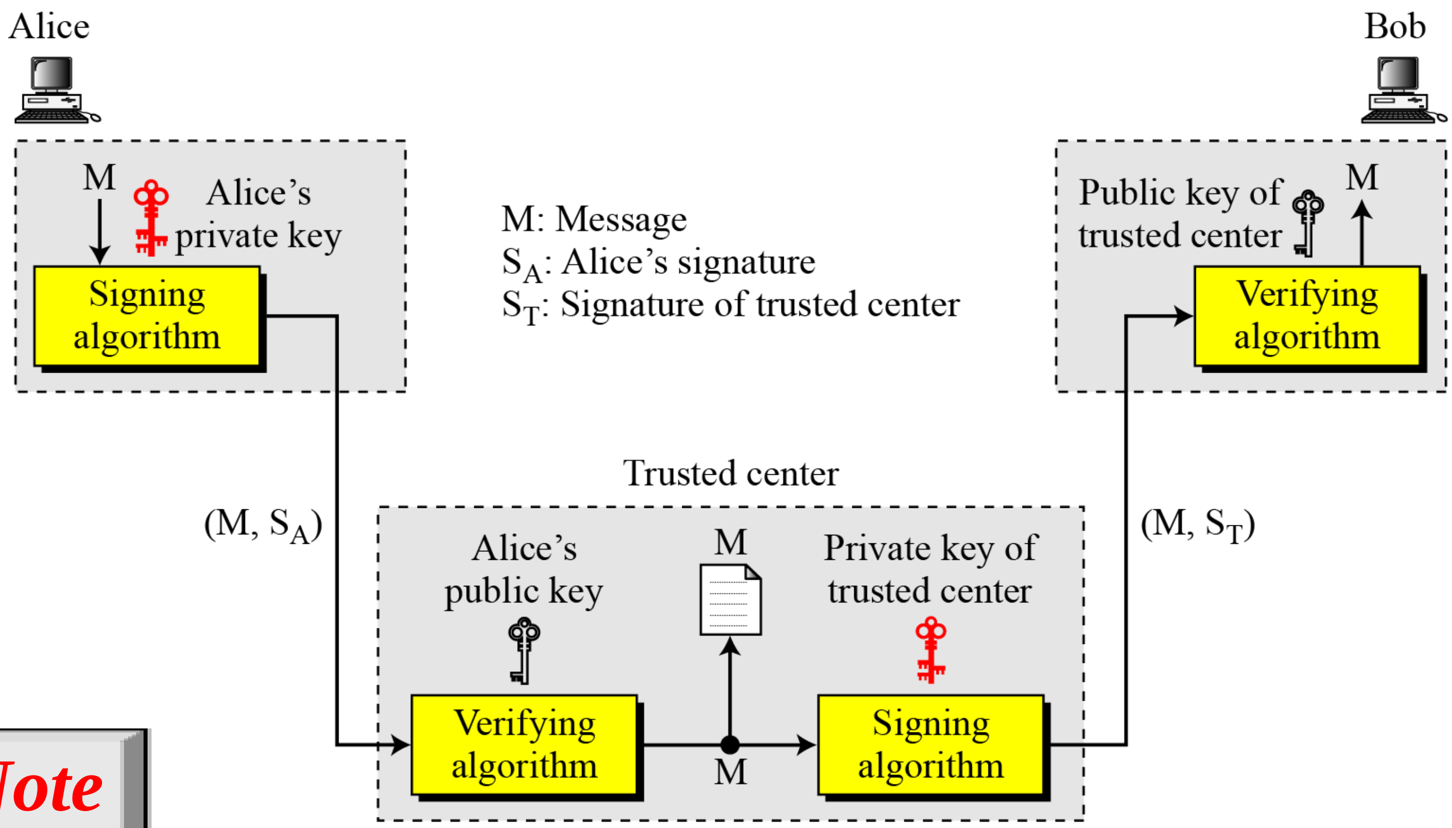
The integrity of the message is preserved even if we sign the whole message because we cannot get the same signature if the message is changed.

Note

A digital signature provides message integrity.

13.3.3 Nonrepudiation

Figure 13.4 *Using a trusted center for nonrepudiation*

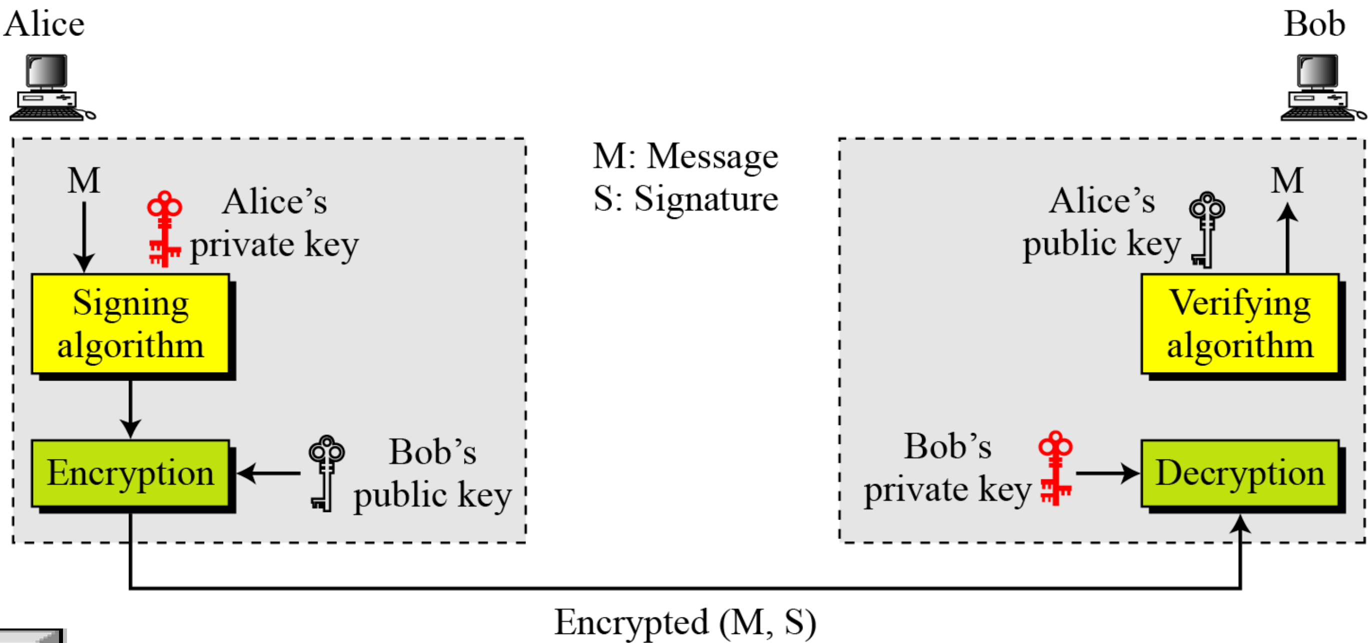


Note

Nonrepudiation can be provided using a trusted party.

13.3.4 Confidentiality

Figure 13.5 *Adding confidentiality to a digital signature scheme*



Note

A digital signature does not provide privacy. If there is a need for privacy, another layer of encryption/decryption must be applied.

13-4 ATTACKS ON DIGITAL SIGNATURE

This section describes some attacks on digital signatures and defines the types of forgery.

Topics discussed in this section:

13.4.1 Attack Types

Key-Only Attack

Known-Message Attack

Chosen-Message Attack

13.4.2 Forgery Types

Existential Forgery

Selective Forgery

13-5 DIGITAL SIGNATURE SCHEMES

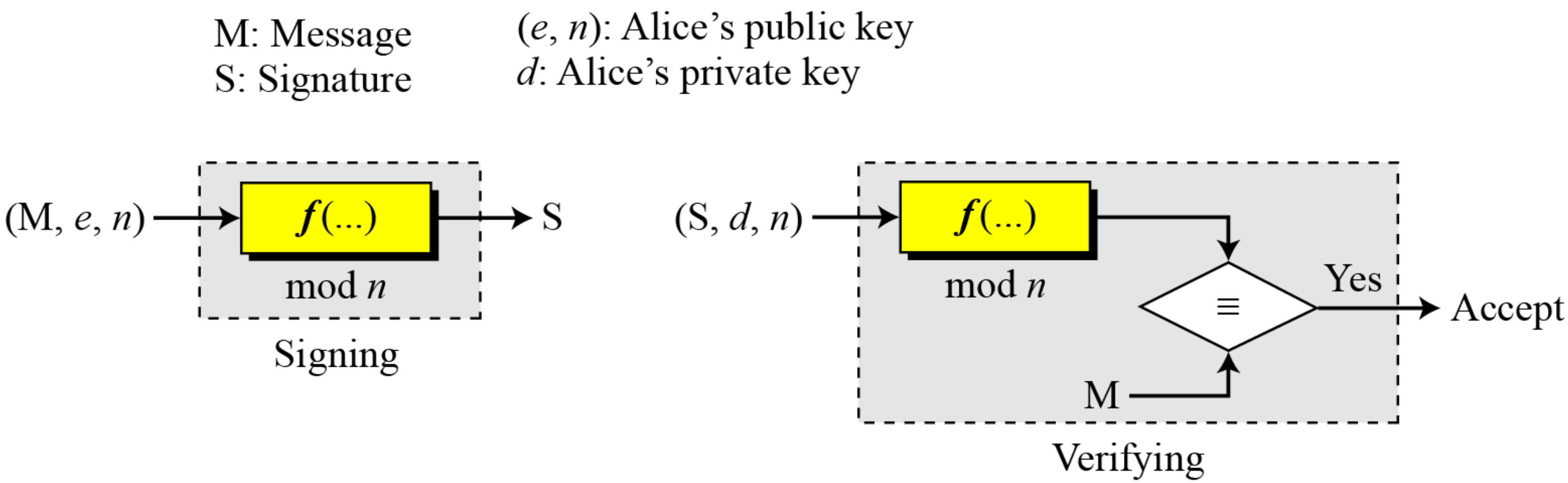
Several digital signature schemes have evolved during the last few decades. Some of them have been implemented.

Topics discussed in this section:

13.5.1 RSA Digital Signature Scheme
13.5.2 ElGamal Digital Signature Scheme
13.5.3 Schnorr Digital Signature Scheme
13.5.4 Digital Signature Standard (DSS)
13.5.5 Elliptic Curve Digital Signature Scheme

13.5.1 RSA Digital Signature Scheme

Figure 13.6 ***General idea behind the RSA digital signature scheme***



13.5.1 Continued

Key Generation

Key generation in the RSA digital signature scheme is exactly the same as key generation in the RSA

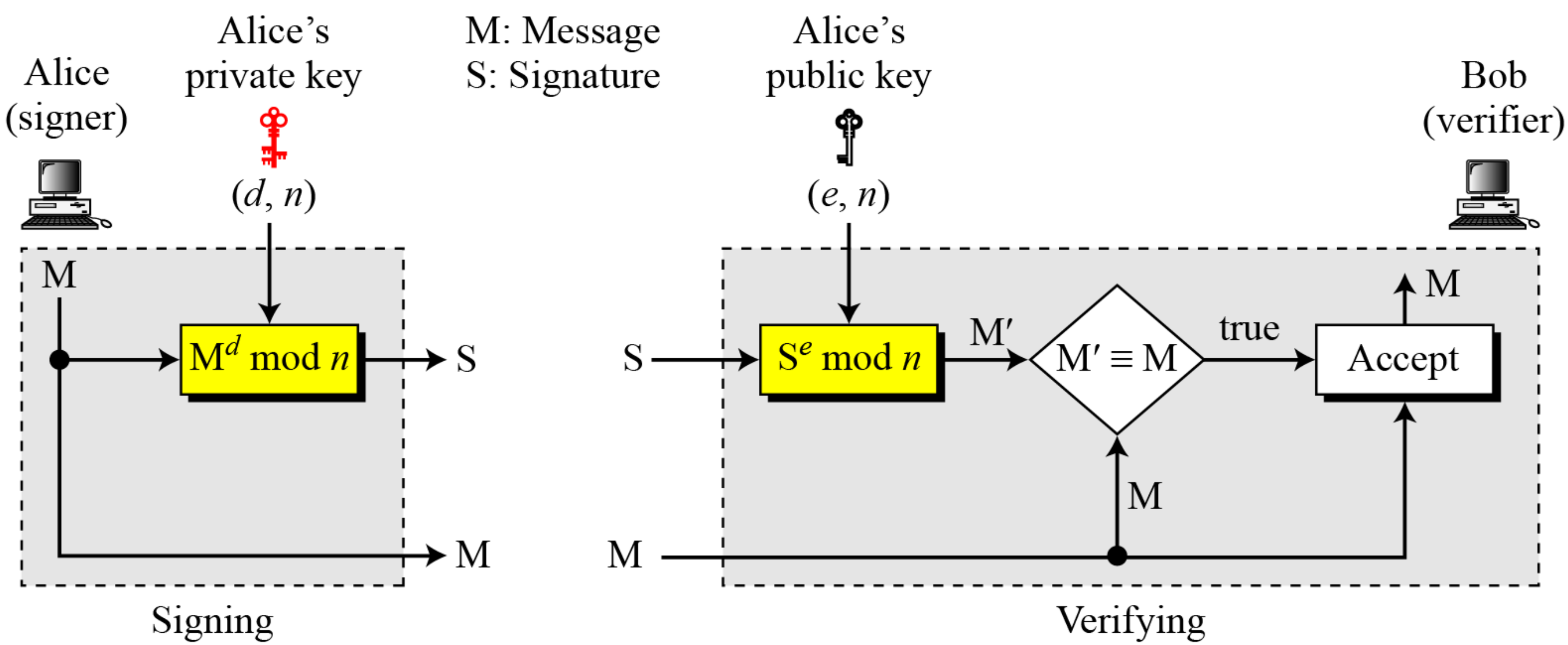
Note

In the RSA digital signature scheme, *d* is private; *e* and *n* are public.

13.5.1 Continued

Signing and Verifying

Figure 13.7 ***RSA digital signature scheme***



13.5.1 Continued

Example 13.1

As a trivial example, suppose that Alice chooses p = 823 and q = 953, and calculates n = 784319. The value of ϕ(n) is 782544. Now she chooses e = 313 and calculates d = 160009. At this point key generation is complete. Now imagine that Alice wants to send a message with the value of M = 19070 to Bob. She uses her private exponent, 160009, to sign the message:

$$\text{M: } 19070 \quad \rightarrow \quad S = (19070^{160009}) \bmod 784319 = 210625 \bmod 784319$$

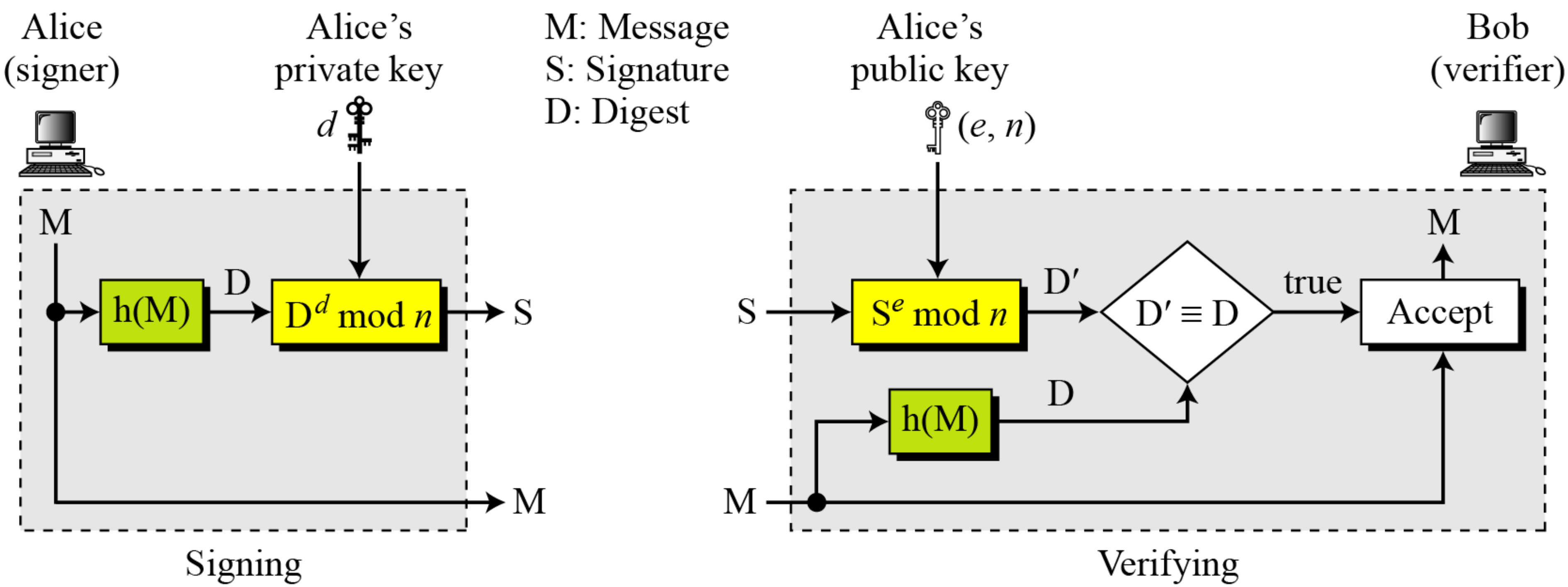
Alice sends the message and the signature to Bob. Bob receives the message and the signature. He calculates

$$M' = 210625^{313} \bmod 784319 = 19070 \bmod 784319 \quad \rightarrow \quad M \equiv M' \bmod n$$

Bob accepts the message because he has verified Alice's signature.

RSA Signature on the Message Digest

Figure 13.8 *The RSA signature on the message digest*



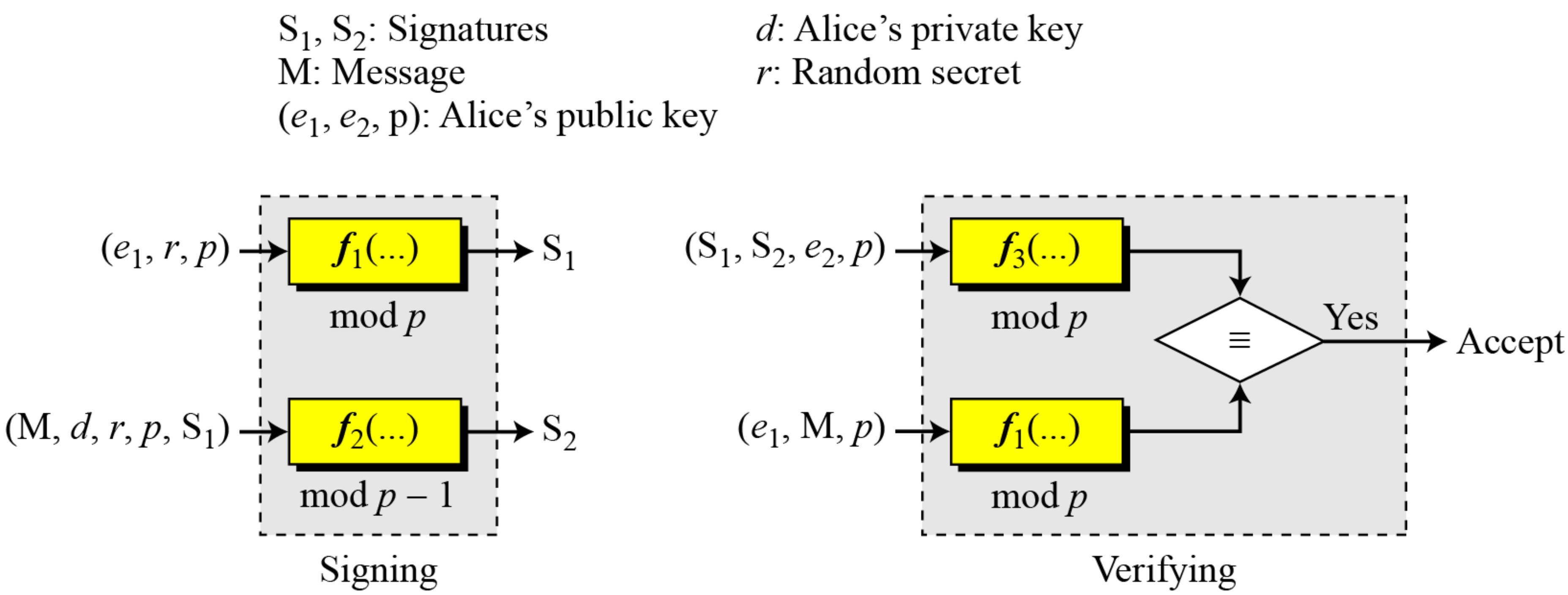
13.5.1 Continued

Note

When the digest is signed instead of the message itself, the susceptibility of the RSA digital signature scheme depends on the strength of the hash algorithm.

13.5.2 ElGamal Digital Signature Scheme

Figure 13.9 ***General idea behind the ElGamal digital signature scheme***



13.5.2 Continued

Key Generation

The key generation procedure here is exactly the same as the one used in the cryptosystem.

Note

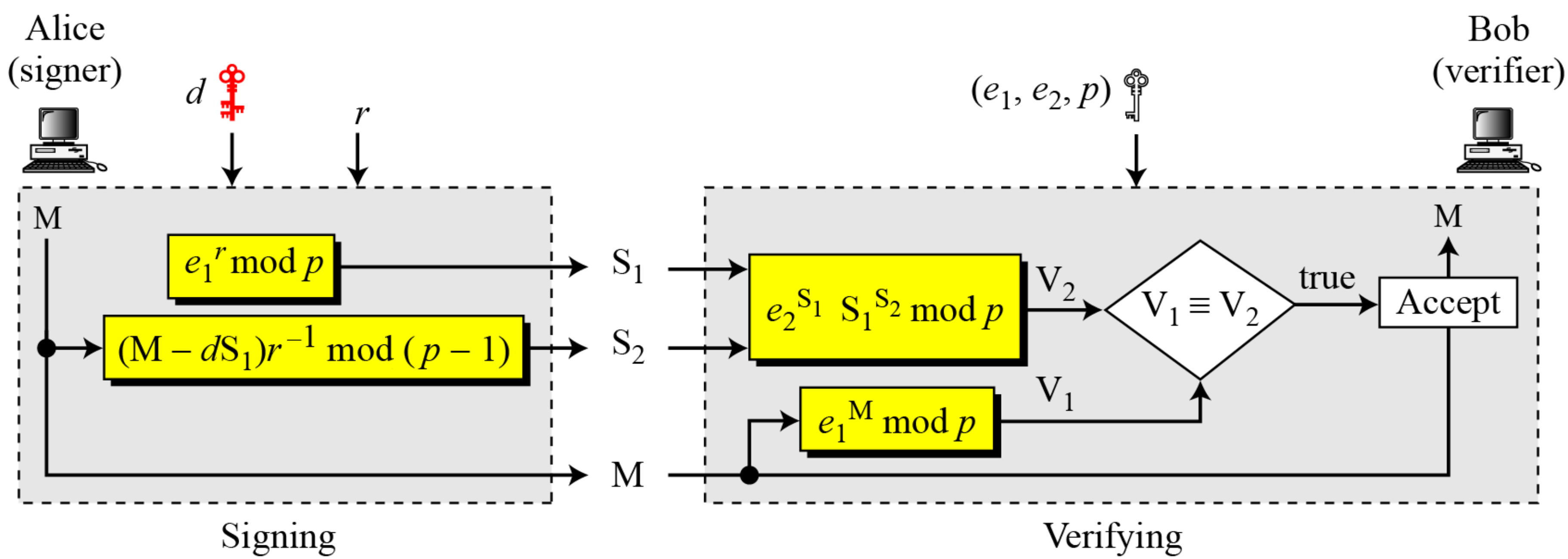
In ElGamal digital signature scheme, (e_1, e_2, p) is Alice's public key; d is her private key.

13.5.2 Continued

Verifying and Signing

Figure 13.10 ***ElGamal digital signature scheme***

M: Message
S_1, S_2: Signatures
V_1, V_2: Verifications
r: Random secret
d: Alice's private key
(e_1, e_2, p): Alice's public key



13.5.1 *Continued*

Example 13.2

Here is a trivial example. Alice chooses *p* = 3119, e_1 = 2, $_d$ = 127 and calculates $e_2 = 2^{127}$ mod 3119 = 1702. She also chooses *r* to be 307. She announces e*1*, e*2*, and *p* publicly; she keeps *d* secret. The following shows how Alice can sign a message.

$$M = 320$$

$$S_1 = {e_1}^r = 2^{307} = 2083 \bmod 3119$$

$$S_2 = (M - d \times S_1) \times r^{-1} = (320 - 127 \times 2083) \times 307^{-1} = 2105 \bmod 3118$$

Alice sends M, S_1, and S_2 to Bob. Bob uses the public key to calculate V_1 and V_2.

$$V_1 = {e_1}^M = 2^{320} = 3006 \bmod 3119$$

$$V_2 = d^{S_1} \times {S_1}^{S_2} = 1702^{2083} \times 2083^{2105} = 3006 \bmod 3119$$

13.5.1 *Continued*

Example 13.3

Now imagine that Alice wants to send another message, M = 3000, to Ted. She chooses a new *r*, 107. Alice sends M, S_1, and S_2 to Ted. Ted uses the public keys to calculate V_1 and V_2.

$$M = 3000$$

$$S_1 = e_1^{\,r} = 2^{107} = 2732 \bmod 3119$$

$$S_2 = (M - d \times S_1)\, r^{-1} = (3000 - 127 \times 2083) \times 107^{-1} = 2526 \bmod 3118$$

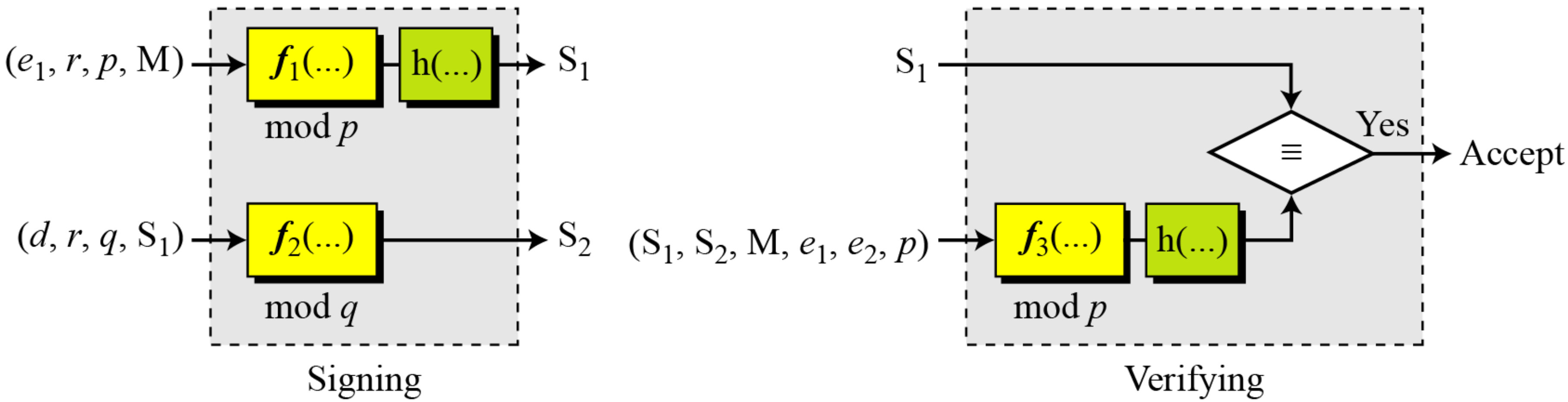
$$V_1 = e_1^{\,M} = 2^{3000} = 704 \bmod 3119$$

$$V_2 = d^{\,S_1} \times S_1^{\,S} = 1702^{2732} \times 2083^{2526} = 704 \bmod 3119$$

13.5.3 Schnorr Digital Signature Scheme

Figure 13.11 ***General idea behind the Schnorr digital signature scheme***

S_1, S_2: Signatures
M: Message
(e_1, e_2, p, q): Alice's public key
(d): Alice's private key
r: Random secret



13.5.3 Continued

Key Generation

1) *Alice selects a prime p, which is usually 1024 bits in length.*
2) *Alice selects another prime q.*
3) *Alice chooses e_1 to be the qth root of 1 modulo p.*
4) *Alice chooses an integer, d, as her private key.*
5) *Alice calculates $e_2 = e_1^d \bmod p$.*
6) *Alice's public key is (e_1, e_2, p, q); her private key is (d).*

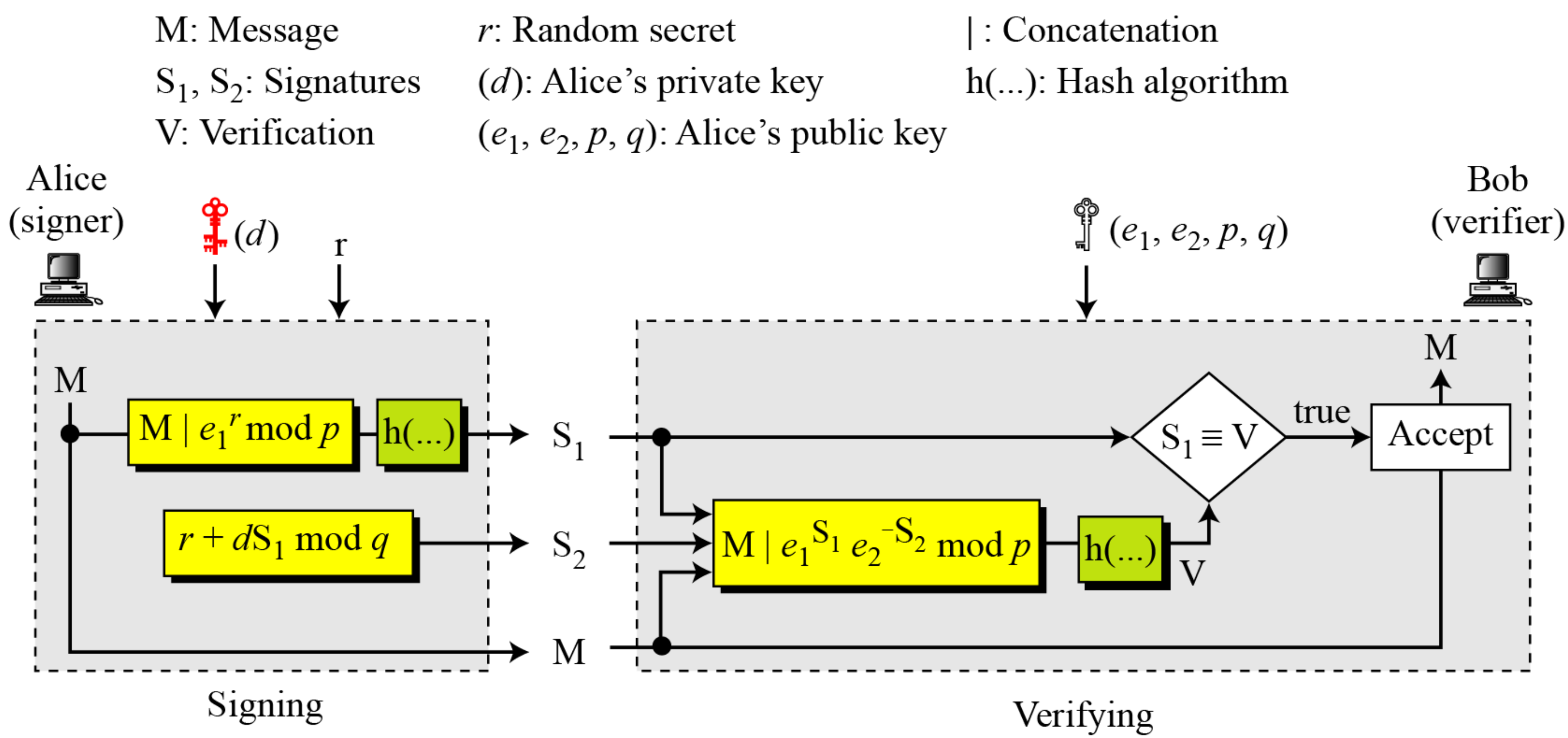
Note

In the Schnorr digital signature scheme, Alice's public key is (e_1, e_2, *p*, *q*); her private key (*d*).

13.5.3 Continued

Signing and Verifying

Figure 13.12 *Schnorr digital signature scheme*



13.5.3 Continued

Signing

1. Alice chooses a random number r.
2. Alice calculates $S_1 = h(M|e_1^r \bmod p)$.
3. Alice calculates $S_2 = r + d \times S_1 \bmod q$.
4. Alice sends M, S_1, and S_2.

Verifying Message

1. Bob calculates $V = h\,(M \mid e_1^{S2}\, e_2^{-S1} \bmod p)$.
2. If S_1 is congruent to V modulo p, the message is accepted;

13.5.1 Continued

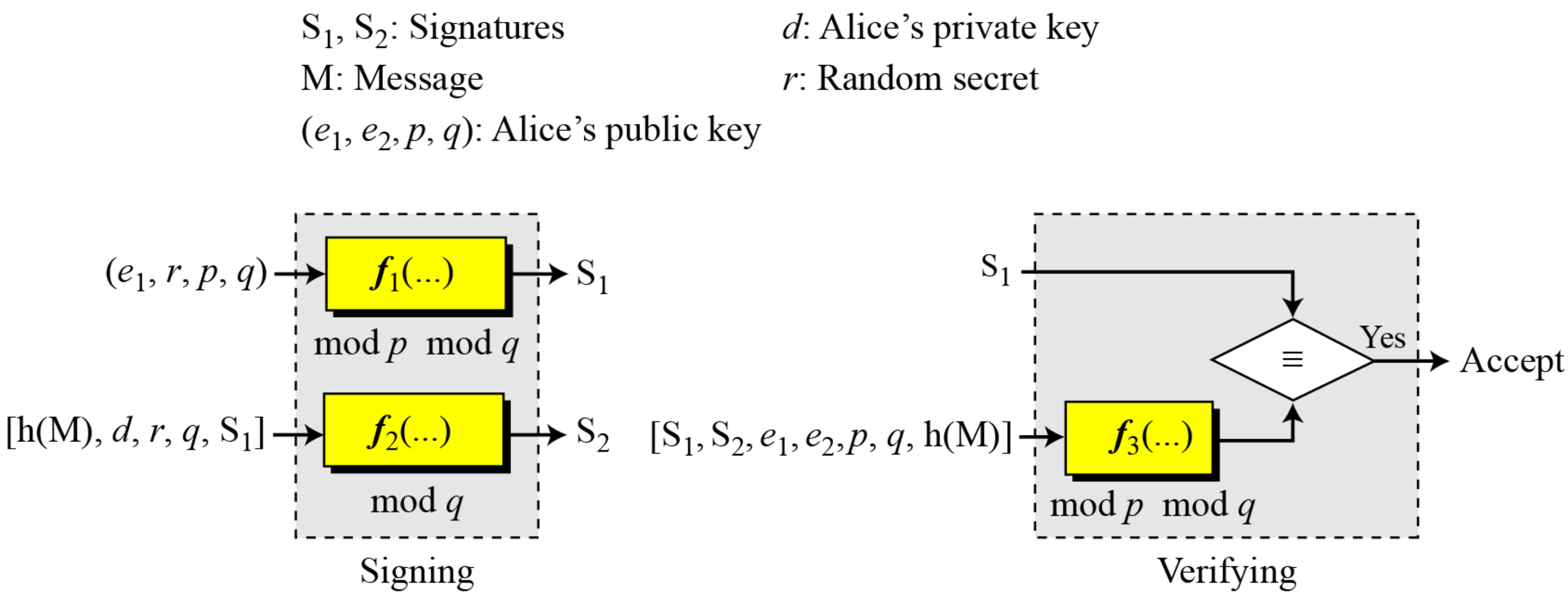
Example 13.4

Here is a trivial example. Suppose we choose $q = 103$ and $p = 2267$. Note that $p = 22 \times q + 1$. We choose $e_0 = 2$, which is a primitive in $Z_{2267}*$. Then $(p - 1) / q = 22$, so we have $e_1 = 2^{22} \bmod 2267 = 354$. We choose $d = 30$, so $e_2 = 354^{30} \bmod 2267 = 1206$. Alice's private key is now (d); her public key is (e_1, e_2, p, q).

Alice wants to send a message M. She chooses $r = 11$ and calculates $e_2{}^r = 354^{11} = 630 \bmod 2267$. Assume that the message is 1000 and concatenation means 1000630. Also assume that the hash of this value gives the digest h(1000630) = 200. This means S1 = 200. Alice calculates $S2 = r + d \times S_1 \bmod q = 11 + 1026 \times 200 \bmod 103 = 35$. Alice sends the message M =1000, S_1 = 200, and S_2 = 35. The verification is left as an exercise.

13.5.4 Digital Signature Standard (DSS)

Figure 13.13 ***General idea behind DSS scheme***



13.5.4 Continued

Key Generation.

1) Alice chooses primes p and q.

2) Alice uses $\langle Z_p^*, \times \rangle$ and $\langle Z_q^*, \times \rangle$.

3) Alice creates e_1 to be the qth root of 1 modulo p.

4) Alice chooses d and calculates $e_2 = e_1^d$.

5) Alice's public key is (e_1, e_2, p, q); her private key is (d).

13.5.4 Continued

Verifying and Signing

Figure 13.14 ***DSS scheme***

M: Message

S_1, S_2: Signatures

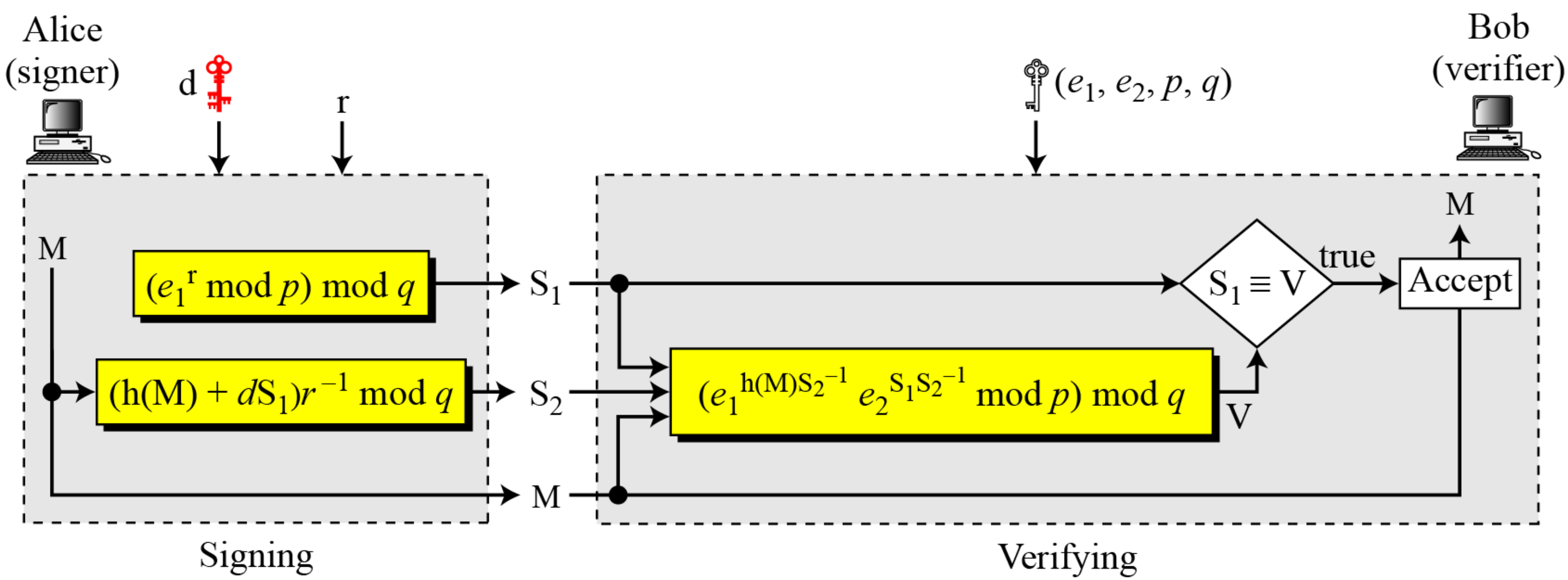
V: Verification

r: Random secret

d: Alice's private key

(e_1, e_2, p, q): Alice's public key

h(M): Message digest



13.5.1 Continued

Example 13.5

Alice chooses q = 101 and p = 8081. Alice *selects* e_0 = 3 and calculates $e^1 = e_0^{(p-1)/q}$ mod p = 6968. Alice chooses d = 61 as the private key and calculates $e_2 = e_1^d$ mod p = 2038. Now Alice can send a message to Bob. Assume that h(M) = 5000 and Alice chooses r = 61:

$$\mathrm{h(M)} = 5000 \qquad r = 61$$

$$S_1 = (e_1{}^r \bmod p) \bmod q = 54$$

$$S_2 = ((\mathrm{h(M)} + d\, S_1)\, r^{-1}) \bmod q = 40$$

Alice sends M, S_1, and S_2 to Bob. Bob uses the public keys to calculate V.

$$S_2{}^{-1} = 48 \bmod 101$$

$$V = [(6968^{5000 \times 48} \times 2038^{54 \times 48}) \bmod 8081] \bmod 101 = 54$$

13.5.4 Continued

DSS Versus RSA

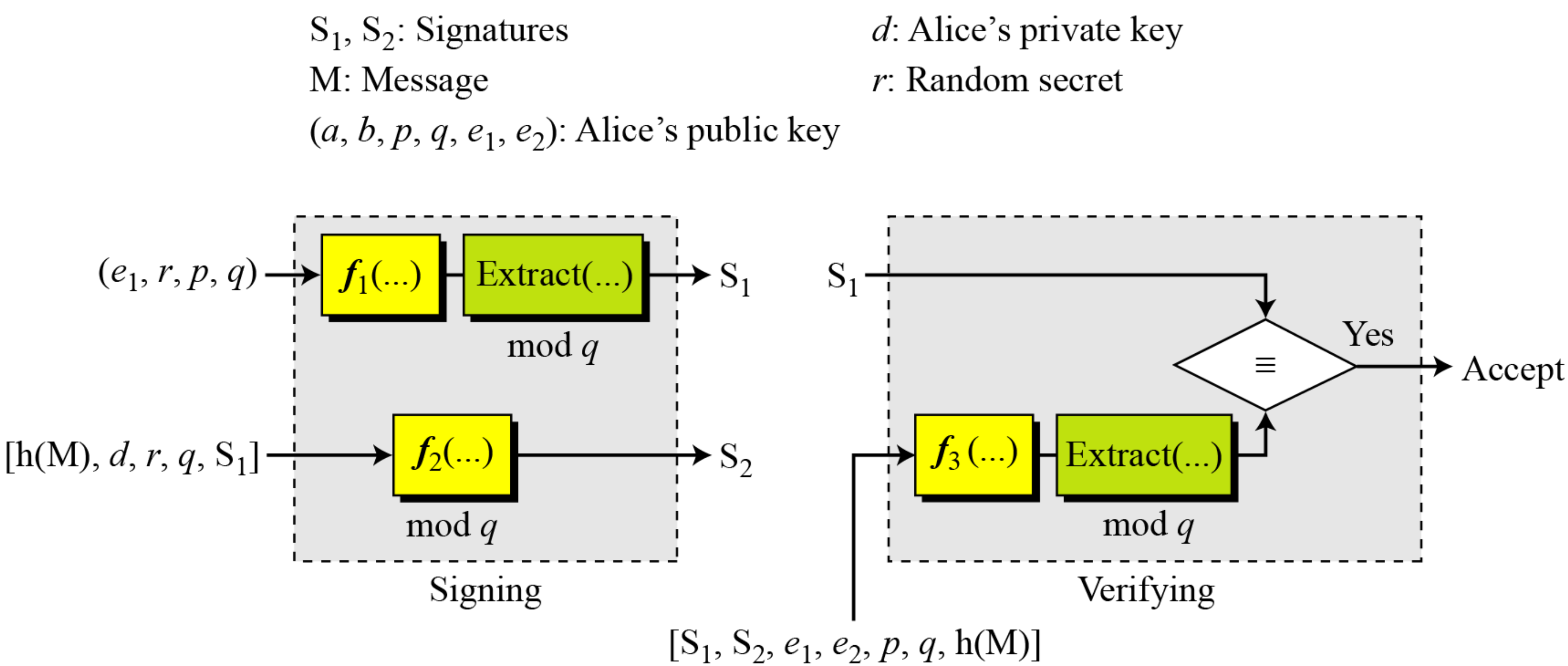
Computation of DSS signatures is faster than computation of RSA signatures when using the same p.

DSS Versus ElGamal

DSS signatures are smaller than ElGamal signatures because q is smaller than p.

13.5.5 Elliptic Curve Digital Signature Scheme

Figure 13.15 ***General idea behind the ECDSS scheme***



13.5.5 Continued

Key Generation

Key generation follows these steps:

1) Alice chooses an elliptic curve $E_p(a, b)$.

2) Alice chooses another prime q the private key d.

3) Alice chooses $e_1(\ldots, \ldots)$, a point on the curve.

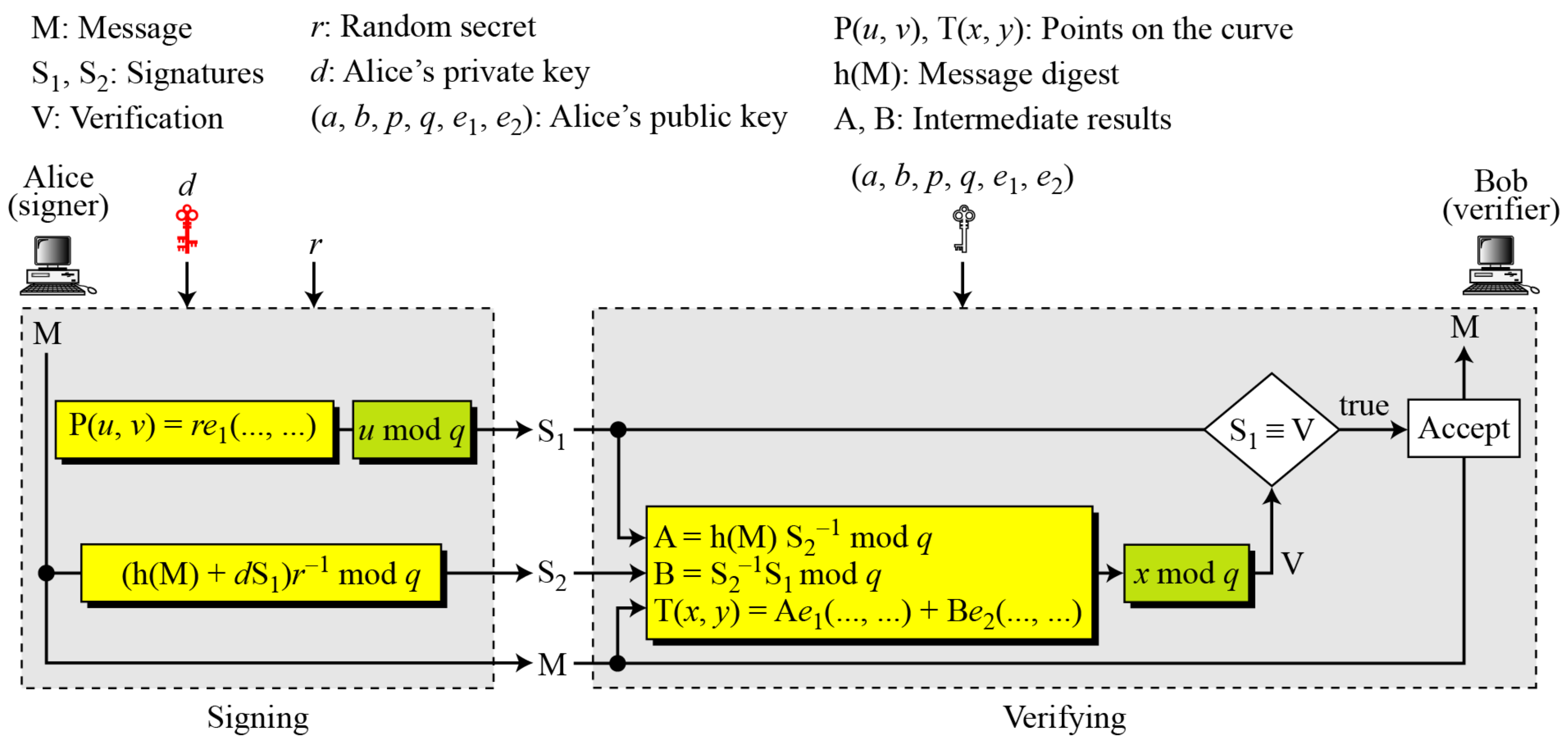
4) Alice calculates $e_2(\ldots, \ldots) = d \times e_1(\ldots, \ldots)$.

5) Alice's public key is (a, b, p, q, e1, e2); her private key is d.

13.5.5 Continued

Signing and Verifying

Figure 13.16 ***The ECDSS scheme***



13-6 VARIATIONS AND APPLICATIONS

This section briefly discusses variations and applications for digital signatures.

Topics discussed in this section:

13.6.1 Variations
13.6.2 Applications

13.6.1 Variations

Time Stamped Signatures

Sometimes a signed document needs to be time stamped to prevent it from being replayed by an adversary. This is called time-stamped digital signature scheme.

Blind Signatures

Sometimes we have a document that we want to get signed without revealing the contents of the document to the signer.