

A decorative graphic on the left side of the slide, consisting of white lines and circles that resemble a circuit board or network diagram. The lines are vertical and horizontal, with some diagonal connections, and the circles are small and white, placed at various points along the lines.

NETWORK THEORY

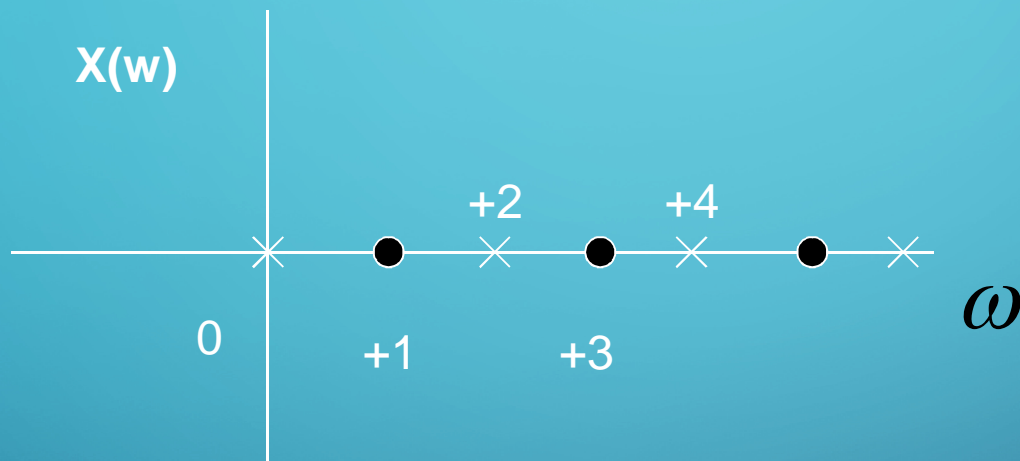


LECTURE 4

SECTION-D: TYPES OF FILTERS AND THEIR CHARACTERISTICS

PROPERTY 3. L-C IMMITTANCE FUNCTION

- 3. The poles and zeros interlace on the $j\omega$ axis.



Highest power: $2n$ \rightarrow next highest power must be $2n-2$

They cannot be missing term. Unless?

if $b_5 s^5 + b_1 s = 0 \rightarrow s=0, s_k = \left(\frac{b_1}{b_5}\right)^{1/4} e^{i(2k-1)\pi/4}$

We can write a general L-C impedance or admittance as

$$Z(s) = \frac{K(s^2 + \omega_1^2)(s^2 + \omega_3^2)\dots(s^2 + \omega_i^2)\dots}{s(s^2 + \omega_2^2)(s^2 + \omega_4^2)\dots(s^2 + \omega_j^2)\dots}$$

$$Z(s) = \frac{K_0}{s} + \frac{2K_2s}{s^2 + \omega_2^2} + \frac{2K_4s}{s^2 + \omega_4^2} + \dots + K_\infty s$$

Since these poles are all on the $j\omega$ axis, the residues must be real and positive in order for $Z(s)$ to be positive real .

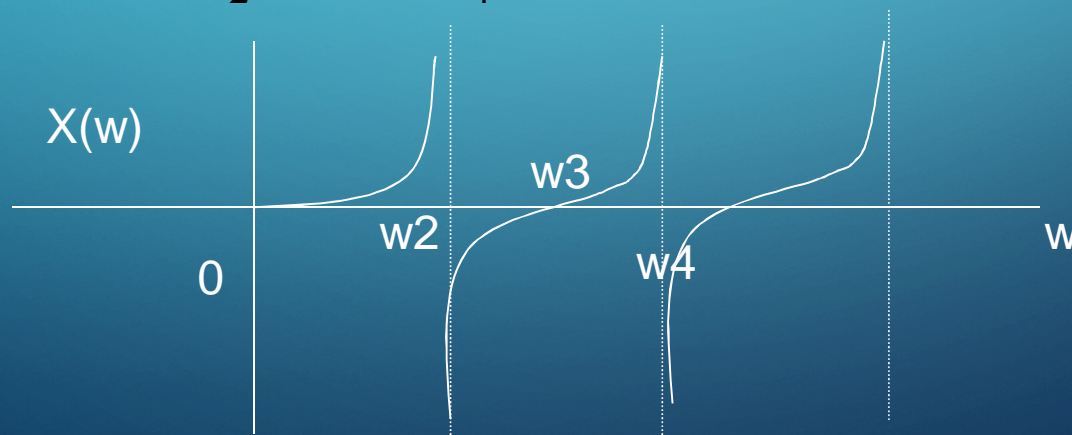
$S=j\omega \rightarrow Z(j\omega)=jX(\omega)$ (no real part)

$$\frac{dX(\omega)}{d\omega} = \frac{K_0}{\omega^2} + K_\infty + \frac{K_2(\omega^2 + \omega_2^2)}{(\omega_2^2 + \omega^2)} + \dots$$

Since all the residues K_i are positive, it is easy to see that for an L-C function

$$\frac{dX(\omega)}{d\omega} \geq 0$$

$$\text{Ex) } Z(s) = \frac{Ks(s^2 + \omega_3^2)}{(s^2 + \omega_2^2)(s^2 + \omega_4^2)} \quad jX(\omega) = +j \frac{K\omega(-\omega^2 + \omega_3^2)}{(-\omega^2 + \omega_2^2)(-\omega^2 + \omega_4^2)}$$



PROPERTIES 4 AND 5. L-C IMMITTANCE FUNCTION

- The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.
- There must be either a zero or a pole at the origin and infinity.

SUMMARY OF PROPERTIES

1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the $j\omega$ axis
3. The poles and zeros interlace on the $j\omega$ axis.
4. The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.
5. There must be either a zero or a pole at the origin and infinity.

EXAMPLES

$$Z(s) = \frac{Ks(s^2 + 4)}{(s^2 + 1)(s^2 + 3)}$$

$$Z(s) = \frac{s^5 + 4s^3 + 5s}{3s^4 + 6s^2}$$

$$Z(s) = \frac{K(s^2 + 1)(s^2 + 9)}{(s^2 + 2)(s^2 + 10)}$$

$$Z(s) = \frac{2(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)}$$