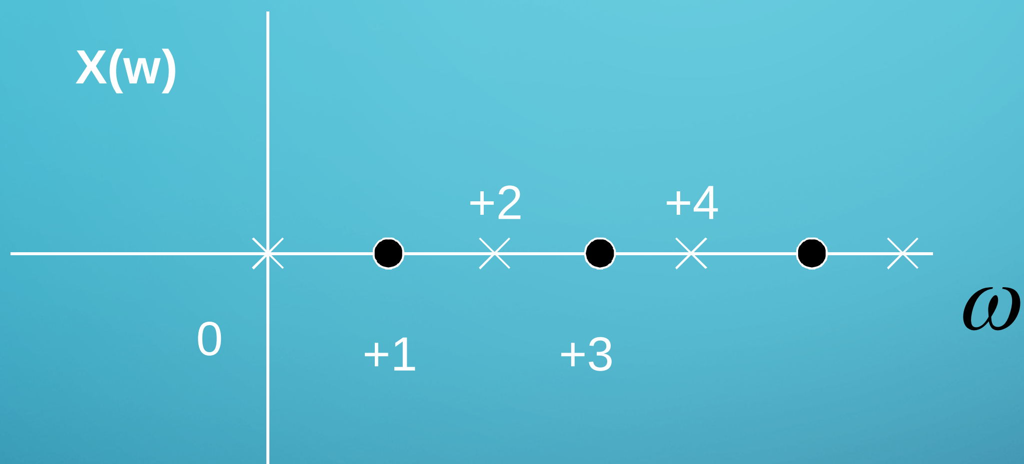
# NETWORK THEORY

# LECTURE 4

## SECTION-D:TYPES OF FILTERS AND THEIR CHARACTERISTICS

# ROPERTY 3. L-C IMMITTANCE FUNCTION

- 3.The poles and zeros interlace on the $j\omega$ axis.




**Highest power: 2n -> next highest power must be 2n-2**

**They cannot be missing term. Unless?**

**if** $b_5 s^5 + b_1 s = 0$ → **s=0,** $s_k = (\frac{b_1}{b_5})^{1/4} e^{i(2k-1)\pi/4}$

**We can write a general L-C impedance or admittance as**

$$Z(s)=\frac{K(s^2+\omega_1^{\,2})(s^2+\omega_3^{\,2})\ldots(s^2+\omega_i^{\,2})\ldots}{s(s^2+\omega_2^{\,2})(s^2+\omega_4^{\,2})\ldots(s^2+\omega_j^{\,2})\ldots}$$

$$Z(s)=\frac{K_0}{s}+\frac{2K_2 s}{s^2+\omega_2^{\,2}}+\frac{2K_4 s}{s^2+\omega_4^{\,2}}+\ldots+K_\infty s$$

Since these poles are all on the jw axis, the residues must be real and positive in order for Z(s) to be positive real .

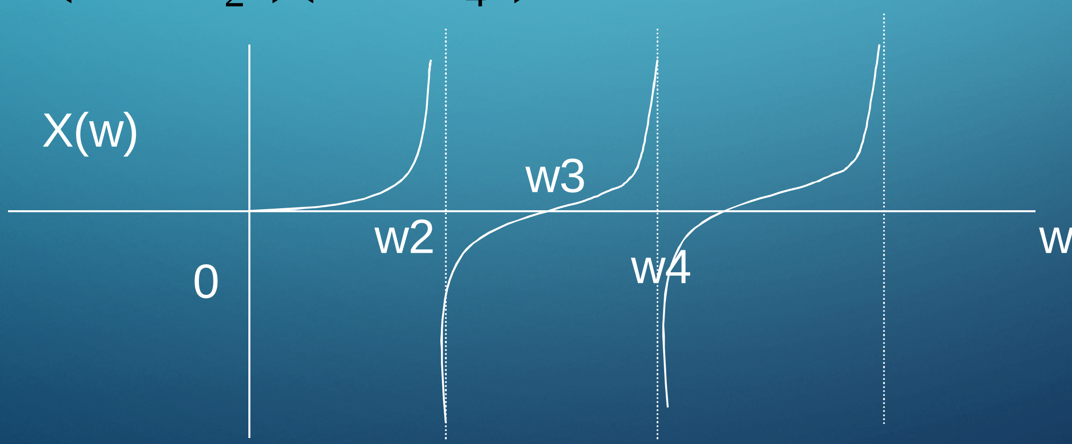
**S=jw → Z(jw)=jX(w) (no real part)**

$$\frac{dX(\omega)}{d\omega} = \frac{K_0}{\omega^2} + K_\infty + \frac{K_2(\omega^2 + \omega_2{}^2)}{(\omega_2{}^2 + \omega^2)} + \ldots$$

Since all the residues Ki are positive, it is eay to see that for an L-C function

$$\frac{dX(\omega)}{d\omega} \geq 0$$

Ex) $Z(s) = \frac{Ks(s^2 + \omega_3{}^2)}{(s^2 + \omega_2{}^2)(s^2 + \omega_4{}^2)}$ $jX(\omega) = +j\frac{K\omega(-\omega^2 + \omega_3{}^2)}{(-\omega^2 + \omega_2{}^2)(-\omega^2 + \omega_4{}^2)}$



# PROPERTIES 4 AND 5. L-C IMMITTANCE FUNCTION

- The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.

- There must be either a zero or a pole at the origin and infinity.

# UMMARY OF PROPERTIES

1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the jw axis
3. The poles and zeros interlace on the jw axis.
4. The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.
5. There must be either a zero or a pole at the origin and infinity.

# XAMPLES

$$Z(s) = \frac{Ks(s^2+4)}{(s^2+1)(s^2+3)}$$

$$Z(s) = \frac{s^5+4s^3+5s}{3s^4+6s^2}$$

$$Z(s) = \frac{K(s^2+1)(s^2+9)}{(s^2+2)(s^2+10)}$$

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$