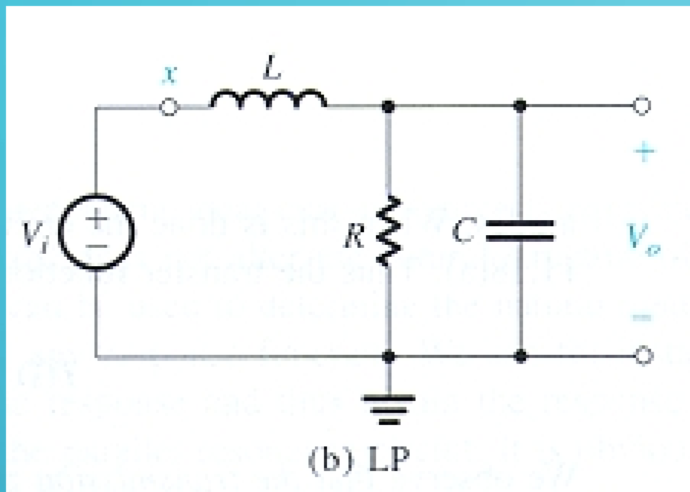
NETWORK THEORY

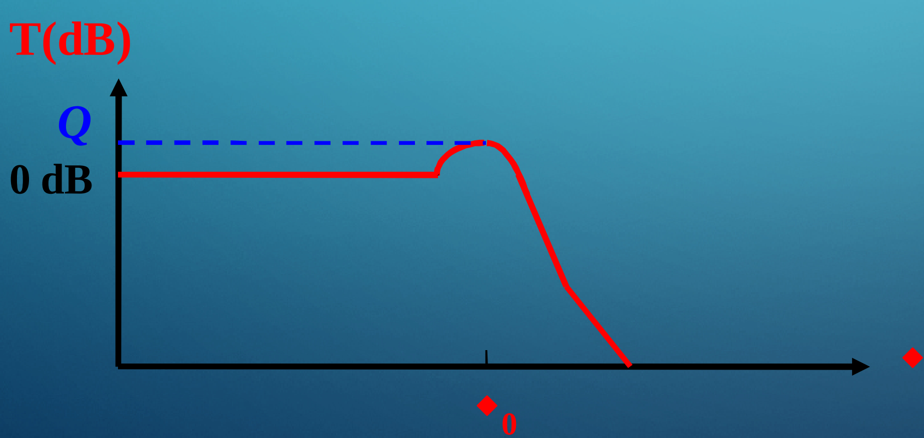
LECTURE 3

SECTION-D:TYPES OF FILTERS AND THEIR CHARACTERISTICS

Example - Passive Second Order Filter Function



$$T(s) = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} = \begin{cases} \to 0 \;\; as \;\; s \to \infty \\ \to Q \;\; as \;\; s = j\omega_o \\ \to 1 \;\; as \;\; s \to 0 \end{cases}$$



* Low pass filter

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_C \| R}{Z_C \| R + Z_L}$$

$$where \quad Z_C \| R = \frac{1}{\frac{1}{Z_C} + \frac{1}{R}} = \frac{1}{sC + \frac{1}{R}} = \frac{R}{1 + sRC} \quad so$$

$$T(s) = \frac{Z_C \| R}{Z_C \| R + Z_L} = \frac{\left(\frac{R}{1+sRC}\right)}{sL + \frac{R}{1+sRC}} = \frac{1}{\left(\frac{1+sRC}{R}\right)\left(sL + \frac{R}{1+sRC}\right)}$$

$$= \frac{1}{1 + s\frac{L}{R} + s^2 LC} = \frac{\left(\frac{1}{LC}\right)}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}} = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

$$where \quad \omega_o = \frac{1}{\sqrt{LC}} \quad and \quad Q = RC\omega_o = R\sqrt{\frac{C}{L}}$$

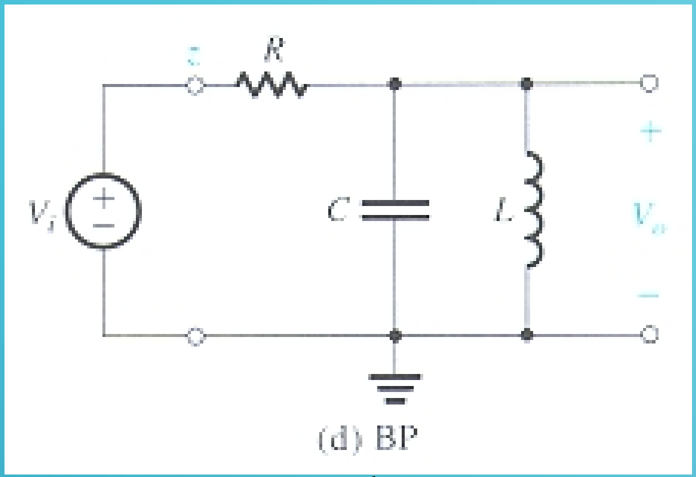
This has the form of a second order filter with

$a_2 = 0, \; a_1 = 0 \quad a_0 = \omega_0^2$

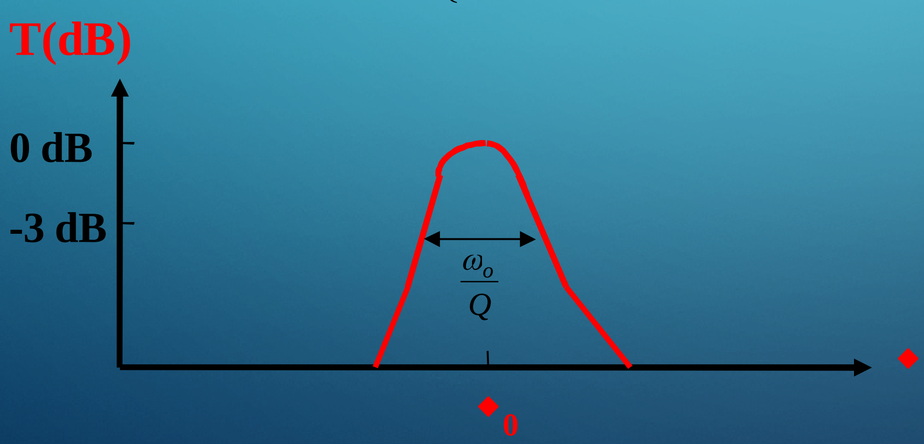
General form of transfer function

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Example - Passive Second Order Filter Function



$$T(s)=\frac{s\frac{\omega_0}{Q}}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}=\begin{cases}\rightarrow \frac{s}{s^2}\rightarrow 0 \ \ as\ s\rightarrow\infty \\ \rightarrow 1\ \ at\ s=j\omega_o \\ \rightarrow \frac{s}{\omega_o^2}\rightarrow 0\ \ as\ s\rightarrow 0\end{cases}$$



* Bandpass filter

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{Z_C\|Z_L}{Z_C\|Z_L+R}$$

$$where \quad Z_C\|Z_L=\frac{1}{\frac{1}{Z_C}+\frac{1}{Z_L}}=\frac{1}{sC+\frac{1}{sL}}=\frac{sL}{1+s^2LC} \quad so$$

$$T(s)=\frac{Z_C\|Z_L}{Z_C\|Z_L+R}=\frac{\left(\frac{sL}{1+s^2LC}\right)}{R+\frac{sL}{1+s^2LC}}=\frac{sL}{sL+R\left(1+s^2LC\right)}$$

$$=\frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}=\frac{\frac{1}{RC}s}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}$$

$$where \quad \omega_o=\frac{1}{\sqrt{LC}} \quad and \quad Q=RC\omega_o=R\sqrt{\frac{C}{L}}$$

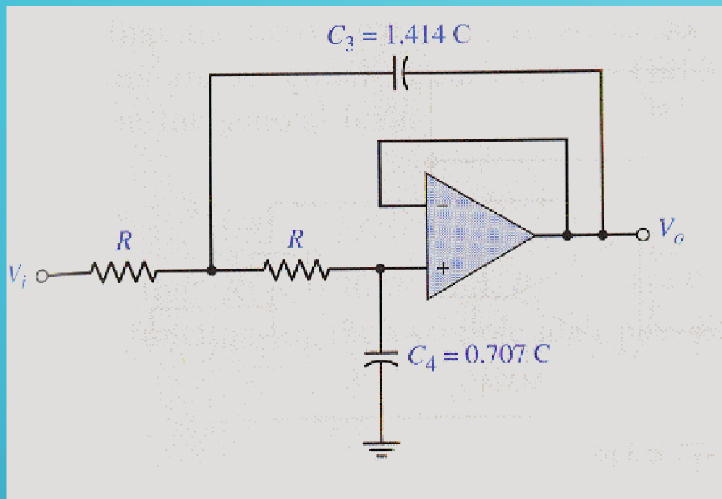
This has the form of a second order filter with

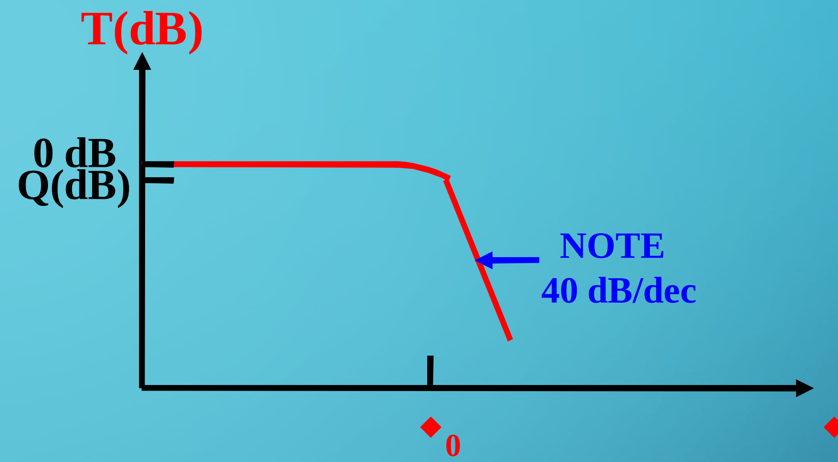
$$a_2=0,\ a_1=\frac{1}{RC}=\frac{\omega_o}{Q} \quad a_0=0$$

General form of transfer function

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{a_2s^2+a_1s+a_o}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

Low Pass Butterworth Filter Design





Hurwitz Polynomials

$$T(s) = \frac{V_o}{V_i} = \frac{1/R^2C^2}{s^2 + s\dfrac{2}{RC} + \dfrac{1}{R^2C^2}} = \begin{cases} \to 1 \;\; as \;\; s \to 0 \\ \to -j/2 \;\; as \;\; s \to j\omega_o \\ \to 0 \;\; as \;\; s \to \infty \end{cases}$$

- * **Given the filter specification (◆$_0$), we can determine the R and C.**
- * **One specification, two parameters – R and C**
- * **Pick a convenient value, say C = 5 nF.**
- * **Calculate R from C and ω_o.**

$$\omega_o = \frac{1}{RC} \quad so \quad R = \frac{1}{C\omega_o}$$

Example: Given $\omega_0 = 2\pi(2MHz) = 1.26x10^7\,rad/\sec$

Choose $C = 5\ nF$, *then*

$$R = \frac{1}{C\omega_o} = \frac{1}{(5x10^{-9}F)(1.26x10^7\,rad/\sec)} = 16\ \Omega$$

ROPERTY 1. L-C IMMITTANCE FUNCTION

- 1. Z_{LC} (s) or Y_{LC} (s) is the ratio of odd to even or even to odd polynomials.

- Consider the impedance Z(s) of passive one-port network.

$$Z(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$$

(M is even N is odd)

As we know, when the input current is *I*, the average power dissipated by one-port network is zero:

Average Power= $\frac{1}{2}\mathrm{Re}[Z(j\omega)]|I|^2$ =0

$$EvZ\ (s) = \frac{M_1(s)M_2(s) - N_1(s)N_2(s)}{M_2(s)^2 + N_2(s)^2} = 0$$

$$M_1(j\omega)M_2(j\omega) - N_1(j\omega)N_2(j\omega) = 0$$

$$M_1 = 0 = N_2 \quad \text{OR} \quad M_2 = 0 = N_1$$

$$Z(s) = \frac{M_1}{N_2}, Z(s) = \frac{N_1}{M_2}$$

Z(s) or Y(s) is the ratio of even to odd or odd to even!!

ROPERTY 2. L-C IMMITTANCE FUNCTION

- 2.The poles and zeros are simple and lie on the $j\omega$ axis.

$$Z(s)=\frac{M_1}{N_2}, Z(s)=\frac{N_1}{M_2}$$

- Since both M and N are Hurwitz, they have only imaginary roots, and it follows that the poles and zeros of Z(s) or Y(s) are on the imaginary axis.

- Consider the example

$$Z(s)=\frac{a_4 s^4 + a_2 s^2 + a_0}{b_5 s^5 + b_3 s^3 + b_1 s}$$

$$Z(s) = \frac{a_4 s^4 + a_2 s^2 + a_0}{b_5 s^5 + b_3 s^3 + b_1 s}$$

In order for the impedance to be positive real → the coefficients must be real and positive.

Impedance function **cannot have multiple poles or zeros** on the $j\omega$ axis.

The highest powers of the numerator and the denominator polynomials can differ by, at most, unity.

Ex) highest order of the numerator : 2n -> highest order of the denominator can either be 2n-1 (simple pole at s= ∞) or

the order can be 2n+1 (simple zero at s= ∞).