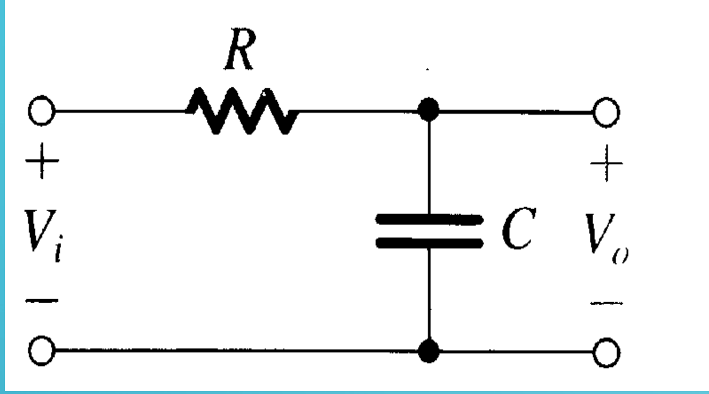
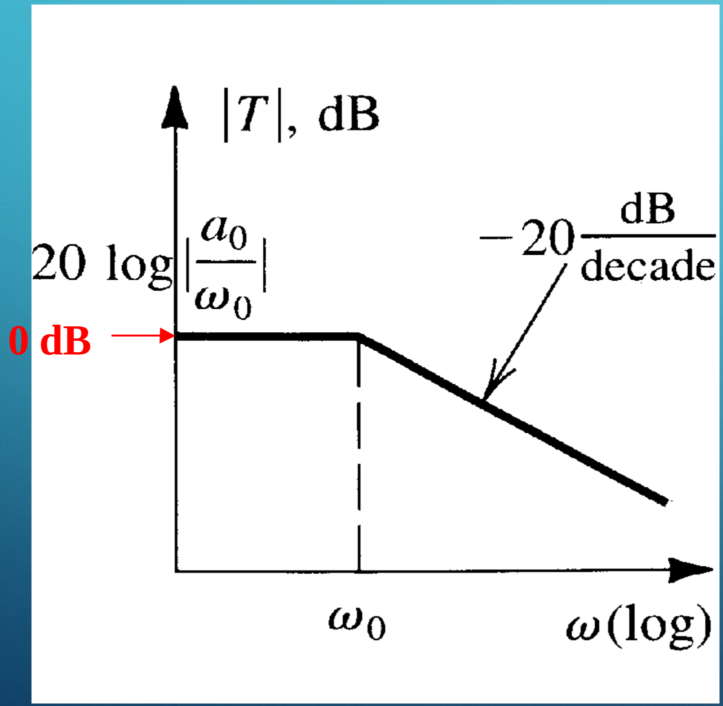
NETWORK THEORY

LECTURE 2

SECTION-D:TYPES OF FILTERS AND THEIR CHARACTERISTICS

Example of First Order Filter - Passive





* Low Pass Filter

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{Z_C}{Z_C+R}=\frac{\left(\frac{1}{sC}\right)}{R+\frac{1}{sC}}=\frac{1}{1+sRC}$$

$$=\frac{1}{1+s\frac{1}{(1/RC)}}=\frac{1}{1+\frac{s}{\omega_0}}=\frac{\omega_0}{s+\omega_0}$$

where $\omega_o=\frac{1}{RC}$

This has the form of a first order (low pass) filter with

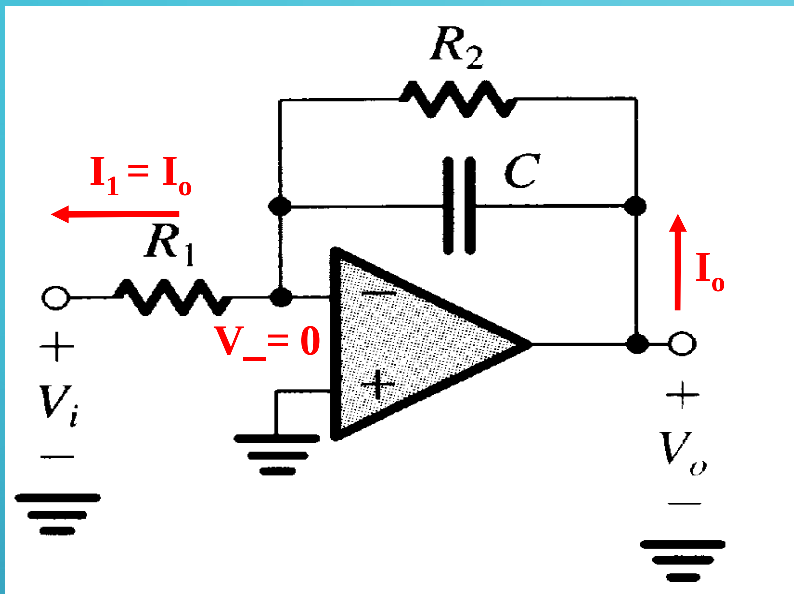
$a_1=0 \quad a_0=\omega_0$

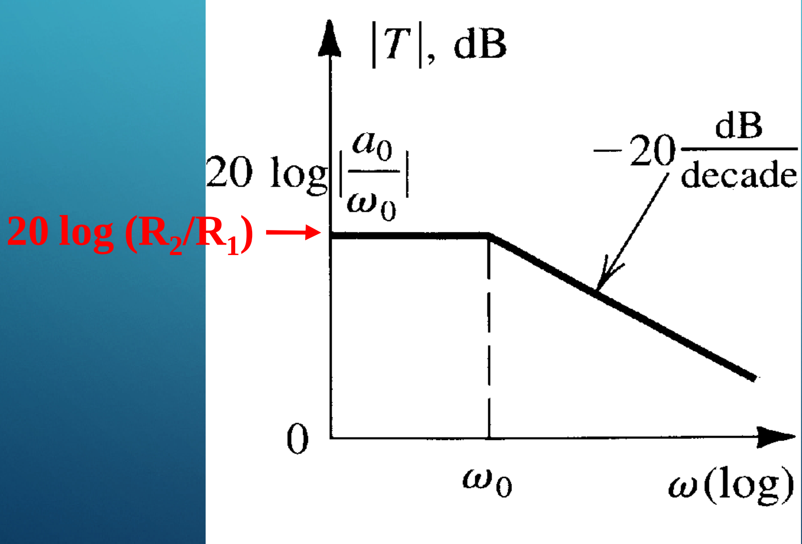
$$\mathrm{T}(\omega)(in\ dB)=20\log\left[\frac{1}{1+\left(\frac{\omega}{\omega_o}\right)^2}\right]^{1/2}=-10\log\left[1+\left(\frac{\omega}{\omega_o}\right)^2\right]$$

For $\omega << \omega_0,\ T(\omega)\approx 1 \longrightarrow 0\ dB$

For $\omega >> \omega_0,\ T(\omega)\approx -20\log\left(\frac{\omega}{\omega_0}\right)$

Example of First Order Filter - Active





* Low Pass Filter

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{I_o(R_2 \| Z_C)}{-I_1 R_1} = \frac{-(R_2 \| Z_C)}{R_1} = \frac{\left(\frac{1}{(1/R_2) + sC}\right)}{R_1} = \frac{-R_2}{R_1(1 + sR_2C)}$$

$$= \frac{-R_2}{R_1} \frac{1}{1 + s\frac{1}{(1/R_2C)}} = \frac{-R_2}{R_1} \frac{1}{1 + \frac{s}{\omega_0}} = \frac{-R_2}{R_1} \frac{\omega_0}{s + \omega_0}$$

where $\omega_o = \frac{1}{R_2 C}$

This has the form of a first order (low pass) filter with

$a_1 = 0 \quad a_0 = \frac{-R_2}{R_1}\omega_0$

Gain   Filter function

$$T(\omega)(in\ dB) = 20\log\left(\frac{R_2}{R_1}\right) + 20\log\left[\frac{1}{1 + \left(\frac{\omega}{\omega_o}\right)^2}\right]^{1/2}$$

$$= 20\log\left(\frac{R_2}{R_1}\right) - 10\log\left[1 + \left(\frac{\omega}{\omega_o}\right)^2\right]$$

For $\omega \ll \omega_0$, $T(\omega) \approx 20\log\left(\frac{R_2}{R_1}\right) > 0\ dB \quad for\ R_2 > R_1$

For $\omega \gg \omega_0$, $T(\omega) \approx 20\log\left(\frac{R_2}{R_1}\right) - 20\log\left(\frac{\omega}{\omega_0}\right)$

Second-Order Filter Functions

*** Second order filter functions are of the form**

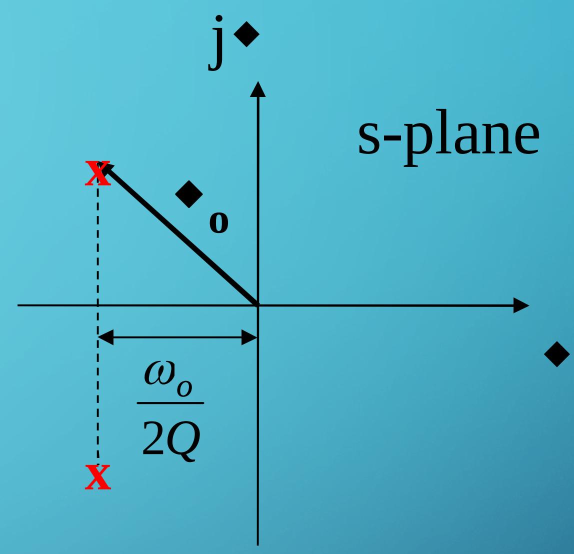
$$T(s) = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + b_1 s + b_o}$$

which we can rewrite as

$$T(s) = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

where ◆$_o$ and Q determine the poles

$$p_1, p_2 = \omega_{P1}, \omega_{P2} = -\frac{\omega_0}{2Q} \pm j \frac{\omega}{2Q_0} \sqrt{1 - 4Q^2}$$



This looks like the expression for the new poles that we had for a feedback amplifier with two poles.

*** There are seven second order filter types:**
Low pass, high pass, bandpass, notch,
Low-pass notch, High-pass notch and
All-pass

Second-Order Filter Functions

$$T(s) = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Low Pass

$a_1 = 0,\ a_2 = 0$

High Pass

$a_0 = 0,\ a_1 = 0$

Bandpass

$a_0 = 0,\ a_2 = 0$

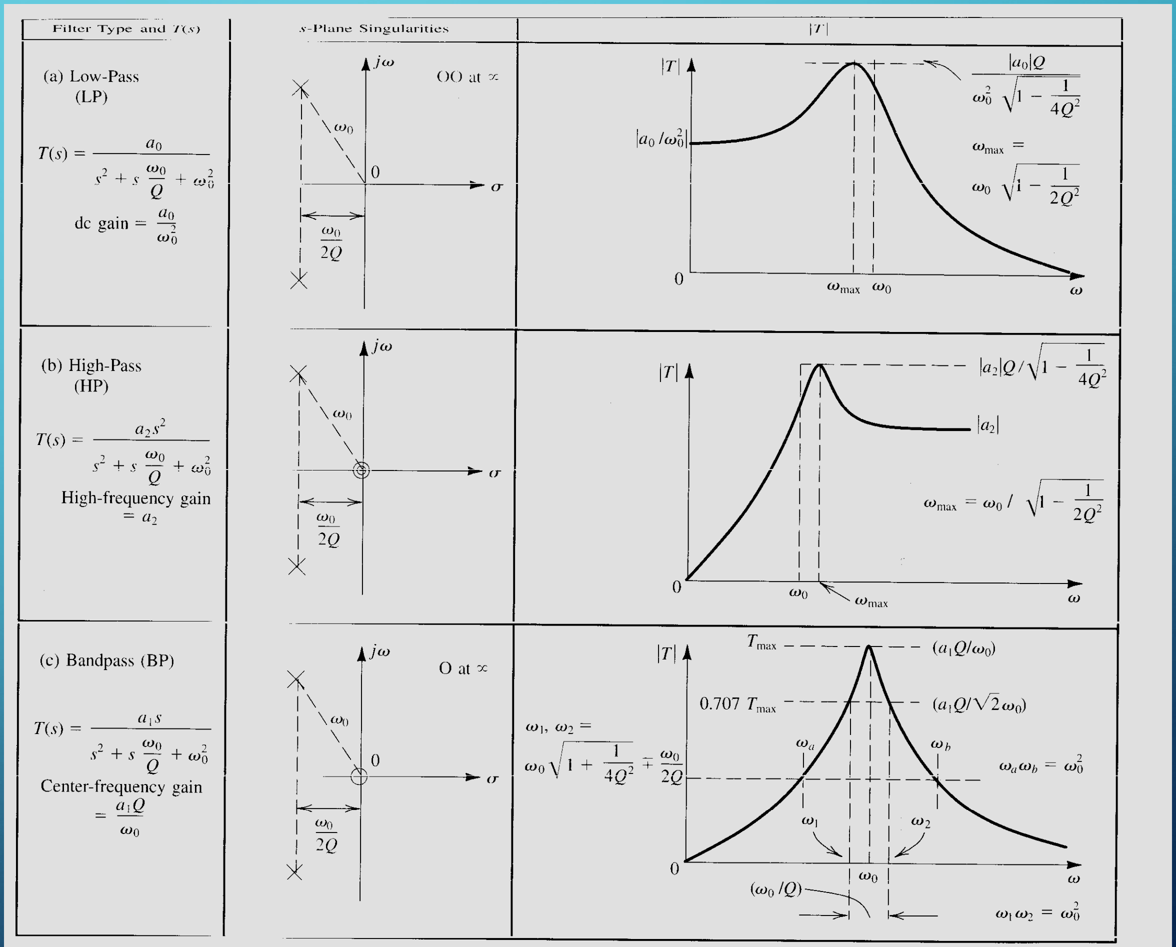


Fig. 11.16 Second-order filtering functions.

Second-Order Filter Functions

$$T(s) = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Notch

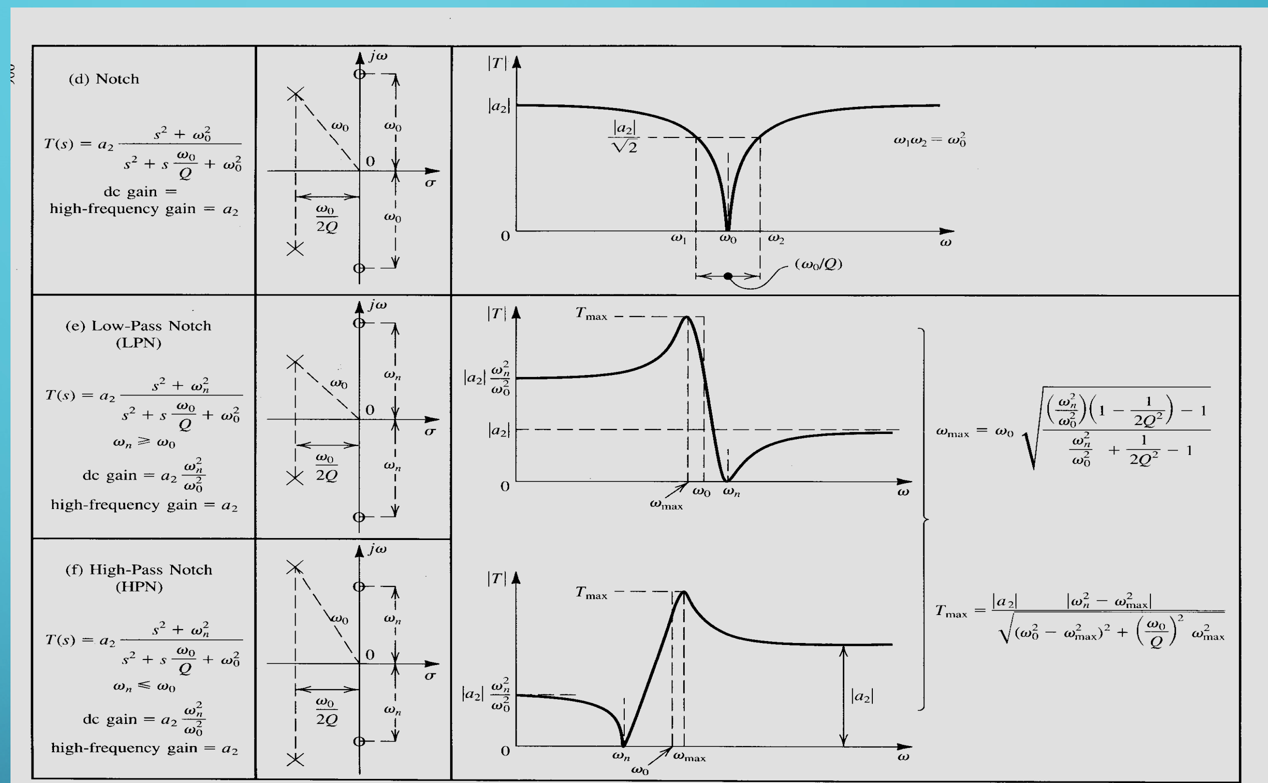
$a_1 = 0,\ a_o = \omega_o^2$

Low Pass Notch

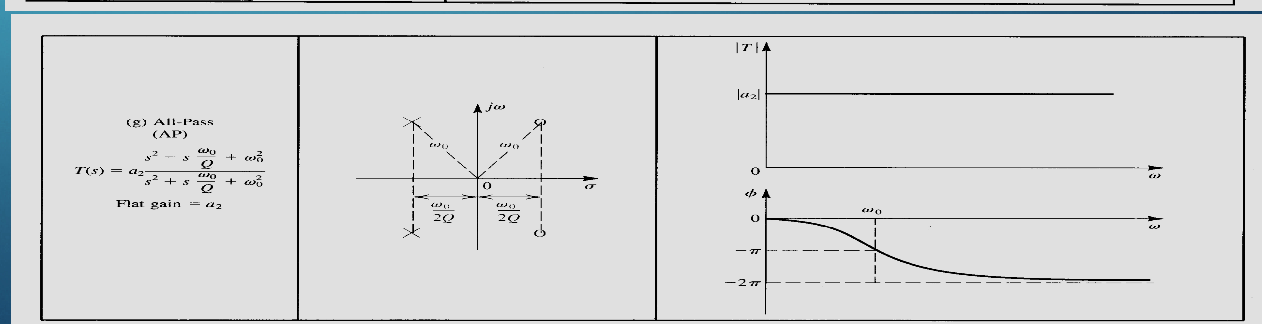
$a_1 = 0,\ a_o > \omega_o^2$

High Pass Notch

$a_1 = 0,\ a_o < \omega_o^2$



All-Pass



Passive Second Order Filter Functions

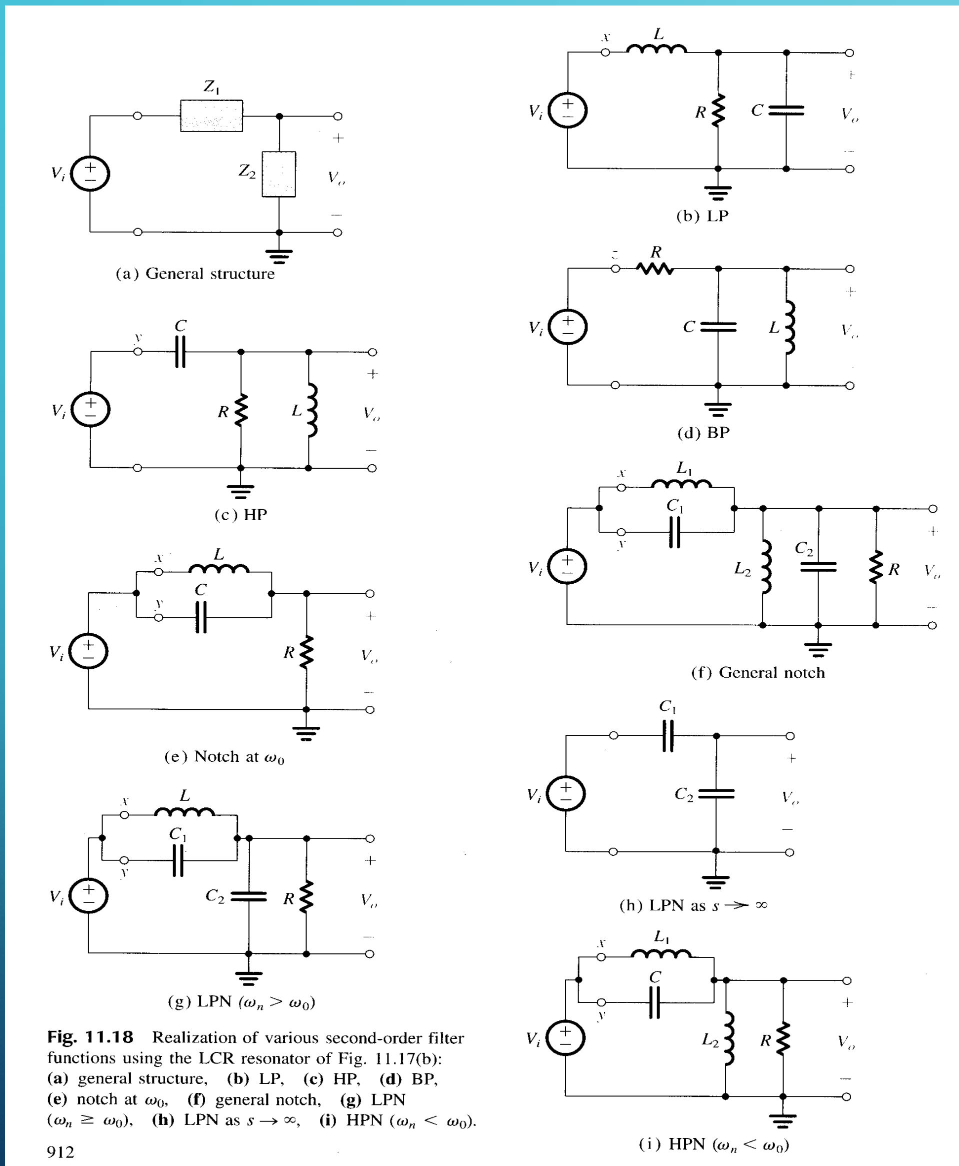


Fig. 11.18 Realization of various second-order filter functions using the LCR resonator of Fig. 11.17(b): **(a)** general structure, **(b)** LP, **(c)** HP, **(d)** BP, **(e)** notch at ω_0, **(f)** general notch, **(g)** LPN ($\omega_n \geq \omega_0$), **(h)** LPN as $s \to \infty$, **(i)** HPN ($\omega_n < \omega_0$).

* Second order filter functions can be implemented with simple RLC circuits

* General form is that of a voltage divider with a transfer function given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

* Seven types of second order filters

- High pass
- Low pass
- Bandpass
- Notch at ω_o
- General notch
- Low pass notch
- High pass notch