NETWORK THEORY

LECTURE 4

Section C

Topic Covered :Principles of network topology, graph matrices, network analysis using graph theory.

INTORDUCTION

- Topology or graph theory is that branch of science which studies the geometrical properties and special relations (for example between voltage and current) which remain unaffected by continuous change of shape or size of figures.
- The concept of topology was first applied to networks of Kirchhoff to study the relationship between the nodes and branches in a network. A circuit or network can be drawn in different shapes and sizes and still maintaining the relationship between the nodes and branches.
- Therefore network topology is the study of the properties of a network which remain unaffected when we stretch, twist or distort the size and shape of the network.

GRAPH

A linear graph or simply graph is defined as a collection of points called **nodes** and line segments called **branches**.

Procedure to draw the Graph: To draw a graph of network the following steps are used:

Step 1: Mark the number of nodes and branches of the network.

Step 2: The network elements are represented by lines.

Step 3: The voltage and current sources are replaced by their internal impedance.

GRAPH MATRICES

- INCIDENCE MATRICES
- CUT SET

INCIDENCE MATRIX [A]

Incidence matrix is defined as a two-dimensional array which provides information regarding the orientation of branches of a graph relative to the nodes of the graph.

2.3.1 Complete Incidence Matrix [A_c]

- for a graph with 'n' nodes and 'b' branches the complete incidence matrix A_c is a rectangular matrix of order $n \times b$ whose elements are defined by

$$a_{nb} = \begin{cases} 0\,; & \text{If branch } b \text{ is not connected with node } n \\ +1; & \text{If branch } b \text{ is connected with node } n \text{ and oriented away from node } n \\ -1; & \text{If branch } b \text{ is connected with node } n \text{ and oriented towards node } n. \end{cases} \qquad \text{...2.1}$$

- **Procedure for Construction of Incidence Matrix:**

1. Mark the nodes of the graph by numerals 1, 2, 3, 4 etc. and the branches of the graph by lowercase letters a, b, c, d etc.
2. Draw the graph and place the arrow mark to obtain the oriented graph (place of arrow mark depends on the direction of orientation)
3. Prepare a table as shown below. In the table, the branches are listed in column and nodes are in a row.

Nodes	*Branches* →				
	a	*b*	*c*	*d*	...
1					
2					
3					
4					
⋮					

4. At the intersection of row and column in table 2.1, write the incidence of the branch to the node by putting '0' or +1 or –1 in a manner as mentioned below.

Consider a branch-*a* and node-1

(i) If the branch-*a* is not connected to node-1 then at the intersection of column-*a* and row-1 enter '0'.

(ii) If the branch *a* is connected to node-1 and arrow in the branch is towards node-1 then at the intersection of column-*a* and row-1 enter –1.

(iii) If the branch-*a* is connected to node 1 and arrow in the branch is away from the node-1 then at the intersection of column-*a* and row-1 enter +1.

Loop Matrix or Circuit Matrix

The relation between the loop currents can be summarized in the form of a matrix called the tie-set matrix.

- Each loop of a graph with an orientation to the loop current by a curved arrow which gives its nodes a cyclic order. The orientation may be arbitrary chosen.
- In *i.e.* complete loop matrix B_c of a linear graph having n nodes and b branches, the number of columns is equal to the number of branches i.e. b and the number of rows is equal to the number of loops in linear graph.

Its elements have the following values.

$$b_{lb} = \begin{cases} 0; & \text{If branch } b \text{ is not in loop } l. \\ +1; & \text{If branch } b \text{ is in loop } l \text{ and their orientation coincide} \\ -1; & \text{If branch } b \text{ is in loop } l \text{ and their orientation donot coincide.} \end{cases} \qquad \text{...(2.2)}$$

TIE SET

A tie-set or fundamental circuits of *f* circuits is a set of branches that form a closed path in a graph such that the closed path contains one link or chord and remainder are tree branches. The closed path is known as loop.

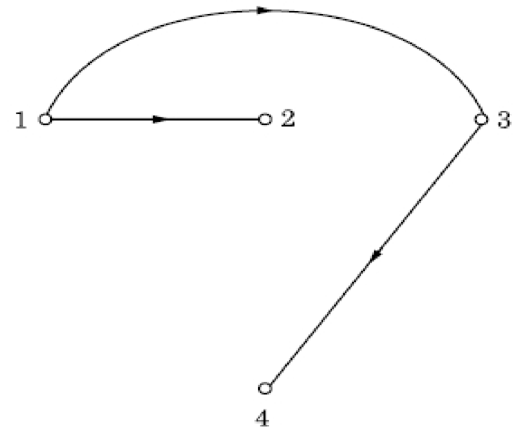

Fig. 2.19 Tree of graph

A tree of a graph does not have any closed path. On adding a link to the tree a closed path is created which is also called a *loop*. Circuits formed in this way will be called **fundamental circuits** or ***f*-circuit** or **tie-set**.

- The orientation of an *f*-circuit will be chosen to coincide with that of its connecting link.
- Also the number of tie-sets will be equal to number of links.

  i.e. Number of tie-sets = Number of links,

  or
- Number of tie-sets = $(b - n + 1)$ where b is the number of branches and n is the number of nodes.

NETWORK ANALYSIS USING GRAPH THEORY.

The following steps are required for nodal analysis of a given network based on graph theory:

1. Choose an arbitary node as the datum node and write down the incidence matrix **A** from the network graph.
2. Determine the branch admittance matrix Y_b.
3. Find the node-admittance matrix from $\boldsymbol{Y}_n = \boldsymbol{A}\ \boldsymbol{Y}_b\ \boldsymbol{A}^T$
4. Obtain the voltage and current source vectors V_s and I_s from the network.
5. Determine $\boldsymbol{I}_n$ from the relation

$$\boldsymbol{I}_n = \boldsymbol{A}\ \boldsymbol{Y}_b\ \boldsymbol{V}_s - \boldsymbol{A}\boldsymbol{I}_s$$

6. Write the node equations of the network from the relation.

$$\boldsymbol{I}_n = \boldsymbol{Y}_n\ \boldsymbol{V}_n$$

7. Use steps 5 and 6 to determine $\boldsymbol{V}_n$ as follows:

$$\boldsymbol{V}_n = Y_n^{-1}\ A Y_b\ V_s - Y^{-1}\ A I_s$$

8. Use the node,tranformation equation

$$V_b = A^T\ V_n \text{ and}$$

$$I_b = I_s + Y_b(V_b - V_s)$$

to determine branch voltage and current vectors V_b and I_b.