NETWORK THEORY

LECTURE 3

SECTION B

TOPIC COVERED :TWO PORT NETWORK

Reciprocal Networks

If a network does not contain non-reciprocal devices or materials* (i.e. ferrites, or active devices), then the network is "reciprocal."

$$\Rightarrow Z_{ij} = Z_{ji} \quad \left(Y_{ij} = Y_{ji}\right)$$

Note: The inverse of a symmetric matrix is symmetric.

$\Rightarrow [Z]$ and $[Y]$ are symmetric

* A reciprocal material is one that has reciprocal permittivity and permeability tensors. A reciprocal device is one that is made from reciprocal materials

Example of a nonreciprocal material: a biased ferrite

(This is very useful for making isolators and circulators.)

Reciprocal Materials

$$\underline{D} = \underline{\underline{\varepsilon}} \cdot \underline{E}$$

$$\underline{B} = \underline{\underline{\mu}} \cdot \underline{H}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \qquad \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix}$$

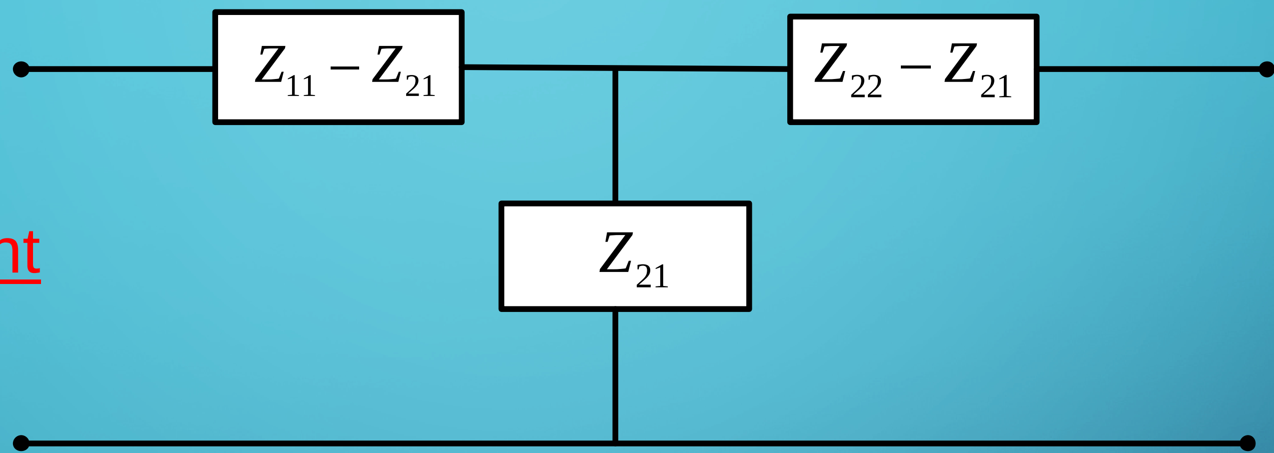
Reciprocal: $\varepsilon_{ij} = \varepsilon_{ji}, \;\; \mu_{ij} = \mu_{ji}$

Ferrite: $\underline{\underline{\mu}} = \mu_0 \begin{bmatrix} \alpha & j\gamma & 0 \\ -j\gamma & \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad \underline{\underline{\mu}}$ is <u>not</u> symmetric!

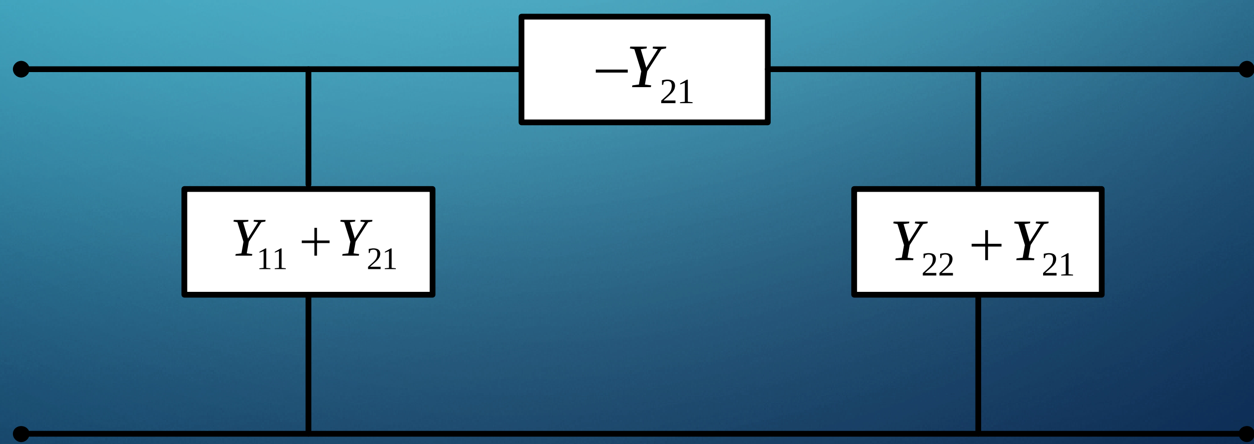
Reciprocal Networks (cont.)

We can show that the equivalent circuits for reciprocal 2-port networks are:

T-equivalent



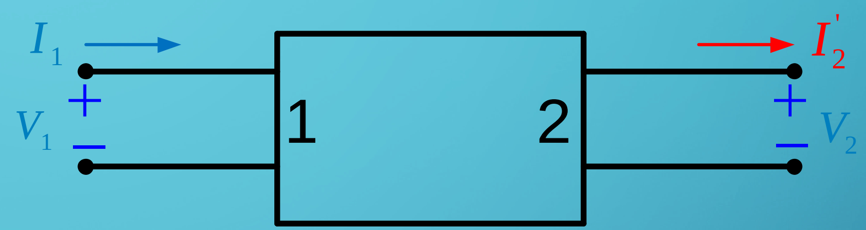
Pi-equivalent



ABCD-Parameters

There are defined only for 2-port networks.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2' \end{bmatrix}$$

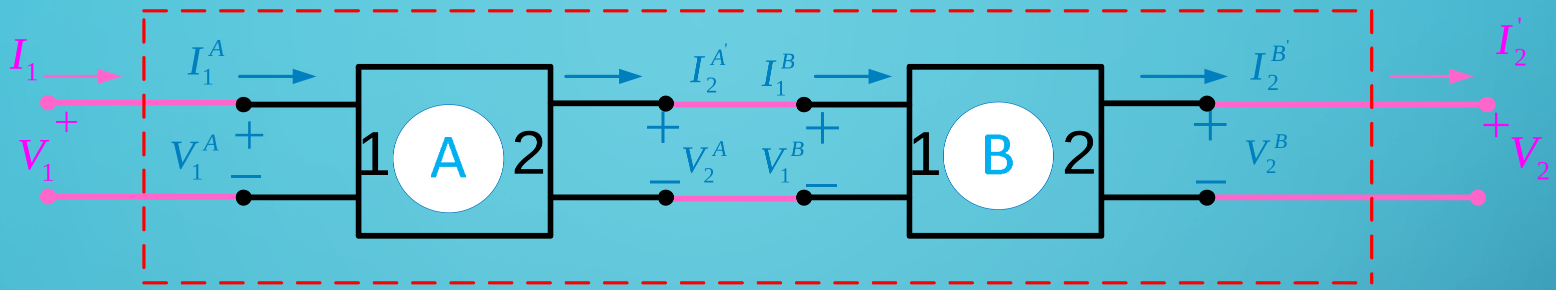


$$I_2' = -I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2'=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2'=0}$$

$$B = \left.\frac{V_1}{I_2'}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{I_2'}\right|_{V_2=0}$$

Cascaded Networks



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1^A \\ I_1^A \end{bmatrix} = \left[ABCD^A \right] \begin{bmatrix} V_2^A \\ I_2^{A'} \end{bmatrix}$$

$$= \left[ABCD^A \right] \begin{bmatrix} V_1^B \\ I_1^B \end{bmatrix}$$

$$= \left[ABCD^A \right] \left[ABCD^B \right] \begin{bmatrix} V_2^B \\ I_2^{B'} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \left[ABCD^{AB} \right] \begin{bmatrix} V_2 \\ I_2^{'} \end{bmatrix}$$

A nice property of the ABCD matrix is that it is easy to use with cascaded networks: you simply multiply the ABCD matrices together.

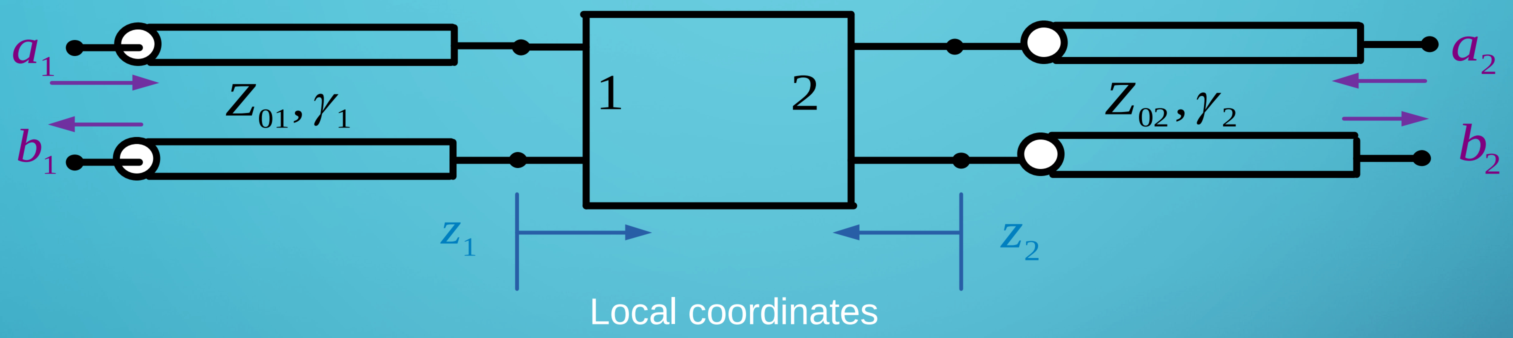
Scattering Parameters

- At high frequencies, Z, Y, h & ABCD parameters are difficult (if not impossible) to measure.
  - V and I are not uniquely defined
  - Even if defined, V and I are extremely difficult to measure (particularly I).
  - Required open and short-circuit conditions are often difficult to achieve.

- Scattering (S) parameters are often the best representation for multi-port networks at high frequency.

Scattering Parameters (cont.)

S-parameters are defined *assuming transmission lines are connected to each port*.



On each transmission line:

$$V_i(z_i) = V_{i0}^{+} e^{-\gamma_i z_i} + V_{i0}^{-} e^{+\gamma_i z_i} = V_i^{+}(z_i) + V_i^{-}(z_i)$$

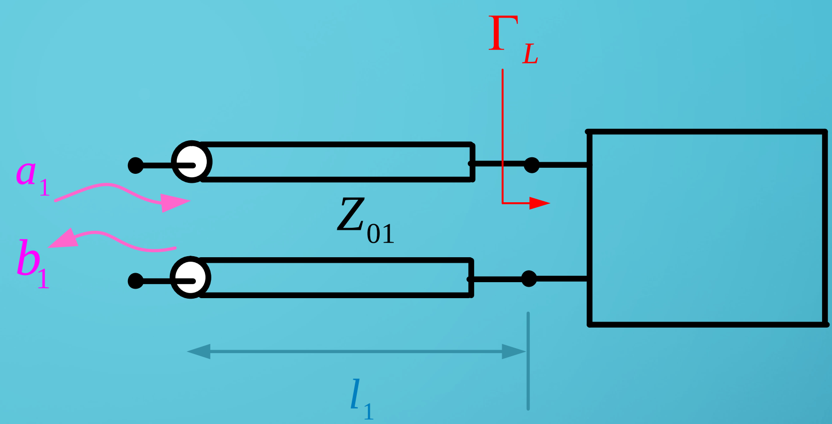
$$I_i(z_i) = \frac{V_i^{+}(z_i)}{Z_{0i}} - \frac{V_i^{-}(z_i)}{Z_{0i}} \qquad i = 1,\ 2$$

$$\text{Incoming wave function} \equiv a_i(z_i) \equiv V_i^{+}(z_i)\Big/\sqrt{Z_{0i}}$$

$$\text{Outgoing wave function} \equiv b_i(z_i) \equiv V_i^{-}(z_i)\Big/\sqrt{Z_{0i}}$$

For a One-Port Network

$$\Gamma_L = \frac{V_1^-(0)/\sqrt{Z_{01}}}{V_1^+(0)/\sqrt{Z_{01}}}$$



$$= \frac{b_1(0)}{a_1(0)}$$

$$\Rightarrow \quad b_1(0) = \Gamma_L a_1(0)$$
$$= S_{11}\, a_1(0)$$

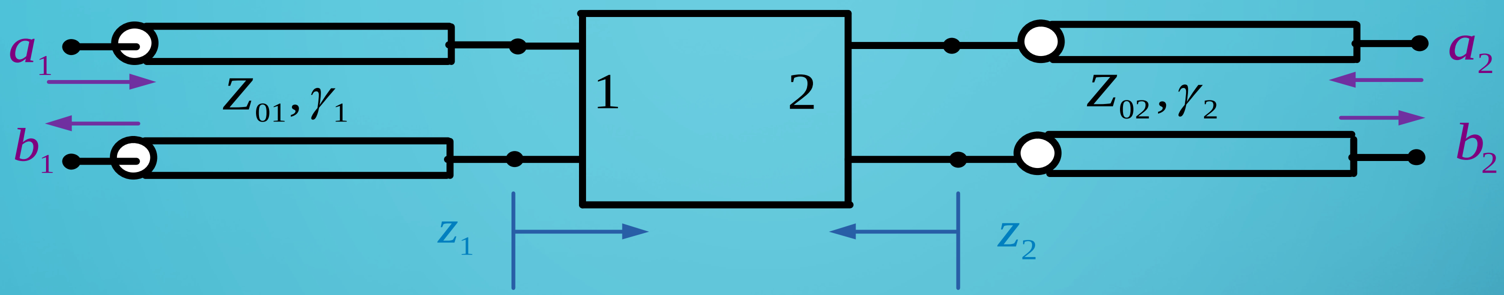
For a one-port network, S_{11} is defined to be the same as Γ_L.

$$= S_{11}$$

Incoming wave function $\equiv a_i(z_i) \equiv V_i^+(z_i)/\sqrt{Z_{0i}}$

Outgoing wave function $\equiv b_i(z_i) \equiv V_i^-(z_i)/\sqrt{Z_{0i}}$

For a Two-Port Network



$$b_1(0) = S_{11}a_1(0) + S_{12}a_2(0)$$
$$b_2(0) = S_{21}a_1(0) + S_{22}a_2(0)$$

Scattering matrix

$$\Rightarrow \begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} \Rightarrow [b] = [S][a]$$