NETWORK THEORY

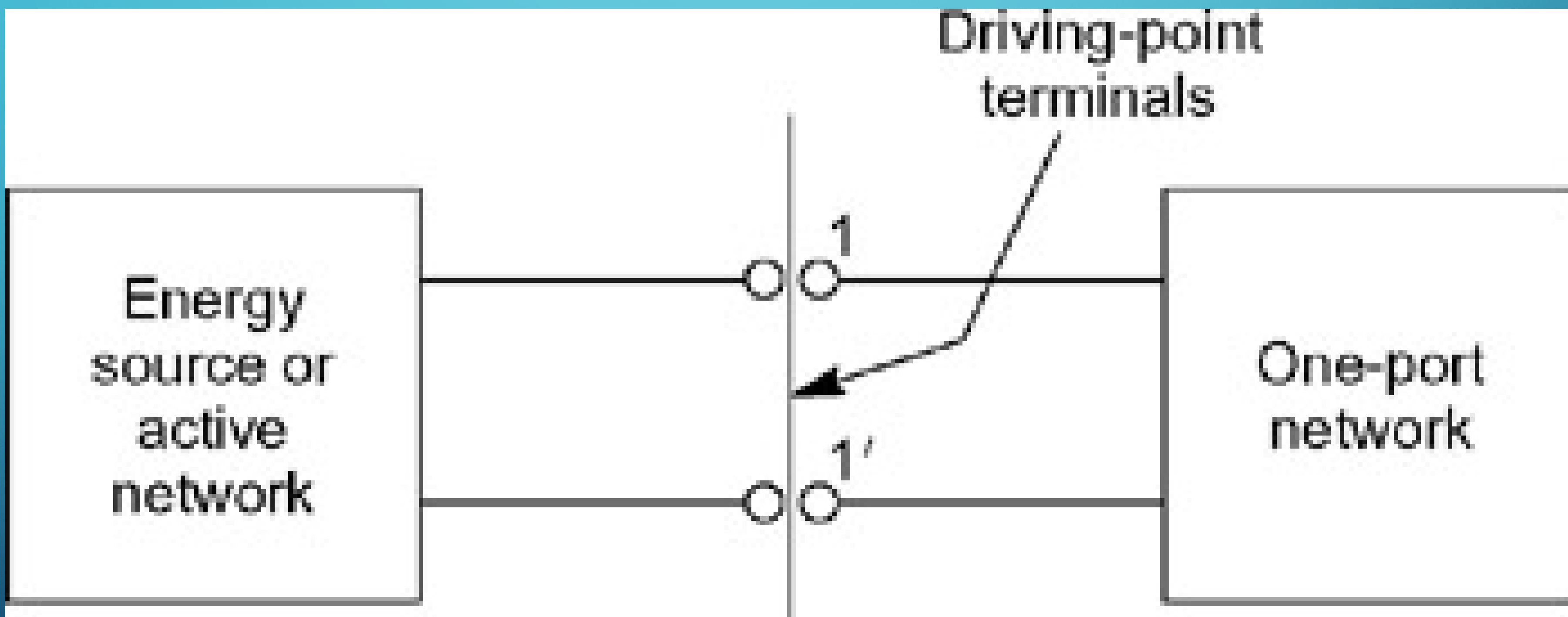
LECTURE 1

SECTION B

TOPIC COVERED: TERMINAL PAIRS OR PORTS

TERMINAL PAIRS OR PORTS

- Any network may be represented schematically by a rectangle or box as shown in Figure 1. A network may be used for a variety of purposes. Thus consider its use as a load connected to some other network. In order to connect it to the active network, there must be available two terminals of this passive network. Figure 1 shows a network with one pair of terminals 1-1 ? or with one port. Such a network may be called a *one-port network or one terminal-pair network.* When such a one-port network is connected to an energy source or an active network at its pair of terminals, the energy source provides the driving force for this one-port network and the pair of terminals constitute the *driving-point* of the network. One pair of terminals is known as a port.


Driving-point
terminals
Energy
source or
active
network
1
1′
One-port
network

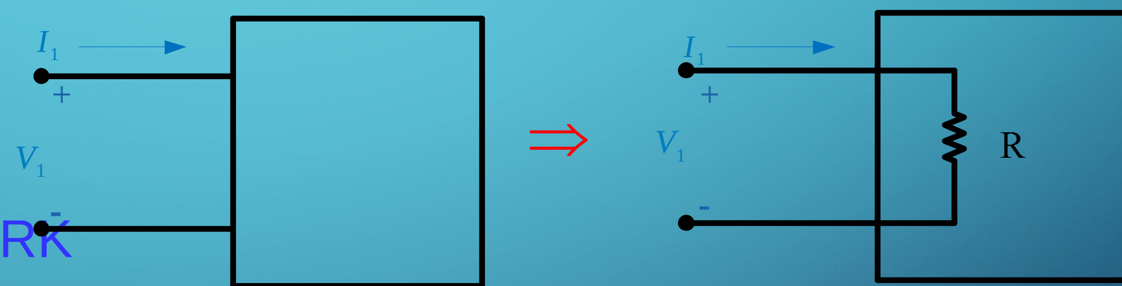
MULTIPORT NETWORKS

A GENERAL CIRCUIT CAN BE REPRESENTED BY A MULTI-PORT NETWORK, WHERE THE "PORTS" ARE DEFINED AS ACCESS TERMINALS AT WHICH WE CAN DEFINE VOLTAGES AND CURRENTS.

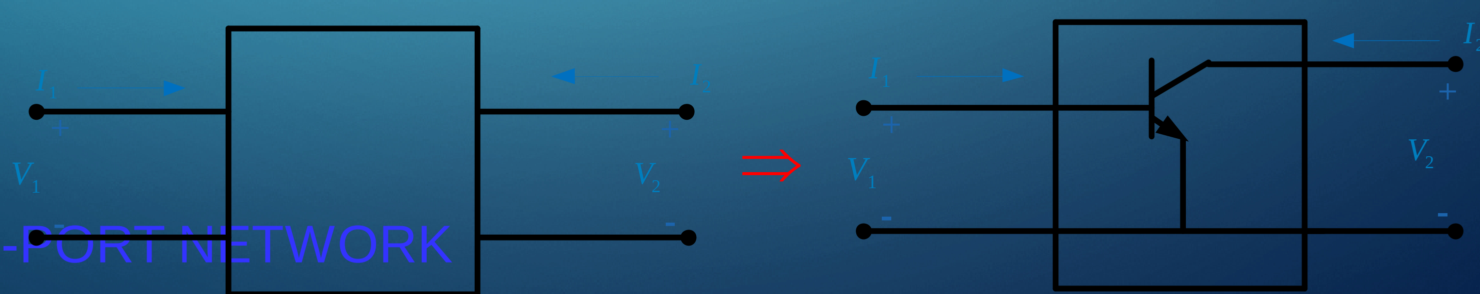
Note: Equal and opposite currents are assumed on the two wires of a port.

EXAMPLES:

1) ONE-PORT NETWORK

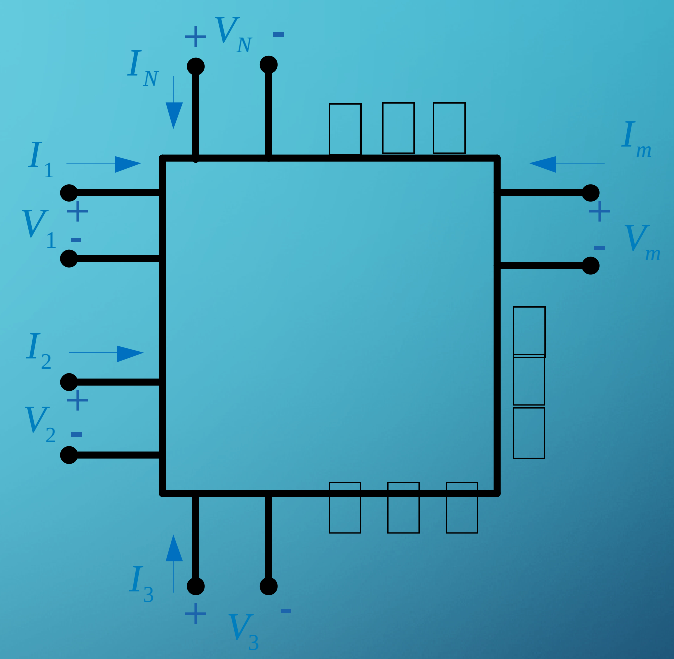


2) TWO-PORT NETWORK



Multiport Networks (cont.)

3) N-port Network



To represent multi-port networks we use:

- Z (impedance) parameters
- Y (admittance) parameters
- h (hybrid) parameters
- $ABCD$ parameters

Not easily measurable at high frequency

- S (scattering) parameters

Measurable at high frequency

Poynting Theorem (Phasor Domain)

$$\int_V -\frac{1}{2}\left(\underline{E}\cdot\underline{J}^{i*}\right)dV = \oint_S \underline{S}\cdot\underline{\hat{n}}\,dS$$

$$+\int_V \left(\frac{1}{2}\omega\varepsilon_c''\left|\underline{E}\right|^2 + \frac{1}{2}\omega\mu''\left|\underline{H}\right|^2\right)dV$$

$$+2j\omega\int_V \left(\frac{1}{4}\mu'\left|\underline{H}\right|^2 - \frac{1}{4}\varepsilon_c'\left|\underline{E}\right|^2\right)dV$$

⇓

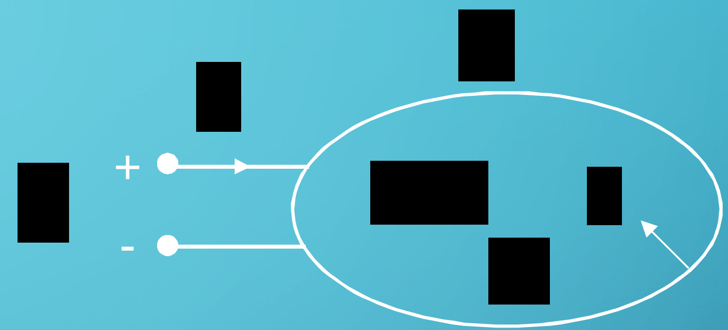
The last term is the VARS consumed by the region.

The notation < > denotes time-average.

$$P_s = P_f + \langle \mathrm{P}_d \rangle + j(2\omega)\left(\langle \mathrm{W}_m \rangle - \langle \mathrm{W}_e \rangle\right)$$

Self Impedance

Consider a general one-port network



Complex power delivered to network:

$$P_{in} = \frac{1}{2}\oiint_S \left(\underline{E} \times \underline{H}\right) \cdot \underline{\hat{n}}\; ds = P_d + j2\omega\left(W_m - W_e\right)$$

$$= \frac{1}{2} V_1 I_1^*$$

Average power dissipated in [W]

$P_d = \langle P_d \rangle$

Average magnetic energy (in [J]) stored inside V

$W_m = \langle W_m \rangle$

Average electric energy (in [J]) stored inside V

$W_e = \langle W_e \rangle$

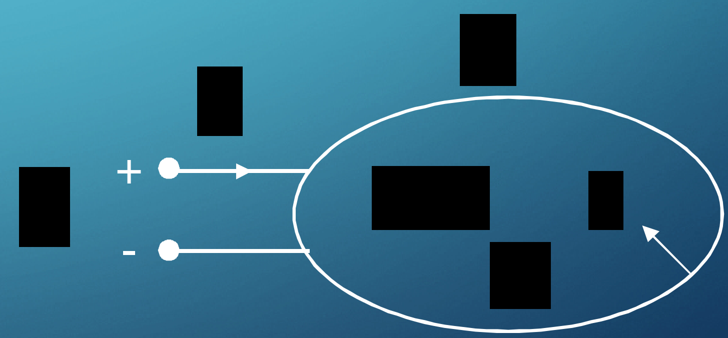
Define Self Impedance (Z_{in})

$$Z_{in} \equiv \frac{V_1}{I_1} = \frac{V_1 I_1^*}{|I_1|^2} = \frac{\frac{1}{2} V_1 I_1^*}{\frac{1}{2}|I_1|^2} = \frac{P_{in}}{\frac{1}{2}|I_1|^2}$$

$$= R_{in} + jX_{in} = \frac{P_d + j2\omega(W_m - W_e)}{\frac{1}{2}|I_1|^2}$$

$$R_{in} = \frac{2P_d}{|I_1|^2}$$

$$X_{in} = \frac{4\omega(W_m - W_e)}{|I_1|^2}$$



Self Impedance (cont.)

We can show that for physically realizable networks the following apply:

$$V_1(-\omega) = V_1^*(\omega)$$

$$\Rightarrow Z_{in}(-\omega) = Z_{in}^*(\omega)$$

$$I_1(-\omega) = I_1^*(\omega)$$

$\Rightarrow$ $R_{in}(\omega)$ is an even function of ω

$$Z_{in}(\omega) = R_{in}(\omega) + jX_{in}(\omega)$$

$X_{in}(\omega)$ is an odd function of ω

Note: Frequency is usually defined as a positive quantity. However, we consider the analytic continuation of the functions into the complex frequency plane.