

A decorative graphic on the left side of the slide, consisting of white lines and circles that resemble a circuit board or network diagram. The lines are vertical and horizontal, with some diagonal connections, and the circles are small and white, placed at various points along the lines.

NETWORK THEORY



LECTURE 4

SECTION A

TOPIC COVERED: TRANSIENT RESPONSE OF RC TO VARIOUS
EXCITATION SIGNALS

THE NATURAL RESPONSE

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation $f(t)$ equal to zero,

$$\tau \frac{dx_N(t)}{dt} + x_N(t) = 0 \text{ or } \frac{dx_N(t)}{dt} = -\frac{x_N(t)}{\tau}$$

$$x_N(t) = \alpha e^{-t/\tau}$$

It is called the natural response.

THE FORCED RESPONSE

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

Setting the excitation $f(t)$ equal to F , a constant for $t \geq 0$

$$\tau \frac{dx_F(t)}{dt} + x_F(t) = K_S F$$

$$x_F(t) = K_S F \text{ for } t \geq 0$$

It is called the forced response.

THE COMPLETE RESPONSE

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

The complete response is:

- the natural response +
- the forced response

$$\begin{aligned}x &= x_N(t) + x_F(t) \\ &= \alpha e^{-t/\tau} + K_S F \\ &= \alpha e^{-t/\tau} + x(\infty)\end{aligned}$$

Solve for α ,

for $t = 0$

$$x(t = 0) = x(0) = \alpha + x(\infty)$$

$$\alpha = x(0) - x(\infty)$$

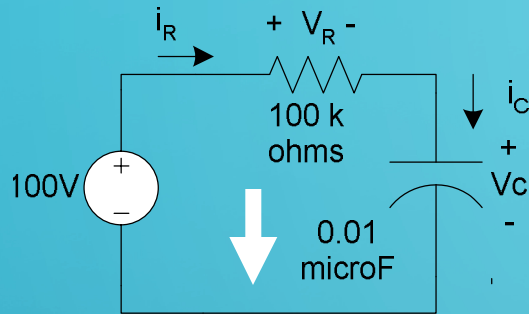
The Complete solution:

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

$[x(0) - x(\infty)]e^{-t/\tau}$ **called transient response**

$x(\infty)$ **called steady state response**

EXAMPLE



Initial condition $V_C(0) = 0V$

$$i_R = i_C$$

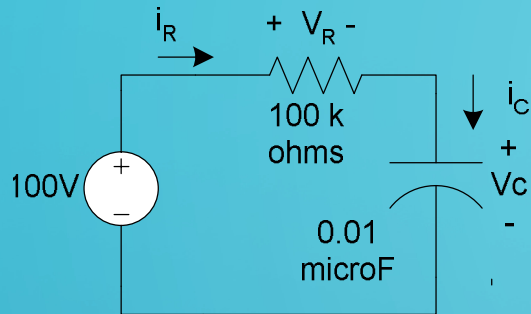
$$i_R = \frac{v_s - v_C}{R}, i_C = C \frac{dv_C}{dt}$$

$$RC \frac{dv_C}{dt} + v_C = v_s$$

$$10^5 \times 0.01 \times 10^{-6} \frac{dv_C}{dt} + v_C = 100$$

$$10^{-3} \frac{dv_C}{dt} + v_C = 100$$

EXAMPLE



Initial condition $v_C(0) = 0V$

$$10^{-3} \frac{dv_C}{dt} + v_C = 100$$

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

and

$$\begin{aligned} x &= x_N(t) + x_F(t) \\ &= \alpha e^{-t/\tau} + K_S F \\ &= \alpha e^{-t/\tau} + x(\infty) \end{aligned}$$

$$v_C = 100 + A e^{-\frac{t}{10^{-3}}}$$

$$\text{As } v_C(0) = 0, 0 = 100 + A$$

$$A = -100$$

$$v_C = 100 - 100 e^{-\frac{t}{10^{-3}}}$$

ENERGY STORED IN CAPACITOR

$$p = vi = Cv \frac{dv}{dt}$$

$$\int_{t_0}^t p dt = \int_{t_0}^t Cv \frac{dv}{dt} dt = C \int_{t_0}^t v dv$$

$$= \frac{1}{2} C \left\{ [v(t)]^2 - [v(t_0)]^2 \right\}$$

If the zero-energy reference is selected at t_0 , implying that the capacitor voltage is also zero at that instant, then

$$w_c(t) = \frac{1}{2} Cv^2$$