NETWORK THEORY

LECTURE 4

SECTION A

TOPIC COVERED: TRANSIENT RESPONSE OF RC TO VARIOUS EXCITATION SIGNALS

THE NATURAL RESPONSE

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation *f* (*t*) equal to zero,

$$\tau \frac{dx_N(t)}{dt} + x_N(t) = 0 \quad or \quad \frac{dx_N(t)}{dt} = -\frac{x_N(t)}{\tau}$$

$$x_N(t) = \alpha\, e^{-t/\tau}$$

It is called the natural response.

THE FORCED RESPONSE

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation *f* (*t*) equal to F, a constant for *t* ≥0

$$\tau \frac{dx_F(t)}{dt} + x_F(t) = K_S F$$

$$x_F(t) = K_S F \quad for \ t \geq 0$$

It is called the forced response.

THE COMPLETE RESPONSE

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete response is:

- the natural response +
- the forced response

$$x = x_N(t) + x_F(t)$$
$$= \alpha e^{-t/\tau} + K_S F$$
$$= \alpha e^{-t/\tau} + x(\infty)$$

Solve for α,

$$for \ t = 0$$
$$x(t=0) = x(0) = \alpha + x(\infty)$$
$$\alpha = x(0) - x(\infty)$$

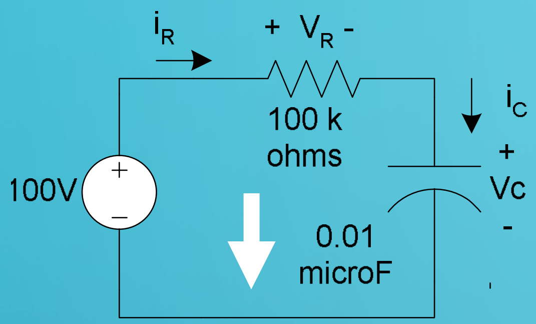
The Complete solution:

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

$[x(0) - x(\infty)]e^{-t/\tau}$ **called transient response**

$x(\infty)$ **called steady state response**

EXAMPLE



Initial condition $Vc(0)$ = 0V

$$i_R = i_C$$

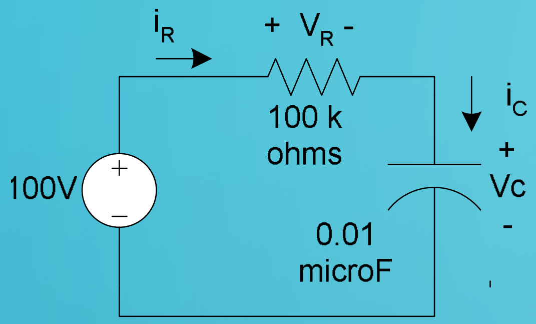
$$i_R = \frac{v_s - v_C}{R}, i_C = C\frac{dv_C}{dt}$$

$$RC\frac{dv_C}{dt} + v_C = v_s$$

$$10^5 \times 0.01 \times 10^{-6}\frac{dv_C}{dt} + v_C = 100$$

$$10^{-3}\frac{dv_C}{dt} + v_C = 100$$

EXAMPLE



Initial condition *Vc*(0) = 0V

$$10^{-3}\frac{dv_C}{dt}+v_C=100$$

$$\tau\frac{dx(t)}{dt}+x(t)=K_s f(t)$$

$$v_c=100+Ae^{-\frac{t}{10^{-3}}}$$

and

$$As\ \ v_c(0)=0,\ 0=100+A$$

$$A=-100$$

$$x=x_N(t)+x_F(t)$$

$$=\alpha e^{-t/\tau}+K_S F$$

$$=\alpha e^{-t/\tau}+x(\infty)$$

$$v_c=100-100e^{-\frac{t}{10^{-3}}}$$

ENERGY STORED IN CAPACITOR

$$p = vi = Cv\frac{dv}{dt}$$

$$\int_{t_o}^{t} p\,dt = \int_{t_o}^{t} Cv\frac{dv}{dt}\,dt = C\int_{t_o}^{t} v\,dv$$

$$= \frac{1}{2}C\left\{[v(t)]^2 - [v(t_o)]^2\right\}$$

If the zero-energy reference is selected at t_o, implying that the capacitor voltage is also zero at that instant, then

$$w_C(t) = \frac{1}{2}Cv^2$$