NETWORK THEORY

LECTURE 3

SECTION A

TOPIC COVERED: TRANSIENT RESPONSE OF RC TO VARIOUS EXCITATION SIGNALS

DISCHARGE OF A CAPACITANCE THROUGH A RESISTANCE

$$C\frac{dv_C(t)}{dt}+\frac{v_C(t)}{R}=0$$

$$RC\frac{dv_C(t)}{dt}+v_C(t)=0$$

$$v_C(t)=Ke^{st}$$

$$RCKse^{st}+Ke^{st}=0$$

$$s=\frac{-1}{RC}$$

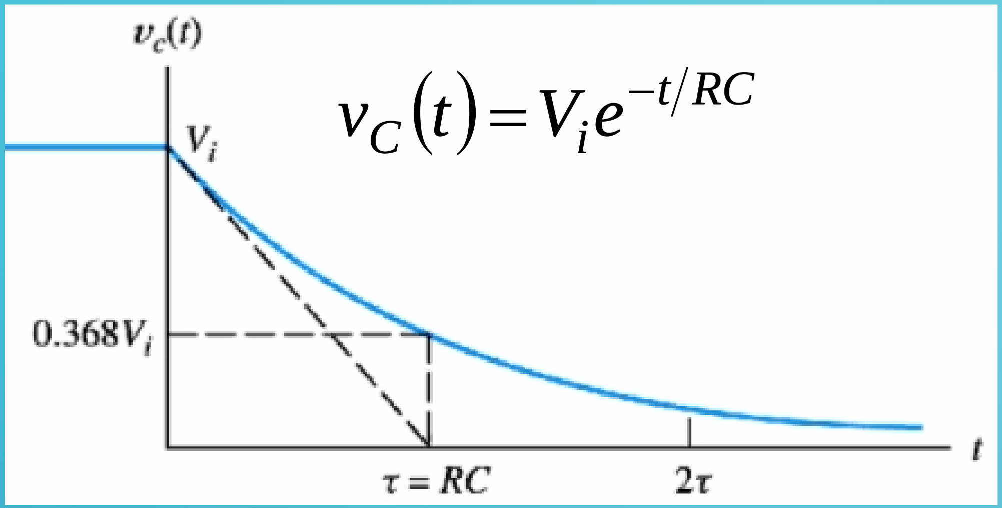
$$v_C(t)=Ke^{-t/RC}$$

$$v_C(0^+)=V_i$$

$$=Ke^{0/RC}$$

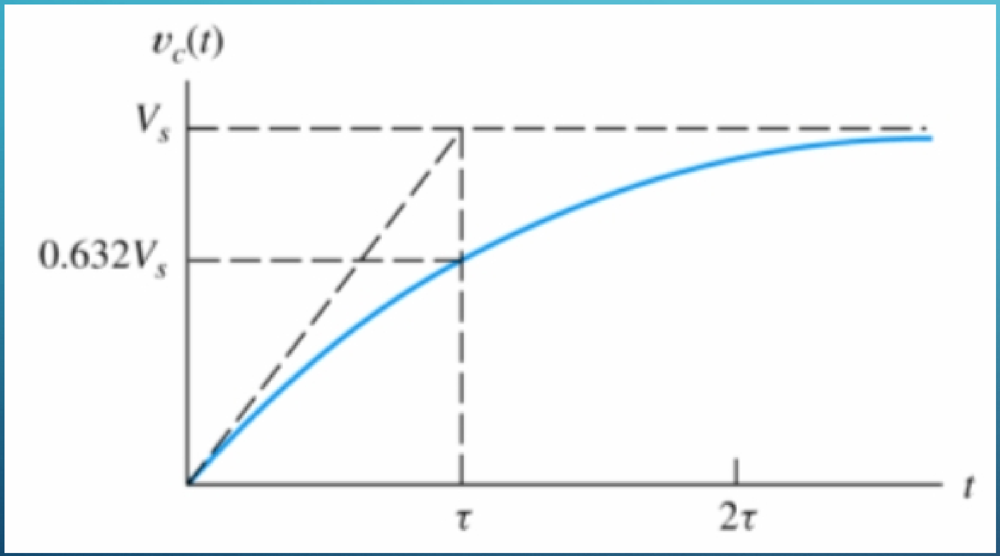
$$=K$$

$$v_C(t)=V_ie^{-t/RC}$$



$$v_C(t) = V_i e^{-t/RC}$$

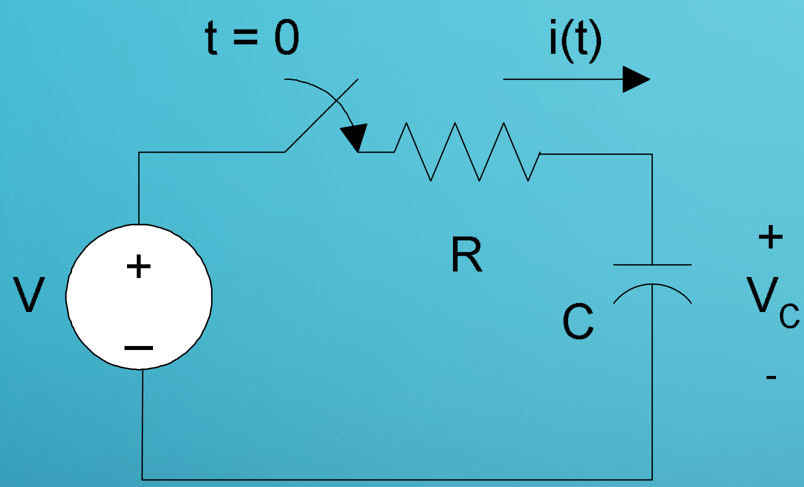
Exponential decay waveform
RC is called the time constant.
At time constant, the voltage is 36.8%
of the initial voltage.

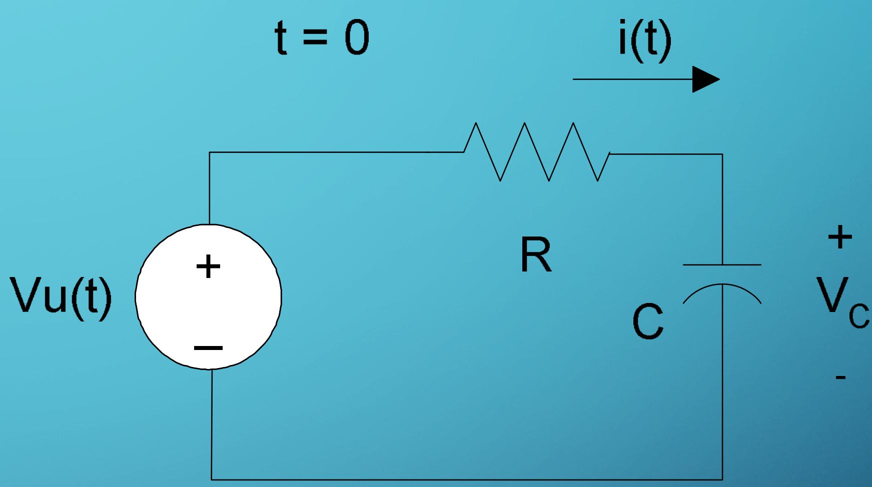


$$v_C(t) = V_i(1 - e^{-t/RC})$$

Exponential rising waveform
RC is called the time constant.
At time constant, the voltage is 63.2%
of the initial voltage.

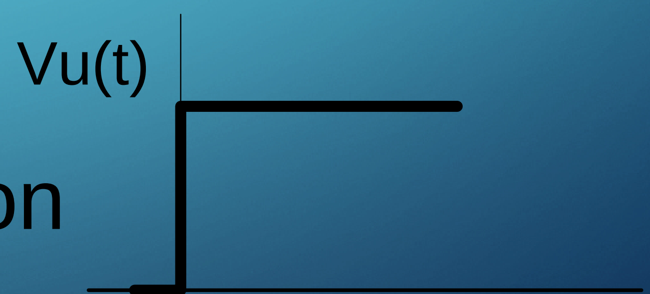
RC CIRCUIT



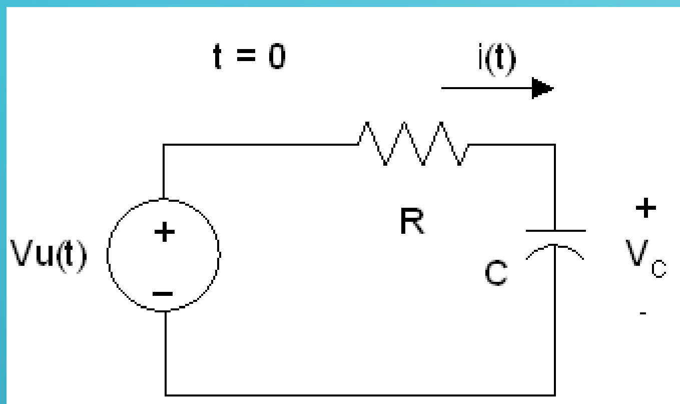


for $t = 0^-$, $i(t) = 0$

$u(t)$ is voltage-step function



RC CIRCUIT

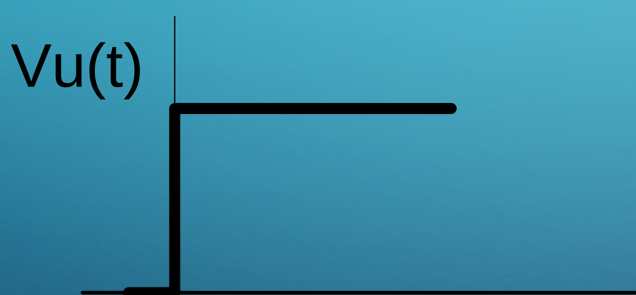


$$i_R = i_C$$

$$i_R = \frac{vu(t) - v_C}{R}, \quad i_C = C\frac{dv_C}{dt}$$

$$RC\frac{dv_C}{dt} + v_C = V, \quad vu(t) = V \; for \; t \geq 0$$

Solving the differential equation

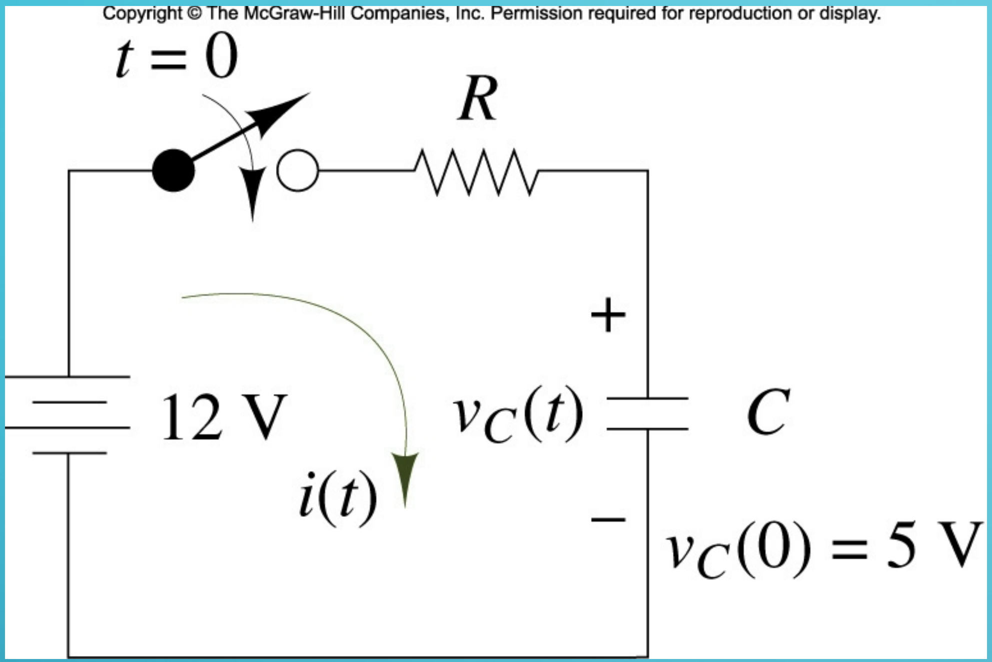


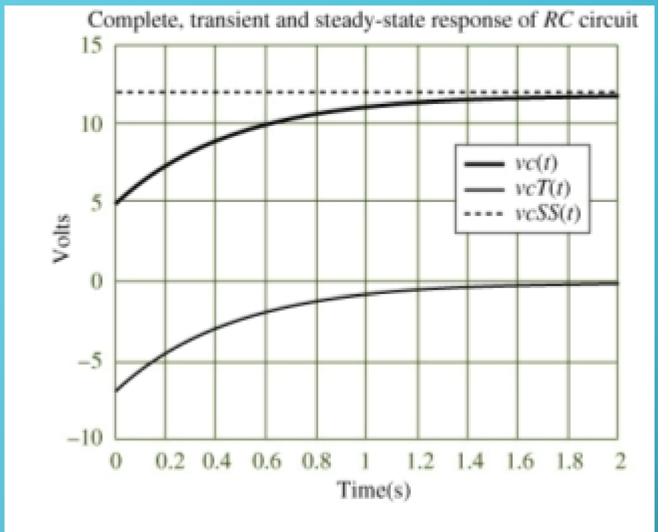
COMPLETE RESPONSE

Complete response

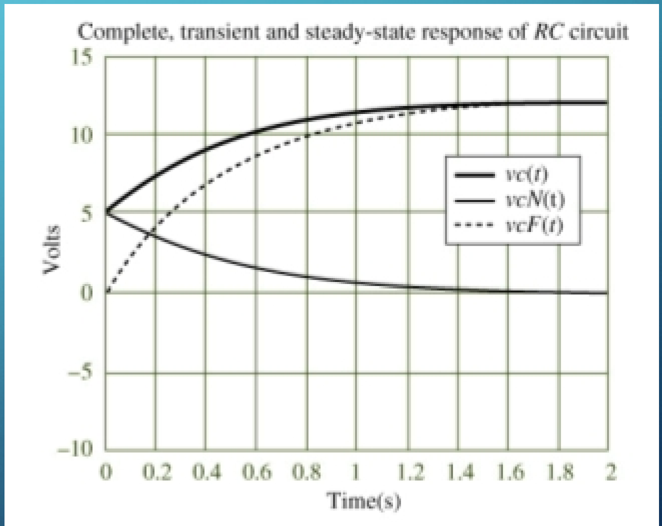
= natural response + forced response

- Natural response (source free response) is due to the initial condition
- Forced response is the due to the external excitation.





(a)

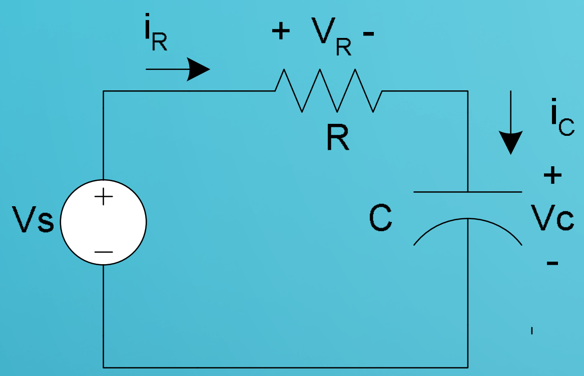


(b)

a). Complete, transient and steady state response

b). Complete, natural, and forced responses of the circuit

CIRCUIT ANALYSIS FOR RC CIRCUIT



Apply KCL

$$i_R = i_C$$

$$i_R = \frac{v_s - v_R}{R}, i_C = C\frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} + \frac{1}{RC}v_R = \frac{1}{RC}v_s$$

v_S is the source applied.

SOLUTION TO FIRST ORDER DIFFERENTIAL EQUATION

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Let the initial condition be $x(t = 0) = x(0)$, then we solve the differential equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete solution consits of two parts:

- the homogeneous solution (natural solution)
- the particular solution (forced solution)