DISCRETE STRUCTURE

Lecture-29

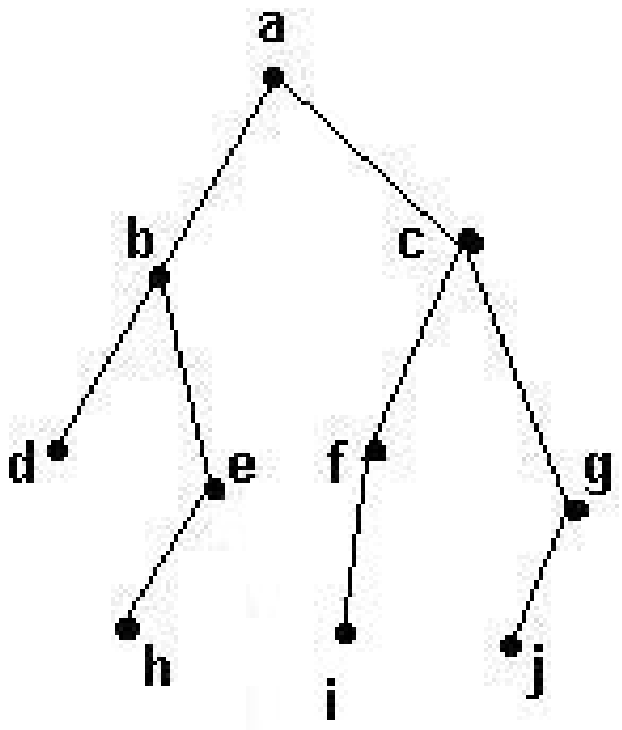
Binary Tree & its traversal

Topics covered

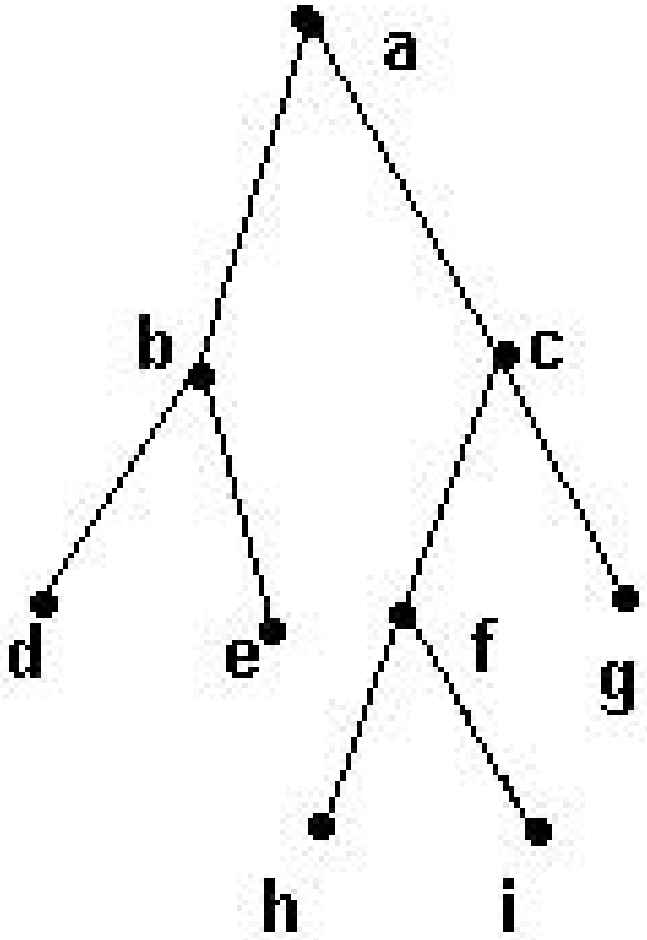
- Introduction to Binary tree
- Full binary tree
- Binary search tree

Introduction to Binary trees

A *binary tree* is a tree where each vertex has zero, one or two children

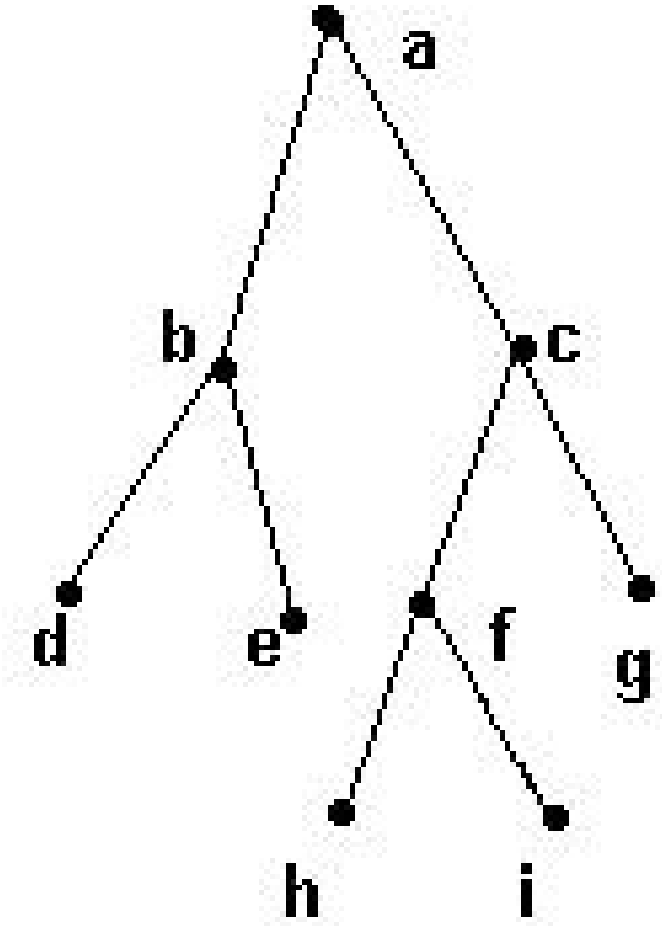


Full binary tree



A *full* binary tree is a binary tree in which each vertex has two or no children.

Full binary tree



Theorem: If T is a *full binary tree* with k internal vertices, then

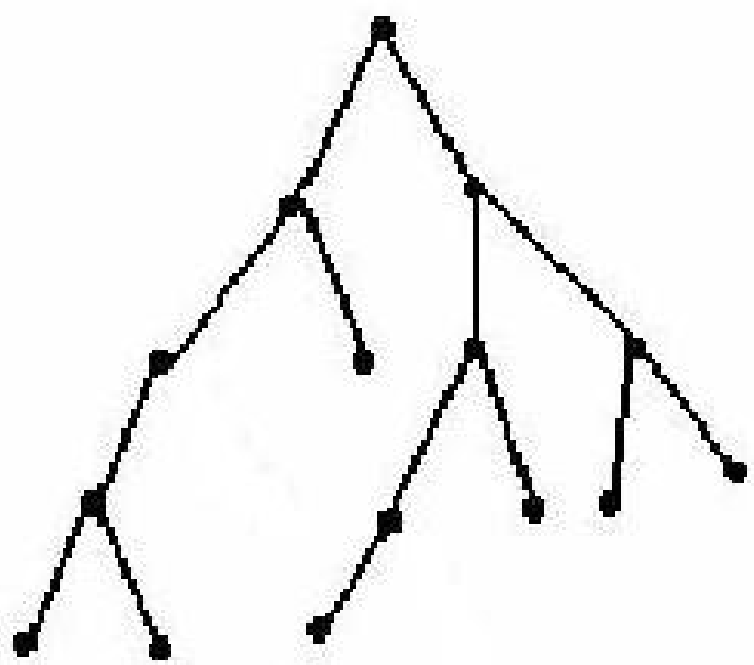
- T has k + 1 terminal vertices and
- the total number of vertices is 2k + 1.
  - Example: there are k = 4 internal vertices (a, b, c and f) and 5 terminal vertices (d, e, g, h and i) for a total of 9 vertices.

Height and terminal vertices

- Theorem : If a binary tree of height h has t terminal vertices, then lg $t \leq h$, where lg is logarithm base 2.

  Equivalently, $t \leq 2^h$.

  - Example, $h = 4$ and $t = 7$. Then: $t = 7 < 16 = 2^4 = 2^h$

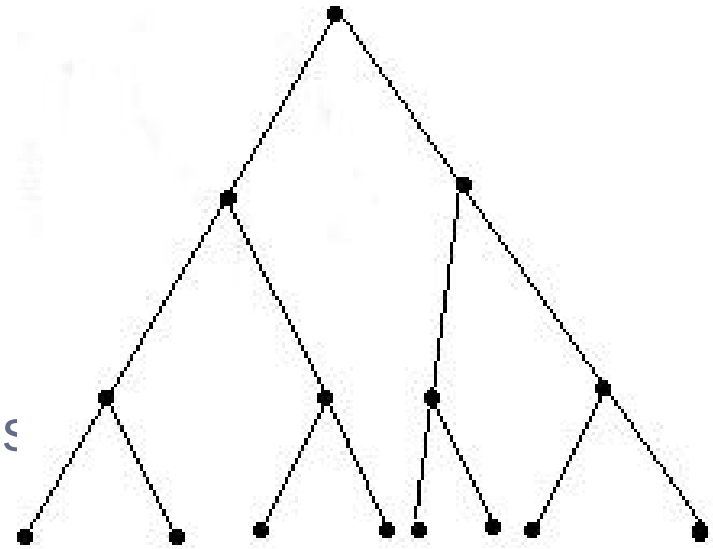


A case of equality

- If all t terminal vertices of a full binary tree T have the same level h = height of T, then

$$t = 2^h.$$

- Example:
  - The height is $h = 3$,
  - and the number of terminal vertices is $t = 8$
  - $t = 8 = 2^3 = 2^h$



Alphabetical order

Alphabetical or *lexicographic order* is the order of the dictionary:

a) start with an ordered set of symbols X = {a,b,c, …}. X can be infinite or finite.

b) Let $\alpha = x_1x_2\ldots x_m$ and $\beta = y_1y_2\ldots y_n$ be strings over X. Then define $\alpha < \beta$ if

- $x_1 < y_1$
- or if $x_j = y_j$ for all j, $1 \leq j \leq k$, for some k such that $1 \leq k \leq \min\{m,n\}$ and $x_{j+1} < y_{j+1}$
- or if $m \leq n$ and $x_j = y_j$ for all j, $1 \leq j \leq m$

Example of alphabetical order

- Let X = set of letters of the alphabet ordered according to precedence, i.e.

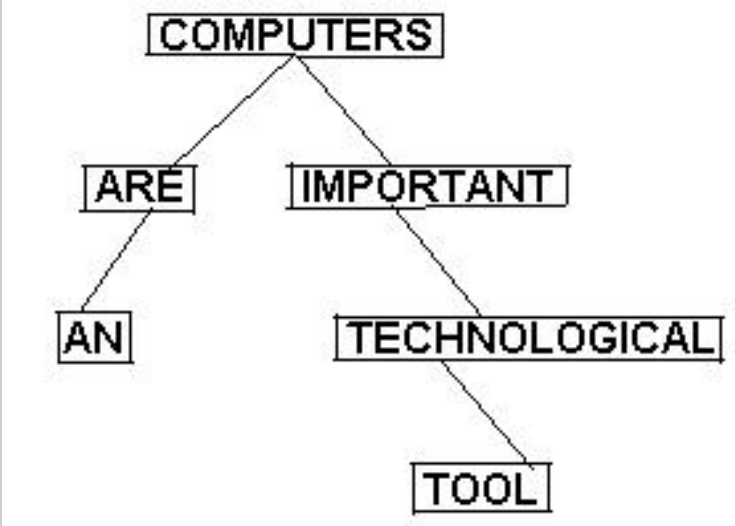
  $$a < b < c < \ldots < x < y < z$$

- Let α = *arboreal* and β = *arbiter*.
- In this case,
  - $x_1 = y_1 = a$,
  - $x_2 = y_2 = r$
  - $x_3 = y_3 = b$.
- So, we go the fourth letter: $x_4 = o$ and $y_4 = i$.
- Since $i < o$ we have that $\beta < \alpha$.

Binary search trees

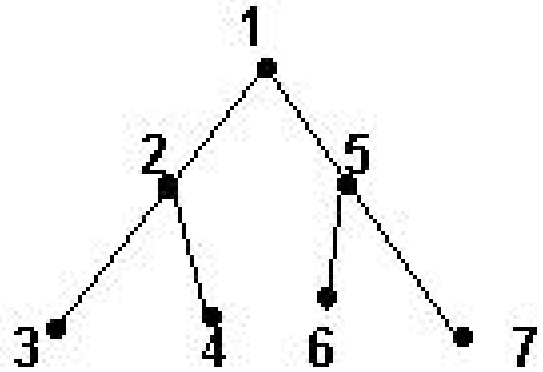
- Data are associated to each vertex
- Order data alphabetically, so that for each vertex v, data to the left of v are less than data in v
- and data to the right of v are greater than data in v

- Example: "Computers are an important technological tool"

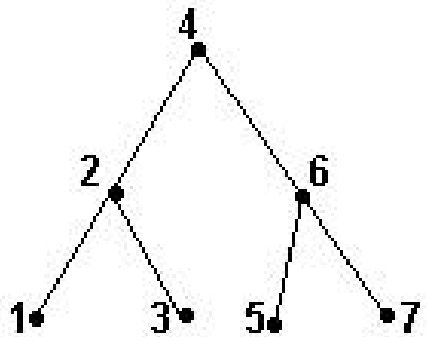


7.6 Tree Traversals

- 1: Pre-order traversal

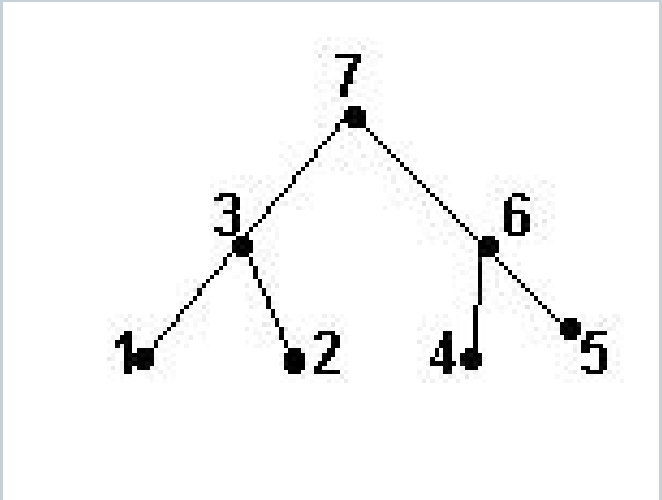


- 2: In-order traversal



Tree traversals

- 3: Post-order traversal



Assignment

Q.1) What is binary tree?

Q.2) What is mean by tree traversal?

Q.3)Give pre-oder & post order traversal of a given tree

Application & Scope of research

- Application

1. Tree structure is used to organize information in database systems. The trees represent the syntactic structure of source programs in compilers. They are used in the analysis of electrical circuits, representation of mathematical formulae and file directory system. Parsing by compilers also use tree structure.

- Scope of research

1. Bluetooth sensing technology