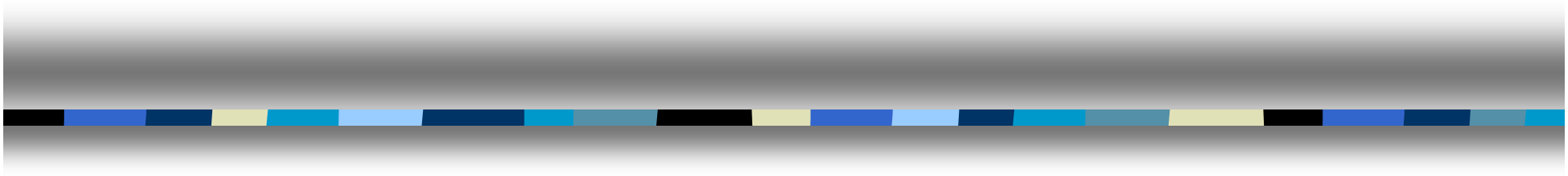
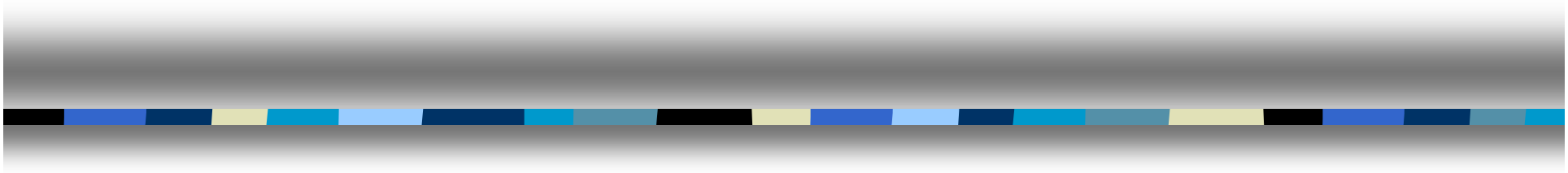
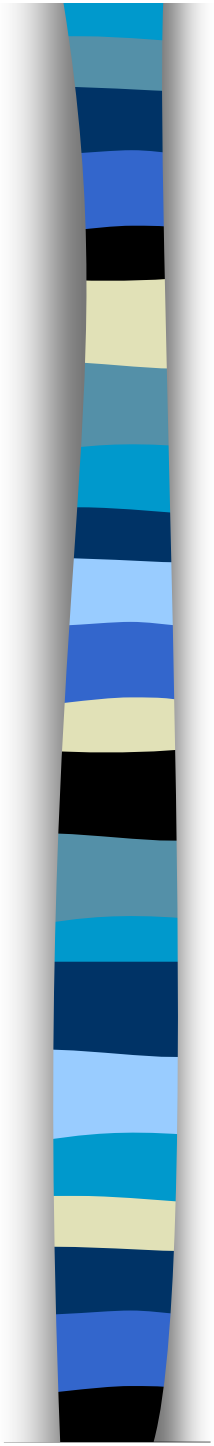
DISCRETE STRUCTURE



Lecture-24



Weighted graph& Shortest path in Weighted

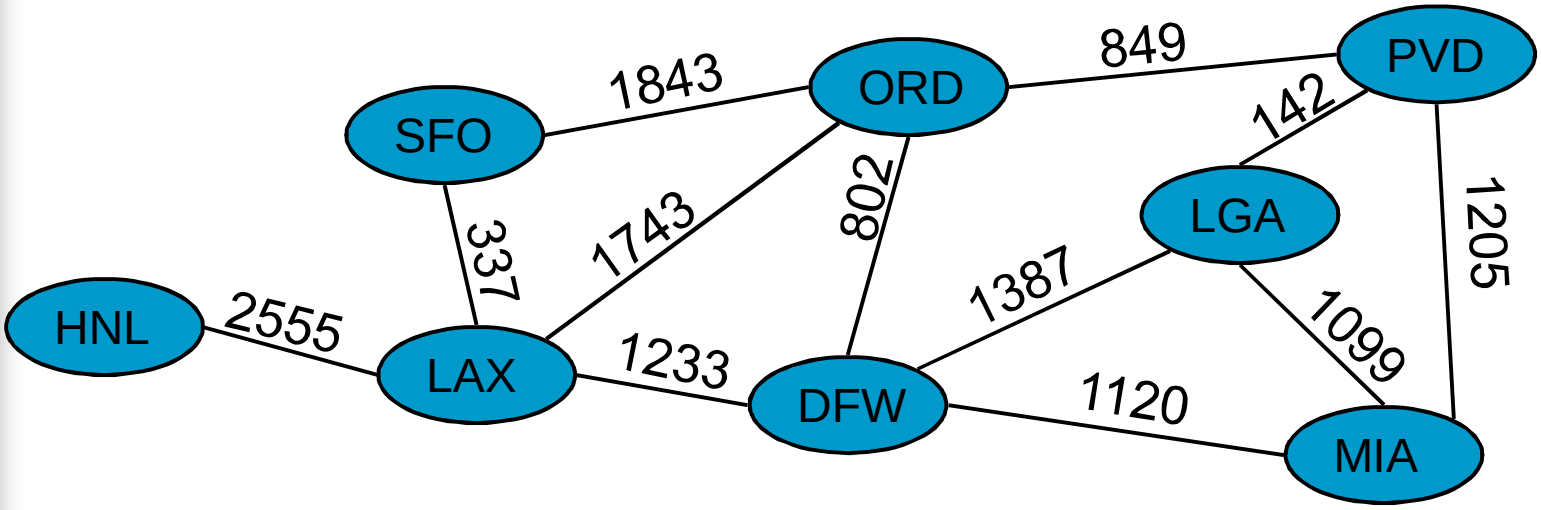


Topics covered

- Introduction to Weighted Graphs
- Shortest path in weighted graphs

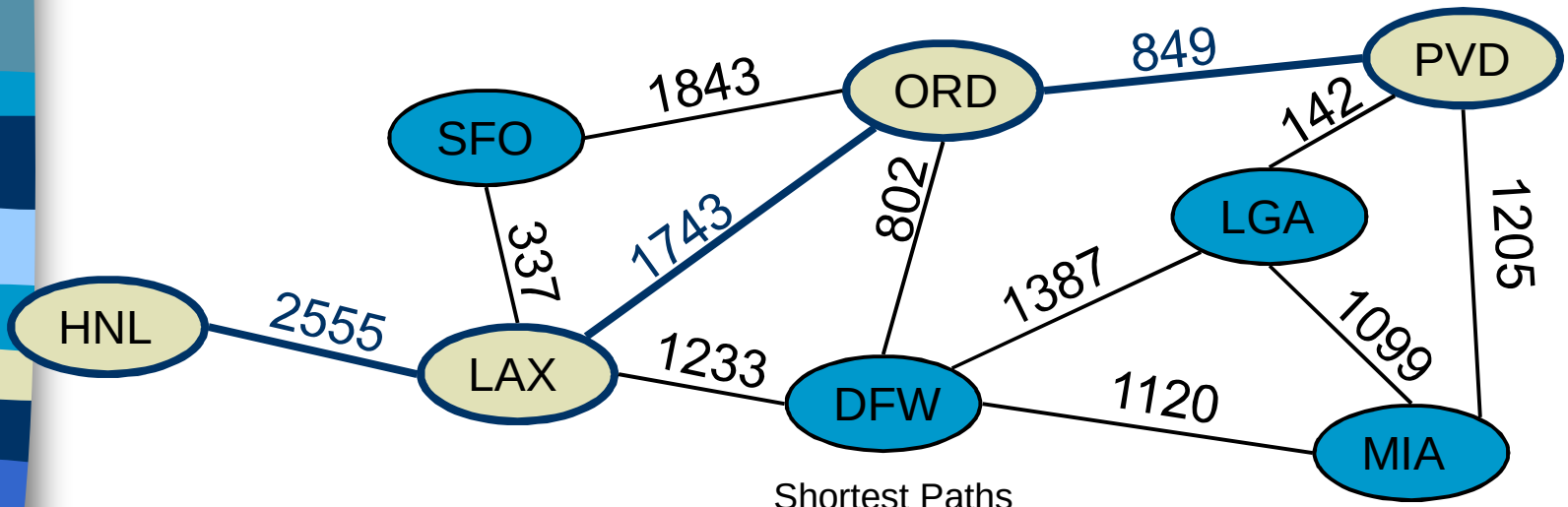
Introduction to Weighted Graphs

- In a weighted graph, each edge has an associated numerical value, called the weight of the edge
- Edge weights may represent, distances, costs, etc.
- Example:
  - In a flight route graph, the weight of an edge represents the distance in miles between the endpoint airports



Shortest Path Problem

- Given a weighted graph and two vertices ***u*** and ***v***, we want to find a path of minimum total weight between ***u*** and ***v*.**
  - Length of a path is the sum of the weights of its edges.
- Example:
  - Shortest path between Providence and Honolulu



Weighted Graphs

- A weighted graph is a graph in which each edge (u, v) has a weight w(u, v). Each weight is a real number.
- Weights can represent distance, cost, time, capacity, etc.
- The length of a path in a weighted graph is the sum of the weights on the edges.
- Dijkstra's Algorithm finds the shortest path between two vertices.

Shortest Path Properties

Property 1:

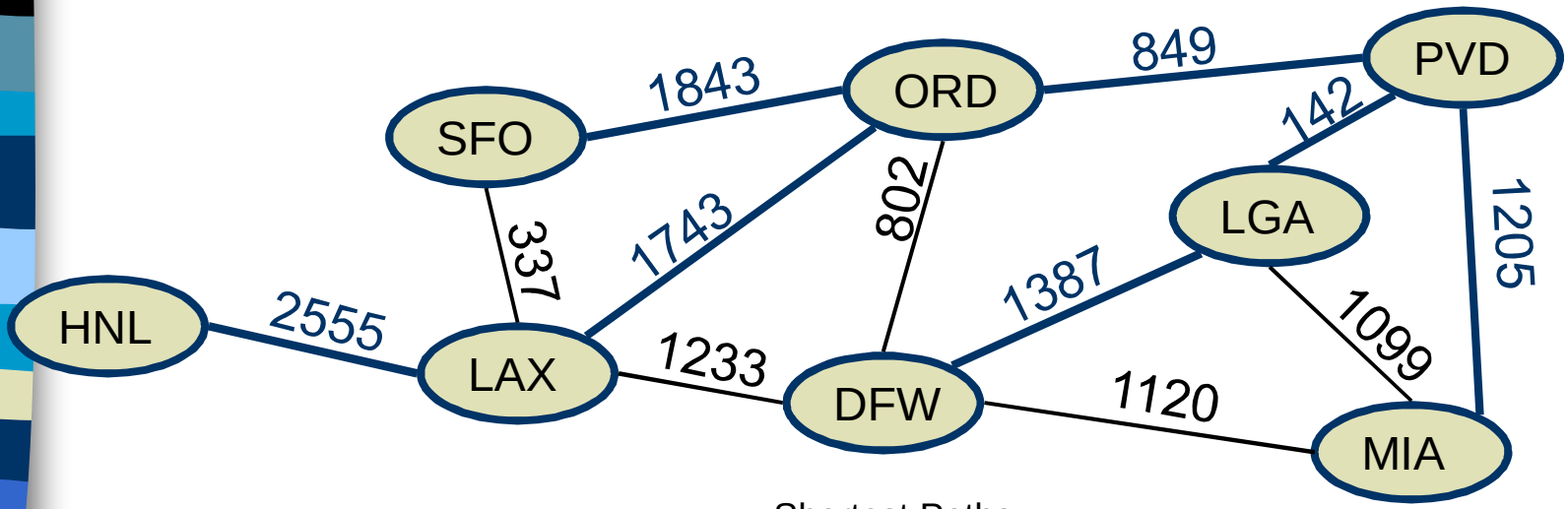
A subpath of a shortest path is itself a shortest path

Property 2:

There is a tree of shortest paths from a start vertex to all the other vertices

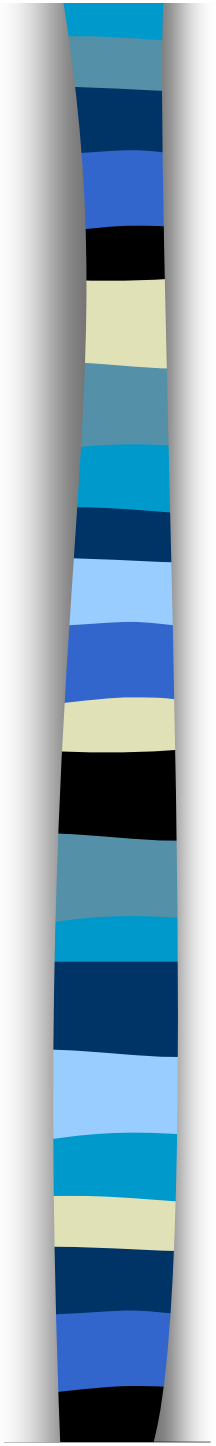
Example:

Tree of shortest paths from Providence



Dijkstra's Algorithm

- The distance of a vertex v from a vertex s is the length of a shortest path between s and v
- Dijkstra's algorithm computes the distances of all the vertices from a given start vertex s
- Assumptions:
  - the graph is connected
  - the edges are undirected
  - the edge weights are nonnegative



Dijkstra's Shortest Path Algorithm

- Due to Edsger W. Dijkstra (1959, when 29).

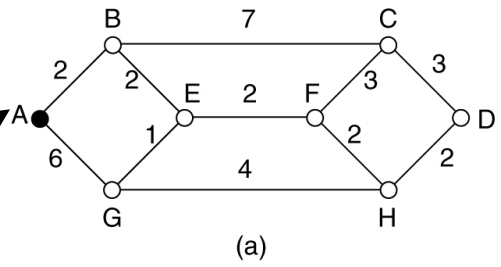
- Assign scores to verticies:
  - S(v) = score of vertex v (some integer)
  - there are *temporary* and *permanent* scores
  - all verticies start with a temporary score of infinity (Inf)

continued

Example of Dijkstra's Algorithm

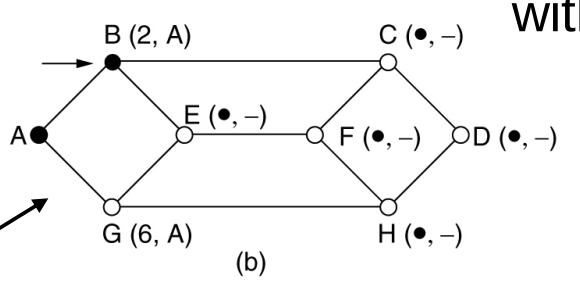
Find shortest-path from A to D:

Start



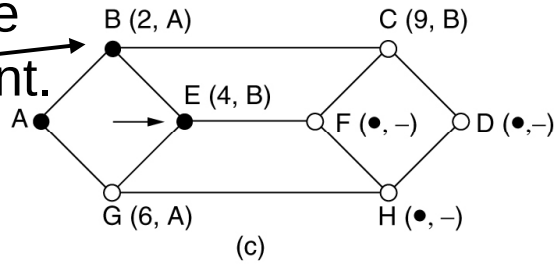
(a)

Label each adjacent node with distance to A.

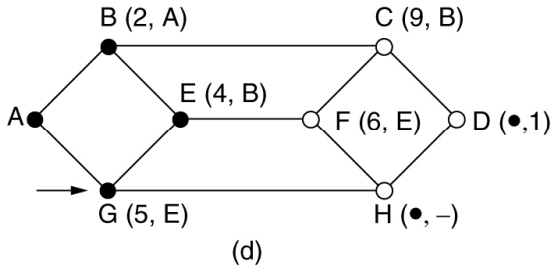


(b)

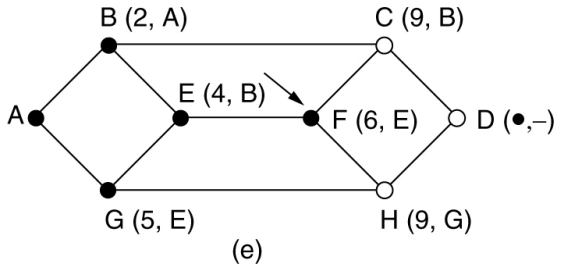
B is made permanent.



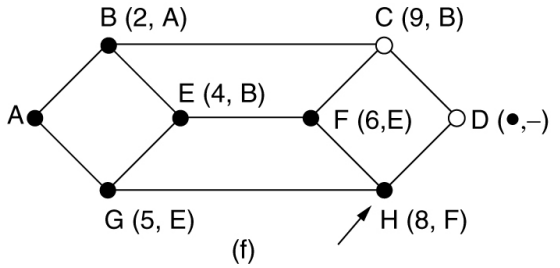
(c)



(d)



(e)



(f)

Application & Scope of research

Application

- Internet packet routing
- Flight reservations
- Driving directions

Scope of research

- To find shortest path from source to destination in a given network s.t. it has minimum cost