DISCRETE STRUCTURE

Lecture-23

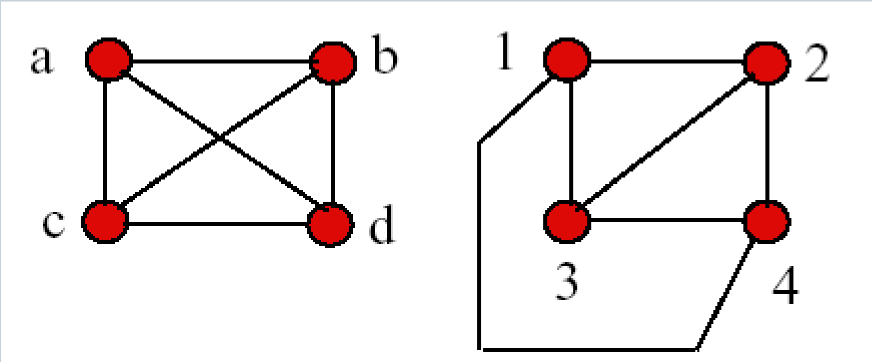
Isomorphic graph

Topics covered

- Introduction to Isomorphic graphs

Introduction to Graph Isomorphism

- The two graphs below are in fact the same graph (structure-wise).
- We say that these graphs are *isomorphic*.



Graph Isomorphism

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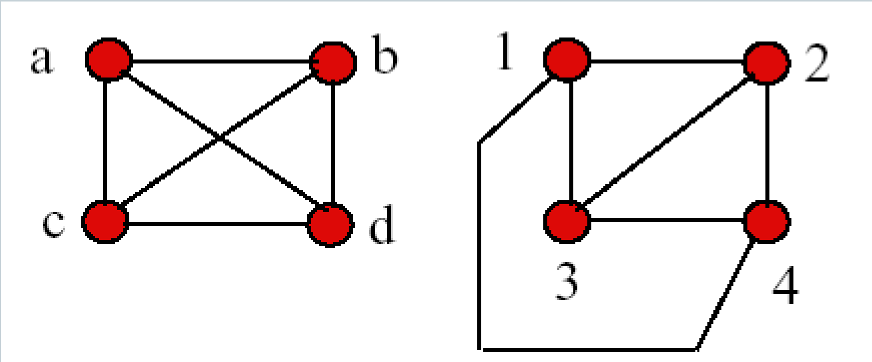


Graph Isomorphism

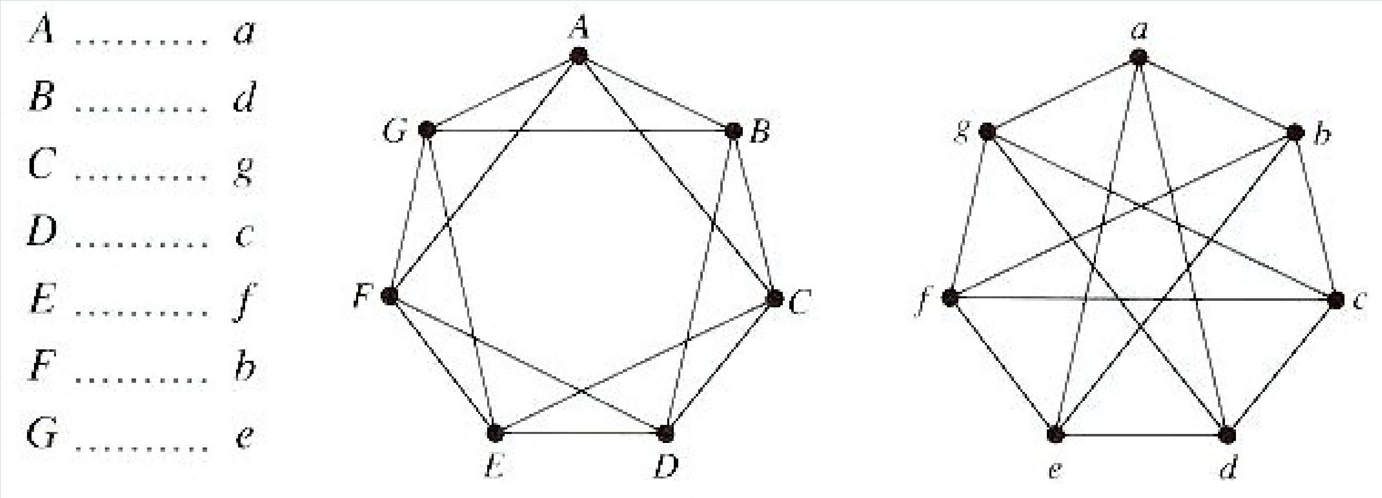
- Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be simple graphs.
- The graphs G_1 and G_2 are *isomorphic* iff
  - There exists a bijection $f : V_1 \to V_2$
    - The function f is called an isomorphism of G_1 with G_2.
  - i.e. For all vertices u and v in V_1,
    - if u and v are adjacent in G_1
    - then $f(u)$ and $f(v)$ are adjacent in G_2.

Graph Isomorphism

- Graph isomorphism invariant properties:
  - Are the properties that G_1 and G_2 must have in common in order to be isomorphic:
    - the same number of vertices.
    - the same number of edges.
    - degrees of corresponding vertices are the same.
    - if one is bipartite, the other must be.
    - if one is complete, the other must be.
    - etc. but these are necessary, not sufficient !

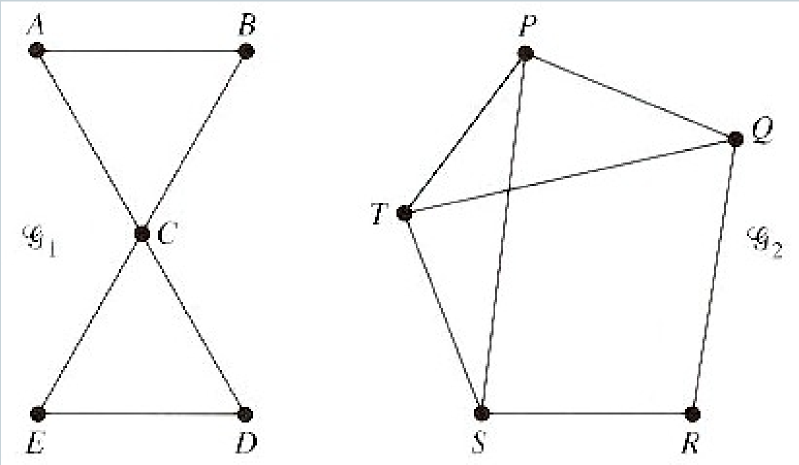
Graph Isomorphism

- Examples:
  - The following graphs are isomorphic.



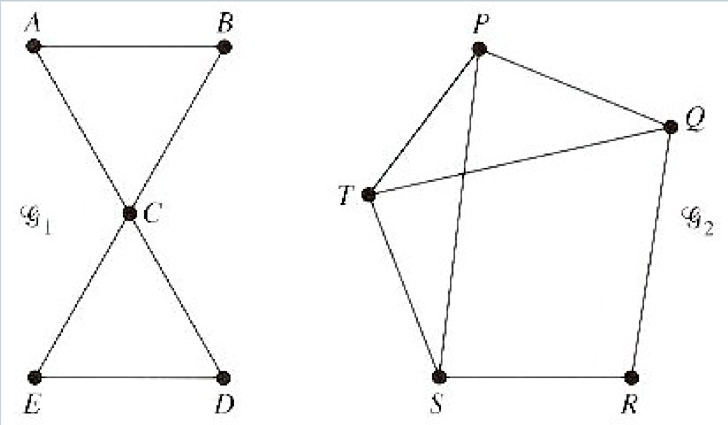
Graph Isomorphism

- Examples:
  - The following graphs are not isomorphic. Why?



Graph Isomorphism

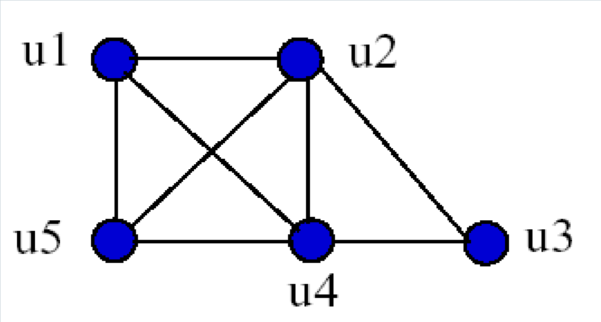
- Examples:
  - The following graphs are not isomorphic.
    - Vertex C in G_1 is adjacent to the vertices A, B, D and E. So it has a degree of 4.
    - But there are no vertices of degree 4 in G_2.



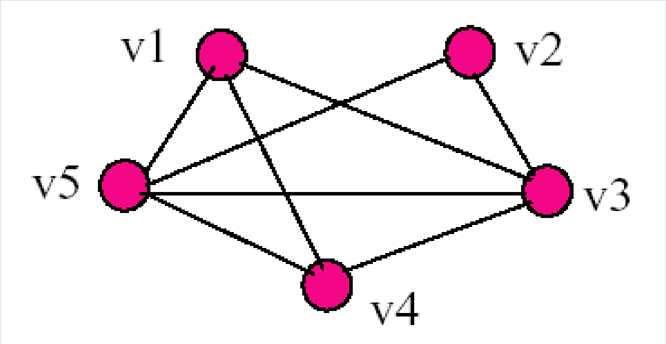
9.6. Graph Isomorphism

- Examples:
  - Determine if the following two graphs G_1 and G_2 are isomorphic.

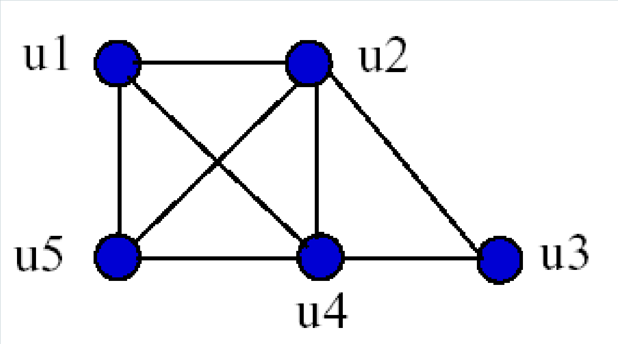
$$G1 = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

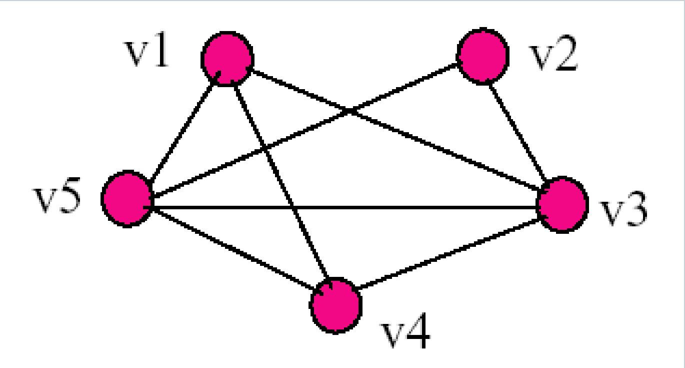


$$G2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$



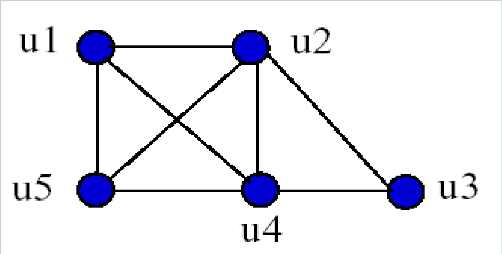
9.6. Graph Isomorphism

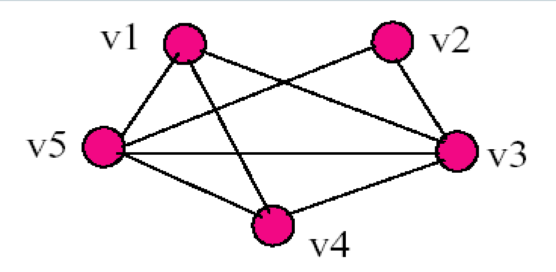




- Solution:
  - Check . . .
    - They have the same number of vertices = 5
    - They have the same number of edges = 8
    - They have the same number of vertices with the same degrees: 2, 3, 3, 4, 4.

9.6. Graph Isomorphism





3→2, 1→1, 5→4, 2→3, 4→5

The problem in general is very difficult, even using a computer !

Application of Isomorphics graph

- In electronic design automation
- graph isomorphism is the basis of the Layout Versus Schematic (LVS) circuit design step, which is a verification whether the electric circuit represented by a circuit schematic and an integrated circuit layout