

DISCRETE STRUCTURE

1

Lecture-22



Introduction to Graph

Topics covered



- Introduction to Graphs
- Directed Graphs
- Undirected Graphs

Introduction to graph

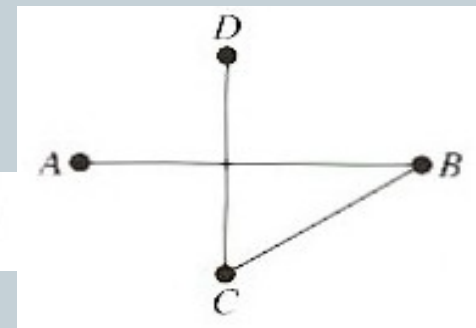
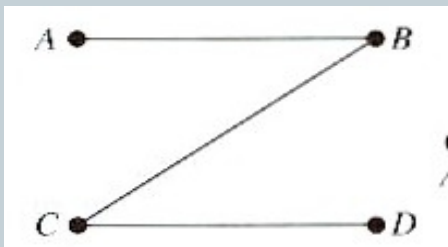


- A graph, G , consists of two sets, V and E .
 - V is a finite, nonempty set of vertices.
 - E is set of pairs of vertices called edges.
- The vertices of a graph G can be represented as $V(G)$.
- Likewise, the edges of a graph, G , can be represented as $E(G)$.

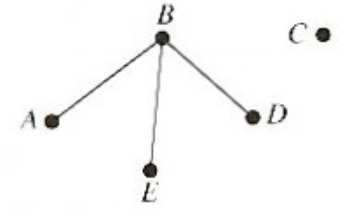
Basic Definitions



- A graph can be described either by the use of
 - ✦ Sets or
 - ✦ a diagram.
 - The diagram can be drawn differently and still represents the same graph.
- Example:
 - ✦ Sets: A graph with vertices $V = \{ A, B, C, D \}$ and edges $E = \{ \{A,B\}, \{B,C\}, \{C,D\} \}$.



Basic Definitions

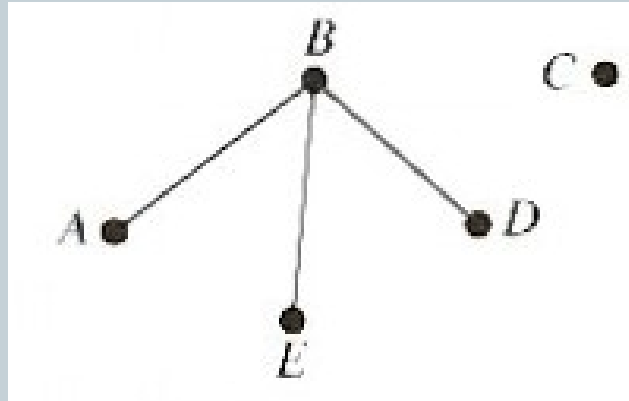


- **Adjacent, incident, and degree:**
 - Two vertices u and v in V are **adjacent** or **neighbors** if there is an edge e between u and v .
 - The edge $e = \{u, v\}$ **joins** (or **connects/links**) the vertices u and v .
 - The vertices u and v are **endpoints** of e .
 - ✦ The edge e is **incident** with the vertex u and v .
 - The **degree of u** , denoted **$\deg(u)$** , is the number of edges incident with a vertex u .

9.1. Basic Definitions

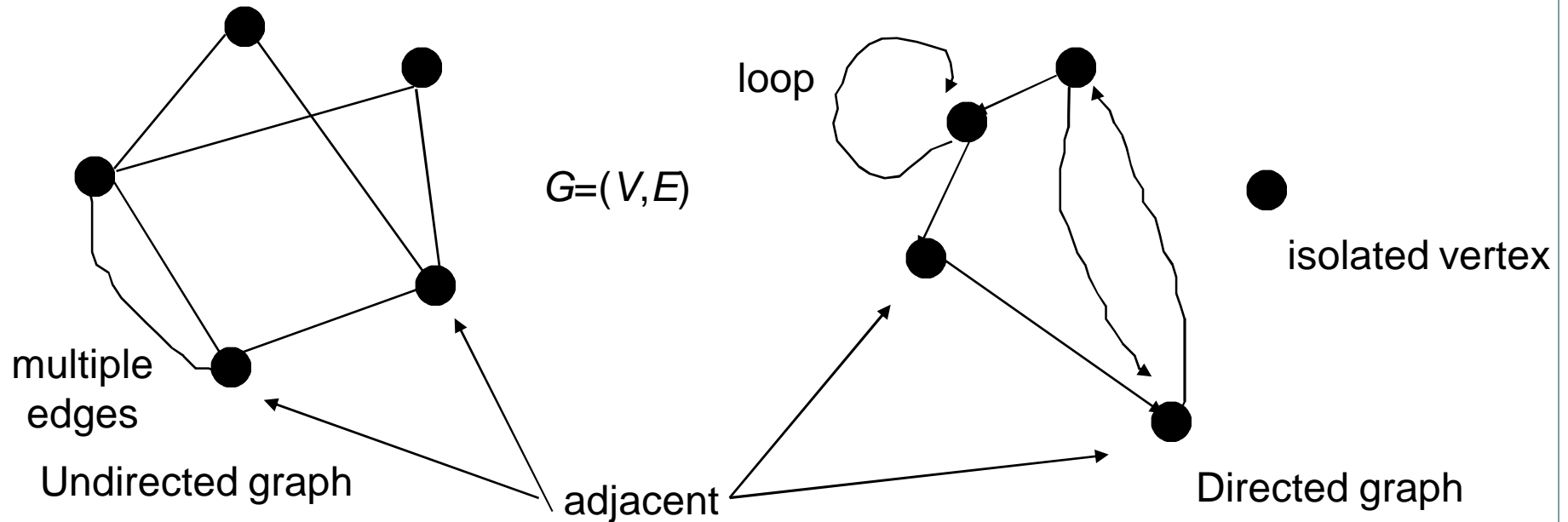


- Example:



- Vertices A and B are adjacent.
- Vertices A and D are not because there is no edge between them.
- $\text{deg}(A) = 1$
- $\text{deg}(B) = 3$
- $\text{deg}(C) = 0$, C is called **isolated**.

Directed & undirected Graph



simple graph: an undirected graph without loop or multiple edges

degree of a vertex: number of edges connected
(indegree, outdegree)

For simple graphs, $\sum_{v_i \in V} \deg(v_i) = 2|E|$

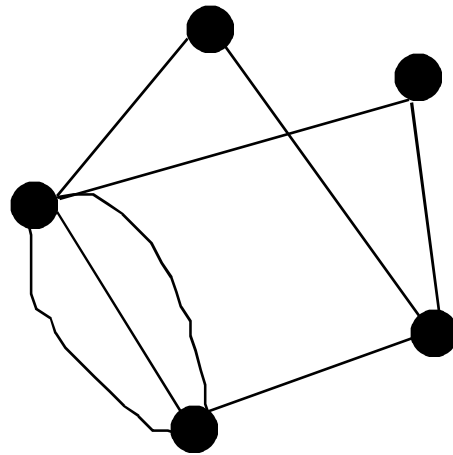
Multigraph

For any graph $G = (V, E)$, the number of components of G is denoted by $\kappa(G)$.

$$1 \leq \kappa(G) \leq |V|$$

Can you think of an algorithm to determine $\kappa(G)$?

multigraphs

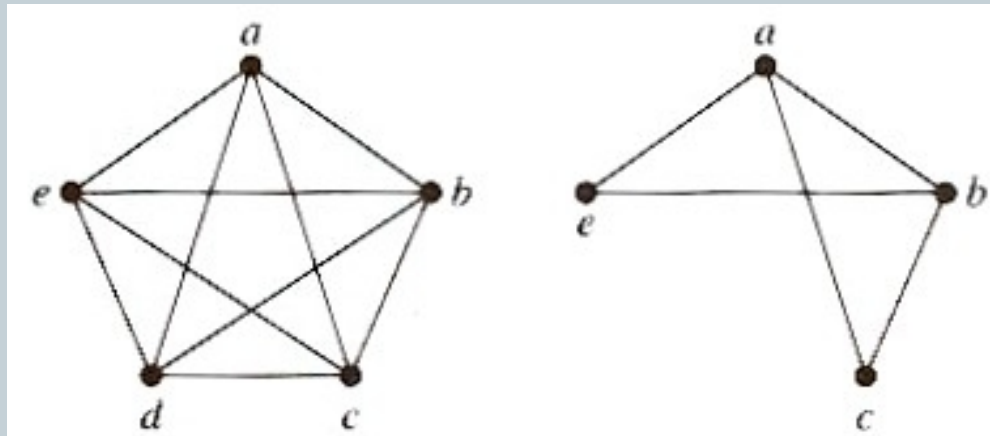


multigraph of *multiplicity 3*

Subgraphs



- A **subgraph** of $G = (V, E)$
 - Is a graph $H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$.
 - Examples:
 - ✦ K_5 and its subgraph:



Application & Scope of Research of Graph

- Graphs are applied widely in our days. They are used in economy, aeronautics, physics, biology. Analysis of electrical circuits
 - Finding shortest routes
 - Project planning
 - Identification of chemical compounds
 - Statistical mechanics
 - Genetics
 - Cybernetics
 - Linguistics
- Social Sciences, and so on Scope of research is construct a network,Game