DISCRETE STRUCTURE

LECTURE-14

Introduction to Combinations

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- Combinations
- R-combination
- Combinatorial proof

INTRODUCTION TO COMBINATIONS

- How many different committees of 3 students can be formed from a group of 4 students?
- We need to find the number of subsets with 3 elements from the set containing 4 students
- We see that there are 4 such subsets, one for each of the 4 students as choosing 4 students is the same as choosing one of the 4 students to leave out of the group
- This means there are 4 ways to choose 3 students for the committee, where th order in which these students are chosen does not matter

R-COMBINATION

- An r-combination of elements of a set is an **unordered** selection of r elements from the set
- An r-combination is simply a subset of the set with r elements
- Denote by C(n,r). Note that C(n,r) is also denoted by and is called a binomial coefficient

$$\binom{n}{r}$$

EXAMPLE

- Let S be the set {1, 2, 3, 4}. Then {1, 3, 4} is a 3-combination from S
- We see that C(4,2)=6, as the 2-combination of {a, b, c, d} are 6 subsets {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, and {c, d}

R-COMBINATION

- We can determine the number of r-combinations of a set with n elements using the formula for the number of r-permutations of a set
- Note that the r-permutations of a set can be obtained by first forming r-combinations and then ordering the elements in these combinations

R-COMBINATION

- The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \le r \le n$ equals

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

- Proof: The r-permutations of the set can be obtained by forming the C(n,r) r-combinations and then ordering the elements in each r-permutation which can be done in P(r,r) ways

$$P(n,r) = C(n,r) \cdot P(r,r)$$

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!/(n-r)!}{r!/(r-r)!} = \frac{n!}{r!(n-r)!}$$

R-COMBINATION

- When computing r-combination

$$C(n,r) = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$$

thus canceling out all the terms in the larger factorial

EXAMPLE

- How many poker hands of 5 cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?
- Choose 5 out of 52 cards: C(52,5)=52!/(5!47!)= (52x51x50x49x48)/(5x4x3x2x1)=26x17x10x49x12= 2,598,960
- C(52,47)=52!/(47!5!)=2,5,98,960

COROLLARY 2

- Let n and r be nonnegative integers with r≤n. Then C(n,r)=C(n,n-r)
- Proof:

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

$$C(n,n-r)=\frac{n!}{(n-r)!(n-(n-r))!}=\frac{n!}{(n-r)!r!}$$

COMBINATORIAL PROOF

- A **combinatorial proof** of an identity is a proof that uses counting arguments to prove that both sides of the identity count the same objects but in different ways
- Proof of Corollary 2: Suppose that S is a set with n elements. Every subset A of S with r elements corresponds to a subset of S with n-r elements, i.e., . Thus, C(n,r)=C(n,n-r)

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APPLICATION & SCOPE OF RESEARCH

- Mathematics
- Find the complexity of probability problem