DISCRETE STRUCTURE

LECTURE-13

Permutation & Combination

TOPICS COVERED

- Permutations
- Combination

Introduction to Permutations

- Counting:
  - Find out the number of ways to select a particular number of elements from a set
  - Sometimes the order of these elements matter
- Example:
  - How many ways we can select 3 students from a group of 5 students?
  - How many different ways they stand in line for picture?

Permutation

- How many ways can we select 3 students from a group of 5 to stand in lines for a picture?
- First note that the order in which we select students matters
- There are 5 ways to select the 1st student
- Once the 1st student is selected, there are 4 ways to select the 2nd student in line. By product rule, there are 5x4x3=60 ways to select 3 student from a group of 5 students to stand line for picture

Permutation

- How many ways can we arrange all 5 in a line for a picture?
- By product rule, we have 5x4x3x2x1=120 ways to arrange all 5 students in a line for a picture

Permutation

- A **permutation** of a set of distinct objects is an **ordered** arrangement of these objects
- An ordered arrangement of r elements of a set is called an r-permutation
- The number of r-permutation of a set with n element is denoted by P(n,r). We can find P(n,r) using the product rule
- Example: Let S={1, 2, 3}. The ordered arrangement 3, 1, 2 is a permutation of S. The ordered arrangement 3, 2, is a 2-permutation of S

Permutation

- Let S={a, b, c}. The 2-permutation of S are the ordered arrangements, a, b; a, c; b, a; b, c; c, a; and c, b
- Consequently, there are 6 2-permutation of this set with 3 elements
- Note that there are 3 ways to choose the 1^{st} element and then 2 ways to choose the 2^{nd} element
- By product rule, there are P(3,2)=3 x 2 = 6

r-permutation

- Theorem 1: If n is a positive and r is an integer with $1 \leq r \leq n$, then there are

  $P(n,r)=n(n-1)(n-2)\ldots(n-r+1)$

  r-permutations of a set with n elements
- Proof: Use the product rule, the first element can be chosen in n ways. There are n-1 ways to chose the 2nd element. Likewise, there are n-2 ways to choose 3rd element, and so on until there are exactly $n-(r-1)=n-r+1$ ways to choose the r-th element. Thus, there are $n\cdot(n-1)\cdot(n-2)\ldots\cdot(n-r+1)$ r-permutations of the set

r-permutation

- Note that $p(n,0)=1$ whenever n is a nonnegative integer as there is exactly one way to order zero element
- Corollary 1: If n and r are integers with $0 \le r \le n$, then $P(n,r)=n!/(n-r)!$
- Proof: When n and r are integers with $1 \le r \le n$, by Theorem 1 we have

$$P(n,r)=n(n-1)\ldots(n-r+1)=n!/(n-r)!$$

- As $n!/(n-0)!=1$ when n is a nonnegative integer, we have $P(n,r)=n!/(n-r)!$ also holds when $r=0$

r-permutation

- By Theorem 1, we know that if n is a positive integer, then P(n,n)=n!
- Example: How many ways are there to select a 1st prize winner, a 2nd prize winner, and a 3rd prize winner from 100 different contestants?
- P(100,3)=100x99x98=970,200

Assignment

- How many permutations of the letters ABCDEFGH contain string ABC?
- As ABC must occur as a block, we can find the answer by finding the permutations of 6 letters, the block ABC and the individual letters, D,E,F,G, and H. As these 6 objects must occur in any order, there are 6!=720 permutations of the letters ABCDEFGH in which ABC occurs in a block

Application & Scope of research

- To find the complexity of sorting algorithm
- To find the Searching algorithm