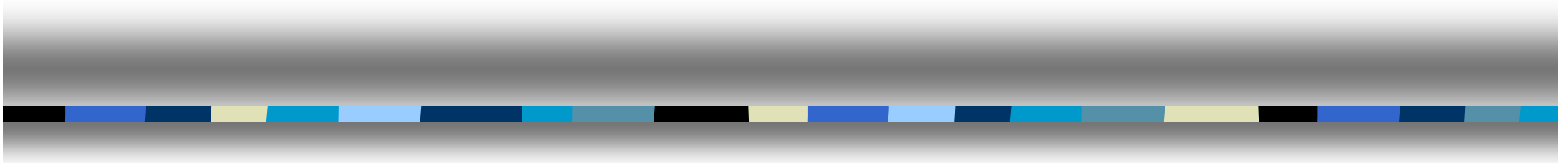


DISCRETE STRUCTURE

LECTURE-13



Permutation & Combination

TOPICS COVERED



n Permutations

n Combination



Introduction to Permutations

n Counting:

- Find out the number of ways to select a particular number of elements from a set
- Sometimes the order of these elements matter

n Example:

- How many ways we can select 3 students from a group of 5 students?
- How many different ways they stand in line₄ for picture?



Permutation

- n How many ways can we select 3 students from a group of 5 to stand in lines for a picture?
- n First note that the order in which we select students matters
- n There are 5 ways to select the 1st student
- n Once the 1st student is selected, there are 4 ways to select the 2nd student in line. By product rule, there are $5 \times 4 \times 3 = 60$ ways to select 3 student from a group of 5 students to stand line for picture



Permutation

- n How many ways can we arrange all 5 in a line for a picture?
- n By product rule, we have $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to arrange all 5 students in a line for a picture



Permutation

- n A **permutation** of a set of distinct objects is an **ordered** arrangement of these objects
- n An ordered arrangement of r elements of a set is called an r -permutation
- n The number of r -permutation of a set with n element is denoted by $P(n,r)$. We can find $P(n,r)$ using the product rule
- n Example: Let $S=\{1, 2, 3\}$. The ordered arrangement 3, 1, 2 is a permutation of S . The ordered arrangement 3, 2, is a 2-permutation of S



Permutation

- n Let $S = \{a, b, c\}$. The 2-permutation of S are the ordered arrangements, a, b ; a, c ; b, a ; b, c ; c, a ; and c, b
- n Consequently, there are 6 2-permutation of this set with 3 elements
- n Note that there are 3 ways to choose the 1st element and then 2 ways to choose the 2nd element
- n By product rule, there are $P(3,2) = 3 \times 2 = 6$



r-permutation

n Theorem 1: If n is a positive and r is an integer with $1 \leq r \leq n$, then there are

$$P(n,r) = n(n-1)(n-2)\dots(n-r+1)$$

r-permutations of a set with n elements

n Proof: Use the product rule, the first element can be chosen in n ways. There are $n-1$ ways to choose the 2nd element. Likewise, there are $n-2$ ways to choose 3rd element, and so on until there are exactly $n-(r-1) = n-r+1$ ways to choose the r -th element. Thus, there are $n \cdot (n-1) \cdot (n-2) \dots \cdot (n-r+1)$ r-permutations of the set



r-permutation

- n Note that $p(n,0)=1$ whenever n is a nonnegative integer as there is exactly one way to order zero element
- n Corollary 1: If n and r are integers with $0 \leq r \leq n$, then $P(n,r) = n! / (n-r)!$
- n Proof: When n and r are integers with $1 \leq r \leq n$, by Theorem 1 we have
$$P(n,r) = n(n-1)\dots(n-r+1) = n! / (n-r)!$$
- n As $n! / (n-0)! = 1$ when n is a nonnegative integer, we have $P(n,r) = n! / (n-r)!$ also holds when $r=0$



r-permutation

- n By Theorem 1, we know that if n is a positive integer, then $P(n,n)=n!$
- n Example: How many ways are there to select a 1st prize winner, a 2nd prize winner, and a 3rd prize winner from 100 different contestants?
- n $P(100,3)=100 \times 99 \times 98 = 970,200$



Assignment

- n How many permutations of the letters ABCDEFGH contain string ABC?
- n As ABC must occur as a block, we can find the answer by finding the permutations of 6 letters, the block ABC and the individual letters, D, E, F, G, and H. As these 6 objects must occur in any order, there are $6! = 720$ permutations of the letters ABCDEFGH in which ABC occurs in a block



Application & Scope of research

- n To find the complexity of sorting algorithm
- n To find the Searching algorithm