DISCRETE STRUCTURE

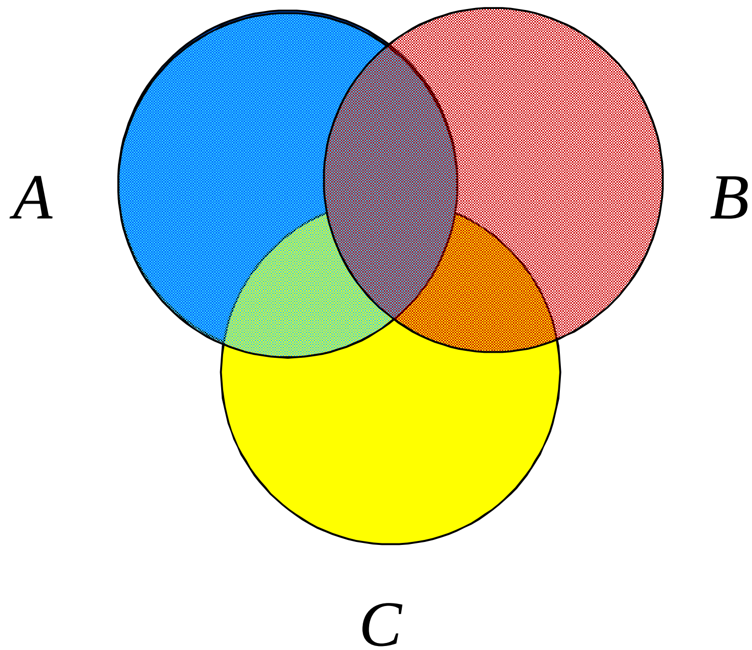
LECTURE-5

Introduction to Set Theory

TOPICS COVERED

- Examples of set
- Types of set

Sets



Introduction to Sets

Definition: A **set** is collection of distinct objects.

The objects in a set are called the **elements** or **members** of the set S, and we say S **contains** its elements.

We can define a set by directly listing all its elements.

e.g. S = {2, 3, 5, 7, 11, 13, 17, 19},

S = {CSC1130, CSC2110, ERG2020, MAT2510}

After we define a set, the set is a single mathematical object, and it can be an element of another set.

e.g. S = {{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}}

Defining Sets by Properties

It is inconvenient, and sometimes impossible,
to define a set by listing all its elements.

Alternatively, we can define by a set by describing
the *properties* that its elements should satisfy.

We use the notation $\{x \in A \mid P(x)\}$

to define the set as the set of elements, *x*,
in *A* such that *x* satisfies property *P*.

e.g. $\{x \mid x \text{ is a prime number and } x < 1000\}$

$\{x \mid x \text{ is a real number and } -2 < x < 5\}$

Examples of Sets

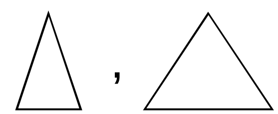
Well known sets:

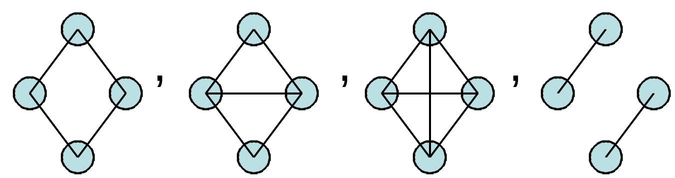
- the set of all real numbers, $\mathbb{R}$
- the set of all complex numbers, $\mathbb{C}$
- the set of all integers, $\mathbb{Z}$
- the set of all positive integers $\mathbb{Z}^+$
- **empty set**, $\emptyset = \{\}$, the set with no elements.

Other examples:

The set of all polynomials with degree at most three: $\{1, x, x^2, x^3, 2x+3x^2, \ldots\}$.

The set of all n-bit strings: {000…0, 000…1, …, 111…1}

The set of all triangles without an obtuse angle: {  ,… }

The set of all graphs with four nodes: {  ,…}

Membership

Order, number of occurence are not important.

e.g. {a,b,c} = {c,b,a} = {a,a,b,c,b}

The most basic question in set theory is whether an element is in a set.

$x \in A$ *x* is an element of *A*

x is in *A*

$x \notin A$ *x* is not an element of *A*

x is not in *A*

e.g. Recall that Z is the set of all integers. So $7 \in Z$ and $2/3 \notin Z$.

Let P be the set of all prime numbers. Then $97 \in P$ and $321 \notin P$

Let Q be the set of all rational numbers. Then $0.5 \in Q$ and $\sqrt{2} \notin Q$

(will prove later)

Size of a Set

In this course we mostly focus on finite sets.

Definition: The **size** of a set S, denoted by **|S|,** is defined as the number of elements contained in S.

e.g. if S = {2, 3, 5, 7, 11, 13, 17, 19}, then |S|=8.

if S = {CSC1130, CSC2110, ERG2020, MAT2510}, then |S|=4.

if S = {{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}}, then |S|=6.

Later we will study how to determine the size of the following sets:

- the set of poker hands which are "full house".
- the set of n-bit strings without three consecutive ones.
- the set of valid ways to add n pairs of parentheses

Application & Scope of research of Set Theory

•Set theory is seen as the foundation from which virtually all of mathematics can be derived. For example, structures in abstract algebra such as group, field & ring, are sets closed under one or more operations.
•Application of set is also found in Database System, Fuzzy logic.

•programming, data structures, data bases, algorithms, formal languages and compilers, operating systems, computer security, operations research
•Scope of research

Data mining
Computer Aided design
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Types of Sets

Empty sets: A set having no element is called an empty set. It is also known as null set or void set. It is denoted by ϕ.

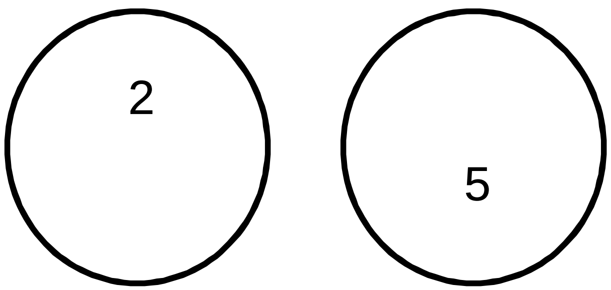
For example:

(a) $A = \{x \in R \,/\, x^2 = -10\} = \phi$

(b) $B = \text{set of immortal man} = \phi$

Singleton set: A set having single element is called singleton set.

For example, {2}, {0}, {5} are singleton set.



Types of Sets

Finite set: A set is called a finite set if it is called either void set or its elements can be counted by natural numbers and process of listing terminates at a certain natural numbers.

For example, {1, 2, 4, 6} is a finite set because it has four elements.

Infinite set: A set which is not a finite set, i.e. the counting up of whose elements is impossible, is called an infinite set.

For example:

(i) The set of all straight line in a given plane.
(ii) The set of all natural numbers.
(iii) The set of real numbers between ‘1’ and ‘2’.

Cardinal number or order of a finite set

The total number of elements in a finite set is called cardinal number or order of a finite set. It is denoted by n(A).
For example, if

$$A = \{1, 2, 3, 4, 5\} \Rightarrow n(A) = 5 \text{ or } o(A) = 5.$$

Set of sets: A set S having all its elements as sets is called set of sets.

For example:

S = { {1, 2}, {2, 4}, {3, 5, 7} }

But S = { {1, 2}, 4, {3, 5, 7} } is not a set of sets as $3 \in S$ is not a set.

Equivalent and Equal Sets

Equivalent sets: Two finite sets A and B are equivalent if their cardinal number is same, i.e. $n(A) = n(B)$.

Equal sets: Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

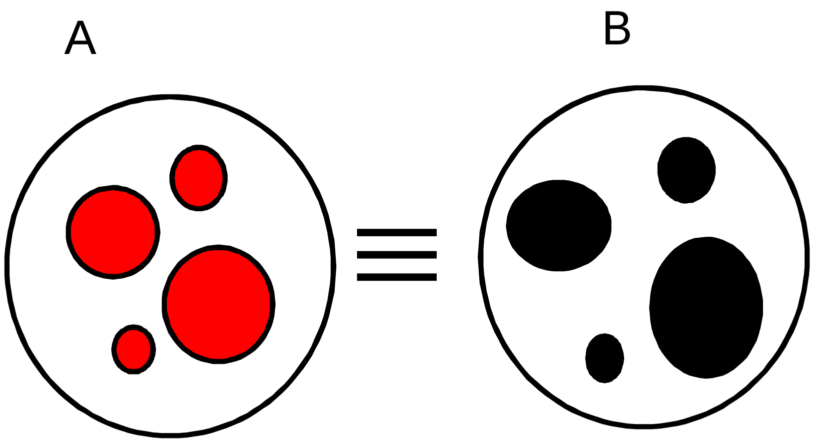
For example:

A = {4, 5, 6} and
B = {a, b, c} are equivalent but

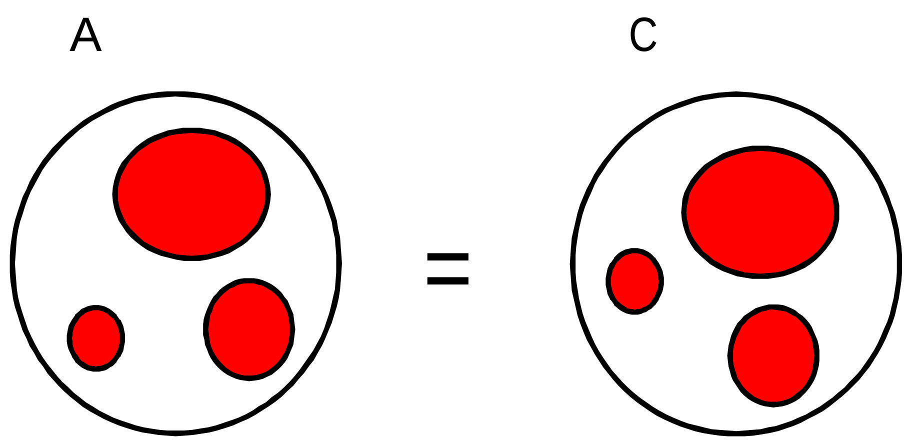
A = {4, 5, 6} and
C = {6, 5, 4} are equal, i.e. A = C.

Equivalent and Equal Sets

A $\equiv$ B if n(A) = n(B)



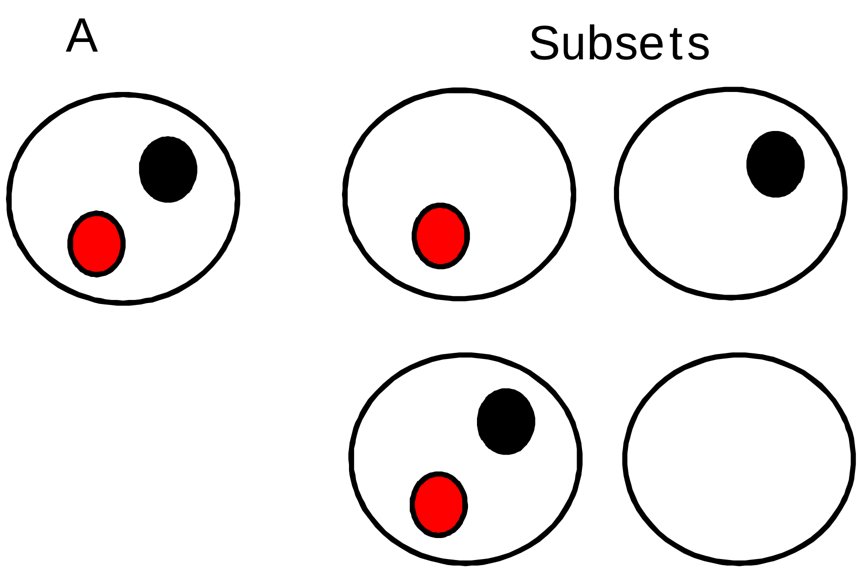
A = C if each element of each set is equal to each other



Subsets and Proper Subsets

Subsets: A set A is said to be a subset of a set B if each element of A is also an element of B.

$$A \subseteq B, \text{ if } x \in A \Rightarrow x \in B$$



For example:

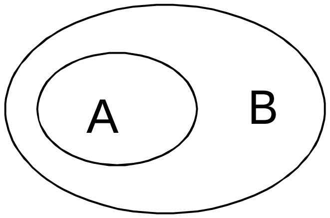
Let A = {2, 4, 6, 8},

B = {2, 4, 6, 8, 10, 12},

then $A \subseteq B$.

Subset

Definition: Given two sets A and B, we say A is a **subset** of B, denoted by $A \subseteq B$, if every element of A is also an element of B.



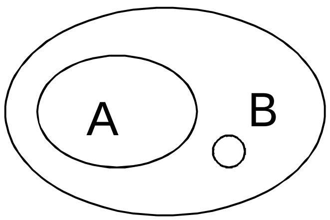
not a subset

- If A={4, 8, 12, 16} and B={2, 4, 6, 8, 10, 12, 14, 16}, then $A \subseteq B$ but $B \not\subseteq A$
- $A \subseteq A$ because every element in A is an element of A.
- $\emptyset \subseteq A$ for any A because the empty set has no elements.
- If A is the set of prime numbers and B is the set of odd numbers, then $A \not\subseteq B$

Fact: If $A \subseteq B$, then |A| <= |B|.

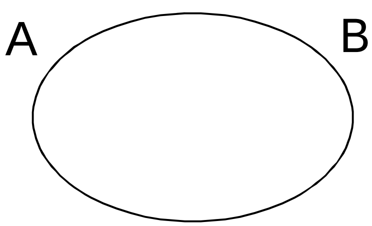
Proper Subset, Equality

Definition: Given two sets A and B, we say A is a **proper subset** of B, denoted by $A \subset B$, if every element of A is an element of B, But there is an element in B that is not contained in A.



Fact: If $A \subset B$, then |A| < |B|.

Definition: Given two sets A and B, we say A = B if $A \subseteq B$ and $B \subseteq A$.



Fact: If A = B, then |A| = |B|.

Subsets and Proper Subsets

Proper subset: A set A is said to be a proper subset of a set B if every element of A is an element of B and B has at least one element which is not an element of A.

It can be written as $A \subset B$

For example:

Let A = {1, 2, 3}, B = {2, 3, 4, 1, 5}, then $A \subset B$.

Thus if A is a proper subset of B, then there exists an element such that $x \in B$ $x \notin A$.

For example, $\{1\} \subset \{1, 2, 3\}$ but $\{1, 4\} \not\subset \{1, 2, 3\}$.

Procedure for Proving Equality of Sets

As we have discussed earlier that two sets A and B are said to be equal if every element of set A is an element of set B and every element of B is an element of A.

It is clear that $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

i.e. $A = B \Leftrightarrow [x \in A \Leftrightarrow x \in B]$.

Some Results on Subsets

(i) Every set is a subset of itself.

(ii) The empty set is a subset of every set.

(iii) The total number of subsets of a finite set containing n elements is 2^n.

Proof : We know that ${}^nC_r$ denotes the number of ways for choosing r things from n different things. Therefore each selection of r things gives a subset of the set A containing r elements.

$\therefore$ The number of subsets of A having no element ${}^nC_0$

The number of subsets of A having one element ${}^nC_1$

Some Results on Subsets

The number of subsets of A having two elements = $^nC_2$

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The number of subsets of A having n elements = ^nC_n.

Hence, the total number of subsets of A

$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

[$\because$ We know that $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$.

Putting x = 1, we get

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n.$$

]

Power Set

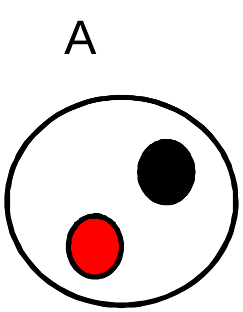
The set of all the subsets of a given set A is said to be the power set of A and is denoted by P(A).

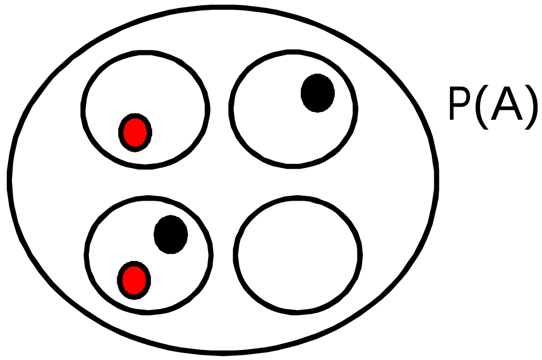
i.e. $P(A) = \{S | S \subseteq A\} \Rightarrow S \in P(A) \Leftrightarrow S \subseteq A$

Also, $\phi \in P(A)$ and $A \in P(A)$ for all sets A.

For example, if A = {a, b, c}, then

$$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}.$$





Universal Set

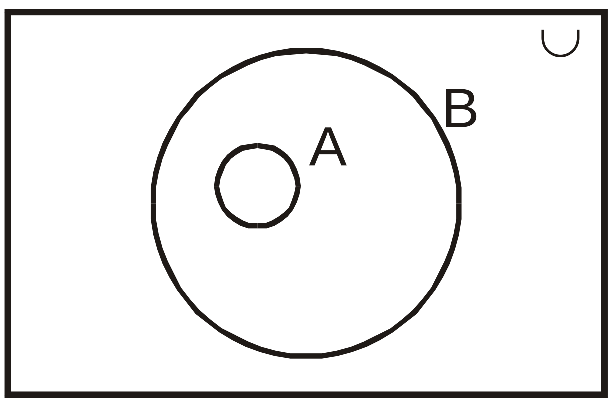
Any set which is super set of all the sets under consideration is called the universal set and is denoted by

Ω or $\cup$.

For example:

(i) When we are using sets containing natural numbers then N is the universal set.

Euler-Venn Diagram



$A \subset B$

Application & Scope of research of Set

Database System
Fuzzy Logic