# The Basic Concepts of Set Theory

# Chapter 2: The Basic Concepts of Set Theory

# Section 2-1

Symbols and Terminology

# Symbols and Terminology

- Designating Sets
- Sets of Numbers and Cardinality
- Finite and Infinite Sets
- Equality of Sets

# Designating Sets

A **set** is a collection of objects. The objects belonging to the set are called the **elements**, or **members** of the set.

Sets are designated using:

1) *word description*,

2) *the listing method*, and

3) *set-builder notation*.

# Designating Sets

**Word description**

The set of even counting numbers less than 10

**The listing method**

{2, 4, 6, 8}

**Set-builder notation**

$\{x | x$ is an even counting number less than $10\}$

# Designating Sets

Sets are commonly given names (capital letters).

$A = \{1, 2, 3, 4\}$

The set containing no elements is called the **empty set** (*null set*) and denoted by $\{\ \}$ or $\varnothing$.

To show 2 is an element of set $A$ use the symbol $\in$.

$$2 \in \{1, 2, 3, 4\}$$

$$a \notin \{1, 2, 3, 4\}$$

# Example: Listing Elements of Sets

Give a complete listing of all of the elements of the set $\{x|x$ is a natural number between 3 and 8$\}$

Solution

{4, 5, 6, 7}

# Sets of Numbers

**Natural** (*counting*) {1, 2, 3, 4, ...}
**Whole numbers** {0, 1, 2, 3, 4, ...}
**Integers** {...,–3, –2, –1, 0, 1, 2, 3, ...}

**Rational numbers** $\left\{ \frac{p}{q} \middle| \, p \text{ and } q \text{ are integers, with } q \neq 0 \right\}$

May be written as a terminating decimal, like 0.25, or a repeating decimal like 0.333...
**Irrational** {$x$ | $x$ is not expressible as a quotient of integers}
Decimal representations never terminate and never repeat.
**Real numbers** {$x$ | $x$ can be expressed as a decimal}

# Cardinality

The number of elements in a set is called the **cardinal number**, or **cardinality** of the set.

The symbol ***n*(*A*)**, read "***n*** of ***A***," represents the cardinal number of set *A*.

# Example: Cardinality

Find the cardinal number of each set.

a) $K = \{a, l, g, e, b, r\}$

b) $M = \{2\}$

c) $\varnothing$

Solution

a) $n(K) = 6$

b) $n(M) = 1$

c) $n(\varnothing) = 0$

# Finite and Infinite Sets

If the cardinal number of a set is a particular whole number, we call that set a **finite set**.

Whenever a set is so large that its cardinal number is not found among the whole numbers, we call that set an **infinite set**.

# Example: Infinite Set

The odd counting numbers are an infinite set.

Word description
The set of all odd counting numbers

Listing method
{1, 3, 5, 7, 9, …}

Set-builder notation
$\{x|x$ is an odd counting number$\}$

# Equality of Sets

Set $A$ is **equal** to set $B$ provided the following two conditions are met:

1. Every element of $A$ is an element of $B$, and
2. Every element of $B$ is an element of $A$.

# Example: Equality of Sets

State whether the sets in each pair are equal.

a) {a, b, c, d} and {a, c, d, b}

b) {2, 4, 6} and $\{x|x$ is an even number$\}$

Solution

a) Yes, order of elements does not matter

b) No, {2, 4, 6} does not represent all the even numbers.