DISCRETE STRUCTURE

LECTURE-3

Sets Operation

Introduction to Set Operation

- Basic operation performed on set are

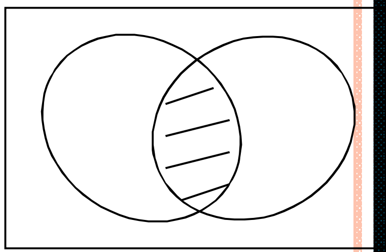
1. Union
2. Intersection
3. Difference
4. Complement

Basic Operations on Sets

Let A, B be two subsets of a *universal* set U

(depending on the context U could be R, Z, or other sets).

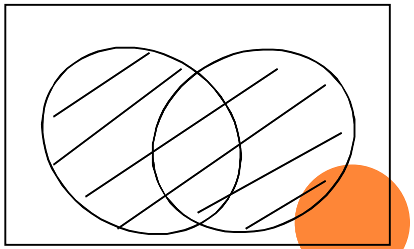
intersection: $A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$



Defintion: Two sets are said to be **disjoint** if their intersection is an empty set.

e.g. Let A be the set of odd numbers, and B be the set of even numbers. Then A and B are disjoint.

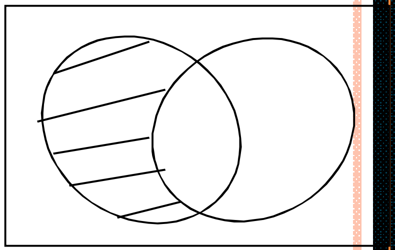
union: $A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$



Fact: $|A \cup B| = |A| + |B| - |A \cap B|$

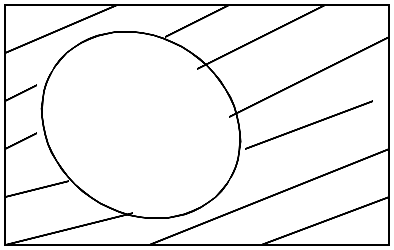
Basic Operations on Sets

difference: $A - B = \{x \in U \mid x \in A \text{ and } x \notin B\}$



Fact: $|A - B| = |A| - |A \cap B|$

complement: $\overline{A} = A^c = \{x \in U \mid x \notin A\}$



e.g. Let U = Z and A be the set of odd numbers.

Then $\overline{A}$ is the set of even numbers.

Fact: If $A \subseteq B$ then $\overline{B} \subseteq \overline{A}$

Examples

$A = \{1, 3, 6, 8, 10\}$ $B = \{2, 4, 6, 7, 10\}$

$A \cap B = \{6, 10\}$, $A \cup B = \{1, 2, 3, 4, 6, 7, 8, 10\}$ $A - B = \{1, 3, 8\}$

Let $U = \{ x \in Z \mid 1 <= x <= 100\}$.

$A = \{ x \in U \mid$ x is divisible by 3$\}$, $B = \{ x \in U \mid$ x is divisible by 5$\}$

$A \cap B = \{ x \in U \mid$ x is divisible by 15$\}$

$A \cup B = \{ x \in U \mid$ x is divisible by 3 or is divisible by 5 (or both)$\}$

$A – B = \{ x \in U \mid$ x is divisible by 3 but is not divisible by 5 $\}$

Exercise: compute $|A|$, $|B|$, $|A \cap B|$, $|A \cup B|$, $|A – B|$.

Cartesian Products

Definition: Given two sets A and B, the **Cartesian product** A x B is the set of all **ordered** pairs (a,b), where a is in A and b is in B. Formally,

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Ordered pairs means the ordering is important, e.g. $(1,2) \neq (2,1)$

e.g. Let A be the set of letters, i.e. {a,b,c,…,x,y,z}.

Let B be the set of digits, i.e. {0,1,…,9}.

AxA is just the set of strings with two letters.

BxB is just the set of strings with two digits.

AxB is the set of strings where the first character is a letter and the second character is a digit.

Cartesian Products

The definition can be generalized to any number of sets, e.g.

$$A \times B \times C = \{(a, b, c) \mid a \in A \text{ and } b \in B \text{ and } c \in C\}$$

Using the above examples, AxAxA is the set of strings with three letters.

Our ID card number has one letter and then six digits,
so the set of ID card numbers is the set AxBxBxBxBxBxB.

Fact: If |A| = n and |B| = m, then |AxB| = nm.

Fact: If |A| = n and |B| = m and |C| = l, then |AxBxC| = nml.

Fact: $|A_1 x A_2 x \ldots x A_k| = |A_1| x |A_2| x \ldots x |A_k|$.

Application & Scope of research

- Database system
- Fuzzy system
- Automata theory