Discrete Structure

Lecture-1

INTRODUCTION TO SETS

TOPICS COVERED

- Set
- Basic, Essential, and Important
- Properties of Sets
- Venn Diagram

Definitions

- A <u>set</u> is a collection of objects.
- Objects in the collection are called <u>elements</u> of the set.

Examples - set

The collection of persons living in Arnold is a set.

- Each person living in Arnold is an element of the set.

The collection of all counties in the state of Texas is a set.

- Each county in Texas is an element of the set.

Examples - set

The collection of all quadrupeds is a set.

- Each quadruped is an element of the set.

The collection of all four-legged dogs is a set.

- Each four-legged dog is an element of the set.

Examples - set

The collection of counting numbers is a set.

- Each counting number is an element of the set.

The collection of pencils in your briefcase is a set.

- Each pencil in your briefcase is an element of the set.

Notation

- Sets are usually designated with capital letters.
- Elements of a set are usually designated with lower case letters.
  - We might talk of the set B. An individual element of B might then be designated by b.

Notation

- The roster method of specifying a set consists of surrounding the collection of elements with braces.

Example – roster method

For example the set of counting numbers from 1 to 5 would be written as

{1, 2, 3, 4, 5}.

Example – roster method

A variation of the simple roster method uses the ellipsis (...) when the pattern is obvious and the set is large.

{1, 3, 5, 7, ... , 9007} is the set of odd counting numbers less than or equal to 9007.

{1, 2, 3, ... } is the set of all counting numbers.

Notation

Set builder notation has the general form

{variable | descriptive statement }.

The vertical bar (in set builder notation) is always read as “such that”.

Set builder notation is frequently used when the roster method is either inappropriate or inadequate.

Example – set builder notation

{x | x < 6 and x is a counting number} is the set of all counting numbers less than 6. Note this is the same set as {1,2,3,4,5}.

{x | x is a fraction whose numerator is 1 and whose denominator is a counting number }.

Set builder notation will become much more concise and precise as more information is introduced.

Notation – is an element of

- If x is an element of the set A, we write this as $x \in A$. $x \notin A$ means x is not an element of A.

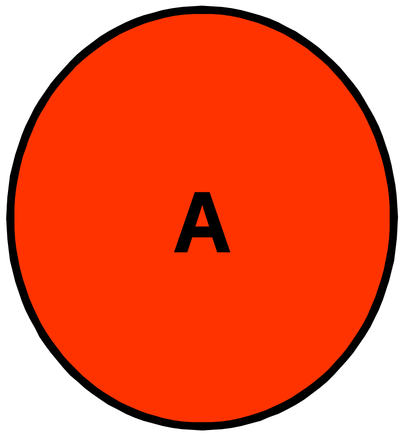
If $A = \{3, 17, 2\}$ then
$3 \in A$, $17 \in A$, $2 \in A$ and $5 \notin A$.

If $A = \{ x \mid x \text{ is a prime number} \}$ then
$5 \in A$, and $6 \notin A$.

Venn Diagrams

It is frequently very helpful to depict a set in the abstract as the points inside a circle (or any other closed shape).

We can picture the set A as the points inside the circle shown here.



Venn Diagrams

Venn Diagrams are used in mathematics, logic, theological ethics, genetics, study of Hamlet, linguistics, reasoning, and many other areas.

Definition

- The set with no elements is called the <u>empty set</u> or the null set and is designated with the symbol $\emptyset$.

Examples – empty set

The set of even prime numbers greater than 2 is the empty set.

The set $\{x \mid x < 3 \text{ and } x > 5\}$ is the empty set.

Definition - subset

- The set A is a subset of the set B if every element of A is an element of B.
- If A is a subset of B and B contains elements which are not in A, then A is a proper subset of B.

Notation - subset

If A is a subset of B we write
$A \subseteq B$ to designate that relationship.

If A is a proper subset of B we write
$A \subset B$ to designate that relationship.

If A is not a subset of B we write
$A \not\subset B$ to designate that relationship.

Example - subset

The set $A = \{1, 2, 3\}$ is a subset of the set $B =\{1, 2, 3, 4, 5, 6\}$ because each element of A is an element of B.

We write $A \subseteq B$ to designate this relationship between A and B.

We could also write

$$\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5, 6\}$$

Example - subset

The set A = {3, 5, 7} is not a subset of the set B = {1, 4, 5, 7, 9} because 3 is an element of A but is not an element of B.

The empty set is a subset of every set, because every element of the empty set is an element of every other set.

Example - subset

The set
A = {1, 2, 3, 4, 5} is a subset of the set
B = {x | x < 6 and x is a counting number}
because every element of A is an element
of B.

Notice also that B is a subset of A because every
element of B is an element of A.

Definition

- Two sets A and B are <u>equal</u> if $A \subseteq B$ and $B \subseteq A$. If two sets A and B are equal we write $A = B$ to designate that relationship.

Example - equality

The sets
$A = \{3, 4, 6\}$ and $B = \{6, 3, 4\}$ are equal because $A \subseteq B$ and $B \subseteq A$.

The definition of equality of sets shows that the order in which elements are written does not affect the set.

Example - equality

If A = {1, 2, 3, 4, 5} and
B = {x | x < 6 and x is a counting number}
then <u>A is a subset of B</u> because every element of A is an element of B and <u>B is a subset of A</u> because every element of B is an element of A.

Therefore the two sets are equal and we write A = B.

Example - equality

The sets $A = \{2\}$ and $B = \{2, 5\}$ are not equal because B is not a subset of A. We would write $A \neq B$. Note that $A \subseteq B$.

The sets $A = \{x \mid x \text{ is a fraction}\}$ and $B = \{x \mid x = 3/4\}$ are not equal because A is not a subset of B. We would write $A \neq B$. Note that $B \subseteq A$.

Definition - intersection

The intersection of two sets A and B is the set containing those elements which are elements of A and elements of B.

We write $A \cap B$

Example - intersection

If $A = \{3, 4, 6, 8\}$ and

$B = \{1, 2, 3, 5, 6\}$ then

$A \cap B = \{3, 6\}$

Example - intersection

If A = { ◈, ⁂, ✺, ◒, ⬢, ⟐, ✶, ⊡, ☙ }
and B = { ⁂, ♌, ◒, @, ☙, Ø } then
$A \cap B$ = { ⁂, ◒ }

If A = { ◈, ⁂, ⟐, ✺, ☙, ◒, Ø, ✶, }
and B = { ⁂, ✺, ◒, Ø } then
$A \cap B$ = { ⁂, ✺, ◒, Ø } = B

Example - intersection

If A is the set of prime numbers and
B is the set of even numbers then
$A \cap B = \{ 2 \}$

If $A = \{x \mid x > 5 \}$ and
$B = \{x \mid x < 3 \}$ then
$A \cap B = \varnothing$

Example - intersection

$$\text{If } A = \{x \mid x < 4\} \text{ and}$$
$$B = \{x \mid x > 1\} \text{ then}$$
$$A \cap B = \{x \mid 1 < x < 4\}$$

$$\text{If } A = \{x \mid x > 4\} \text{ and}$$
$$B = \{x \mid x > 7\} \text{ then}$$
$$A \cap B = \{x \mid x < 7\}$$

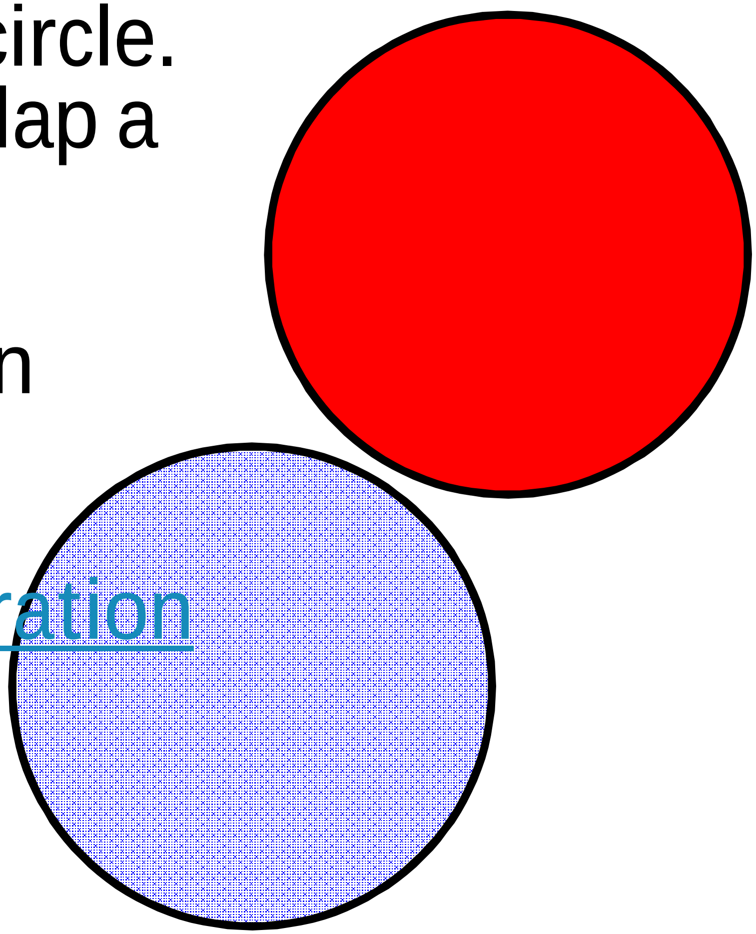
Venn Diagram - intersection

A is represented by the red circle and B is represented by the blue circle. When B is moved to overlap a portion of A, the purple colored region illustrates the intersection

$A \cap B$

of A and B

Excellent online interactive demonstration



Definition - union

The union of two sets A and B is the set containing those elements which are

elements of A **or** elements of B.

We write $A \cup B$

Example - Union

If $A = \{3, 4, 6\}$ and

$B = \{1, 2, 3, 5, 6\}$ then

$A \cup B = \{1, 2, 3, 4, 5, 6\}$.

Example - Union

If A = { ◈, ⁂, ❋, ◒, ⊡, ര }
and B = { ⁂, ♌, ◒, @, ര, Ꝭ } then
$A \cup B$ = {◈, ⁂, ❋, ◒, , ⊡, ര, ♌, @, Ꝭ }

If A = { ◈, ❋, ര, ◒, Ꝭ}
and B = {❋, ◒, Ꝭ } then
$A \cup B$ = {◈, ❋, ര, ◒, Ꝭ } = A

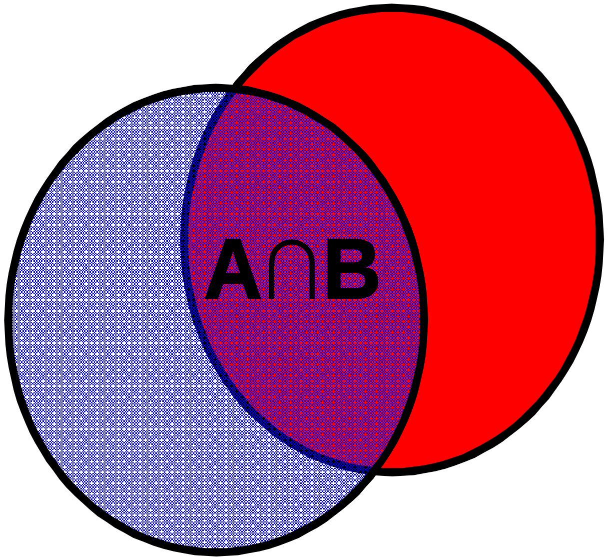
Example - Union

If A is the set of prime numbers and B is the set of even numbers then $A \cup B = \{x \mid x \text{ is even or } x \text{ is prime}\}$.

If $A = \{x \mid x > 5\}$ and
$B = \{x \mid x < 3\}$ then
$A \cup B = \{x \mid x < 3 \text{ or } x > 5\}$.

Venn Diagram - union

A is represented by the red circle and B is represented by the blue circle.
The purple colored region illustrates the intersection.
The union consists of all points which are colored red or blue or purple.



A ∪ B

Algebraic Properties

Union and intersection are <u>commutative</u> operations.

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Algebraic Properties

Union and intersection are <u>associative</u> operations.

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = B \cap (A \cap C)$$

Algebraic Properties

Two distributive laws are true.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

For additional information about the algebra of sets go HERE

Algebraic Properties

A few other elementary properties of intersection and union.

$A \cup \varnothing = A \qquad A \cap \varnothing = \varnothing$

$A \cup A = A \qquad A \cap A = A$

Continued Study

The study of sets is extensive, sophisticated, and quite abstract.

Even at the elementary level many considerations have been omitted from this presentation.

Continued Study

For further references check Amazon
for books about Set Theory.

Google Set Theory
The best online resource seems to be this
Wikipedia page about the Algebra of Sets.
Be sure to follow all the links from that page.
Some Elementary Exercises are HERE