Tautology

Introduction: PL?

- Propositional Logic (PL) = Propositional Calculus = Sentential Logic
- In Propositional Logic, the objects are called propositions
- **Definition**: A proposition is a statement that is either true or false, but not both
- We usually denote a proposition by a letter: *p*, *q*, *r*, *s*, …

1.1 Propositions

- Logic allows consistent mathematical reasoning.
- Many applications in CS: construction and verification computer programs,

Proposition: *A statement that is either true (T) or false (F).*

example: Toronto is the capital of Canada in 2003 (F).
1+1=2 (T).

counter-example: I love this class.

Let "P" be a proposition. Then $\neg P$ ("NOT P") is another one stating that:
It is not the case that "P".

Introduction to Tautologies, contradictions

DEF: A compound proposition is called a ***tautology*** if no matter what truth values its atomic propositions have, its own truth value is **T**.

EG: $p \vee \neg p$ (Law of excluded middle)

The opposite to a tautology, is a compound proposition that's always false –a ***contradiction***.

EG: $p \wedge \neg p$

On the other hand, a compound proposition whose truth value isn't constant is called a ***contingency.***

EG: $p \rightarrow \neg p$

Tautologies and contradictions

The easiest way to see if a compound proposition is a tautology/contradiction is to use a truth table.

p	$\neg p$	$p \vee \neg p$
F	T	T
T	F	T

p	$\neg p$	$p \wedge \neg p$
F	T	F
T	F	F

Tautology example (1.2.8.a) Part 1

Demonstrate that

$$[\neg p \wedge (p \vee q)] \rightarrow q$$

is a tautology in two ways:

1. Using a truth table – show that $[\neg p \wedge (p \vee q)] \rightarrow q$ is always true
2. Using a proof (will get to this later).

Tautology by truth table

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T				
T	F				
F	T				
F	F				

Tautology by truth table

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

Tautology by truth table

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T		
T	F	F	T		
F	T	T	T		
F	F	T	F		

Tautology by truth table

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	
T	F	F	T	F	
F	T	T	T	T	
F	F	T	F	F	

Tautology by truth table

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Application & Scope of research of Tautology

- Circuit-SAT