DISCRETE STRUCTURE

LECTURE-11

Introduction to proposition & basic operation

TOPICS COVERED

Propositional Logic

Application of Propositional logic

Propositional Logic

- Propositional logic is the study of propositions (true or false statements) and ways of combining them (logical operators) to get new propositions. It is effectively an algebra of propositions. In this algebra, the variables stand for unknown propositions (instead of unknown real numbers) and the operators are *and*, *or*, *not*, *implies*, and *if and only if* (rather than *plus*, *minus*, *negative*, *times*, and *divided by*). Just as middle/high school students learn the notation of algebra and how to manipulate it properly, we want to learn the notation of propositional logic and how to manipulate it properly.

Propositional Logic

- Symbol manipulation has a bad reputation in some circles, and rightly so when one learns it without understanding what the symbols and manipulations mean. On the other hand, as you know, the development of good notation is a huge part of the history of mathematics. Good notation greatly facilitates clear thinking, intuition, and insight. It strips away the irrelevant to help us see true relationships that would otherwise be invisible. Good manipulation skills allow us to proceed from one conclusion to the next quickly, confidently, and verifiably. They save us from having to justify routine, familiar steps every time we encounter them, and they suggest new ways of proceeding that we might never have discovered otherwise.

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Propositional Logic

Propositions

- A *proposition* is a statement with a truth value. That is, it is a statement that is true or else a statement that is false. Here are some examples with their truth values.

Propositional Logic

- The Two Elements of Symbolic Logic: Propositions
  - Ayres Hall houses the mathematics department at the University of Tennessee. (true)
  - The main campus of the University of Kentucky is in Athens, Ohio. (false)
  - Homer was the blind poet who composed the Illiad and the Odyssey. (The statement is certainly true or false, but we may not know which. Tradition asserts the truth of this proposition, but some scholars doubt its truth).
  - Jesus of Nazareth was God incarnate. (Again the statement is certainly true or false, even though many people stand on both sides of the question.)

Propositional Logic

- The Two Elements of Symbolic Logic: Propositions
  - It will rain in Knoxville tomorrow. (We cannot know the truth of this statement today, but it is certainly either true or false.)
  - 2+2=4 (true)
  - 2+2=19 (false)
  - There are only finitely many prime numbers. (false)
  - Every positive even integer greater than 2 can be written as the sum of two prime numbers. (This is certainly true or false, but no one knows which. The proposition is known as Goldbach's Conjecture. It holds for every integer yet tested – e.g, 4=2+2, 6=3+3, 8=3+5 – but no one has found a proof that it holds for all even integers greater than two.)

Propositional Logic

- The Two Elements of Symbolic Logic: Propositions
  - On the other hand, here are some examples of expressions that are not propositions.

Propositional Logic

- The Two Elements of Symbolic Logic: Propositions
  - Where is Ayres Hall? (This is neither true nor false. It is a question.)
  - Find Ayres Hall! (This is neither true nor false. It is a command.)
  - Blue is the best color to paint a house. (This is a matter of opinion, not truth. It may be your favorite color, but it is not objective truth.)

Propositional Logic

- The Two Elements of Symbolic Logic: Propositions
  - Coffee tastes better than tea. (Again, this is a matter of taste, not truth.)
  - The integer *n* is even. (Since n has no value, this statement is neither true nor false. If *n* is given a value, this statement becomes a proposition. Later we will call such statements predicates or propositional functions. They are not propositions, but they become propositions when their variables are assigned values.)

Propositional Logic

- The Two Elements of Symbolic Logic: Logical Operators
  - Arithmetic operators (operations) such as addition, subtraction, multiplication, division, and negation act on numbers to give new numbers. Logical operators such conjunction (and), disjunction (or), and negation (not) act on propositions to give new (compound) propositions. Logical operators should be truth functional; that is, the truth value of the compound proposition should depend only on the truth value of the component propositions. This makes it easy to specify the effect of a logical operator: we simply list the truth value of the compound proposition for every combination of truth values of the component compositions. Such a list is a called a *truth table*. Note that no such definition of arithmetic operations is possible because there are infinitely many possible values of numbers.

Propositional Logic

- The Common Logical Operators
  - Conjunction (and): The conjunction of propositions p and q is the compound proposition "p and q". We denote it $p \wedge q$. It is true if p and q are both true and false otherwise. For instance the compound proposition "2+2=4 and Sunday is the first day of the week" is true, but "3+3=7 and the freezing point of water is 32 degrees" false. The truth table that defines conjunction is

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional Logic

- The Common Logical Operators
  - Disjunction (or): In English the word *or* has two senses: inclusive and exclusive. The inclusive sense means “either or both” as in “to be admitted to the university you must have an ACT composite score of at least 17 or a high school GPA of at least 2.5.” The exclusive sense means “one or the other but not both” as in “for dinner I will have a sirloin steak or the fried shrimp platter” or “is the capital of Kentucky Louisville or Lexington?” In mathematics and logic the word *or* always has the inclusive sense; exceptions require alarm bells and warning lights.

Propositional Logic

- The Common Logical Operators
  - The disjunction of propositions p and q is the compound proposition “p or q”. We denote it $p \vee q$. It is true if p is true or q is true or both. For instance the compound proposition “2+2=4 or Sunday is the first day of the week” is true, and “3+3=7 or the freezing point of water is 32 degrees” is also true, but “2+2=5 or UT is in Oklahoma” is false. The truth table that defines disjunction is

p	q	$p \lor q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional Logic

- The Common Logical Operators
  - Negation (not): The negation of a proposition p is "not p". We denote it . It is true if p false and vice versa. This differs from the previous operators in that it is a *unary* operator, acting on a single proposition rather than a pair (the others are *binary* operators). Sometimes there are several ways of expressing a negation in English, and you should be careful to choose a clear one. For instance if p is the proposition "2<5", then reasonable statements of ~p are "it is not the case that 2<5" and "2 is not less than 5" and "". The truth table that defines negation is

p	~p
T	F
F	T

Propositional Logic

- The Common Logical Operators: Conditional (implies, if-then)
  - An implication is a compound proposition of the form “if p then q” or “p implies q”. In English this phrase carries many meanings. Sometimes it means that p causes q as in “if you eat too much you will get fat.” Sometimes it means that p guarantees q and vice versa as in “if you write a book report, I will give you five points extra credit” (tacitly assuring you that if you do not write it, I certainly will not give you extra credit). Sometimes it takes a very weak sense, simply asserting that the truth of p guarantees the truth of q as in “if you resign the chess game, you will lose” (but of course if you play on in a bad position, you will probably lose anyway — continued play does not guarantee winning).

Propositional Logic

- The Common Logical Operators: Conditional (implies, if-then)
  - Mathematics and logic always use implication in this weakest sense. Why? Because this is the very least that implication means in English. It makes our claims as conservative as possible. If we take “if-then” in this very weak sense, then we will never assume the phrase means *more* than it should. We denote the compound proposition “p implies q” by $p \rightarrow q$. From our discussion above the truth table for *implies* is

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Compound Propositions

A compound proposition is a proposition constructed by connecting several propositions together with the following connectives:

negation	$\sim$	$\sim p$ means "not p"
conjunction	$\wedge$	$p \wedge q$ means "p and q"
disjunction	$\vee$	$p \vee q$ means "p or q"
Implication (conditional)	$\rightarrow$	$p \rightarrow q$ means "if p then q"

Truth values for compound propositions

• best summarized by truth tables

Negation

p	~p
T	F
F	T

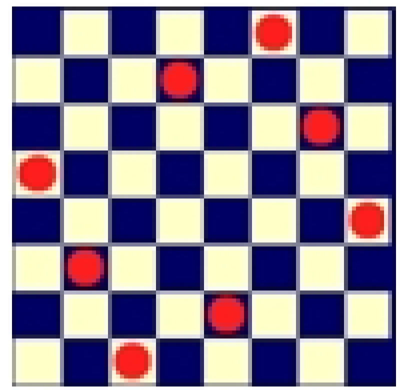
Example:
I live on the top of a mountain.

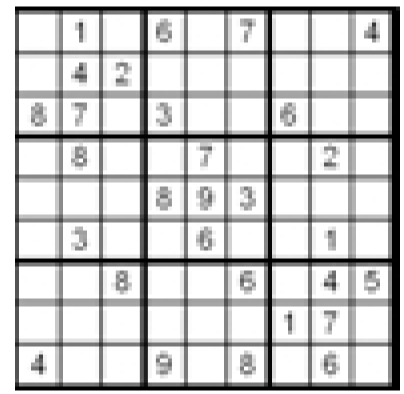
Negation:
I do not live on the top of a mountain.

Applications of Propositional Calculas Logic

Logic programming

solve problems by logic

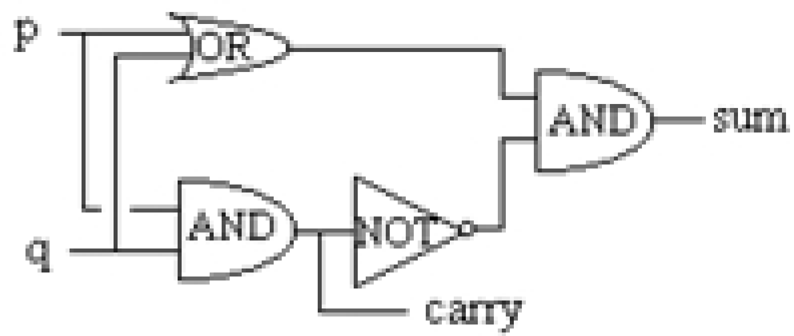




Database

making queries, data mining

Digital circuit



p	q	sum	carry
1	1	0	1
1	0	1	0
0	1	1	0
0	0	0	0