DISCRETE STRUCTURE

Lecture-1

Introduction to sets

TOPICS COVERED

1. Introduction of sets
2. Representation of sets
3. Types of sets
4. Subsets and proper subsets
5. Universal sets
6. Euler-Venn diagram
7. Algebra of sets (i.e. union, intersection, difference etc.)
8. Complement of set
9. Laws of algebra of sets
10. De Morgan's laws
11. Cardinal number of union, intersection, difference and symmetric difference of sets

INTRODUCTION

'Set is any collection of distinct and distinguishable objects of our intuition or thought'.

Following are the some examples of sets:

- The collection of vowels in English alphabets, i.e.
  A = {a, e, i, o, u}.
- The collection of all states in the Indian Union.
- The collection of all past presidents of the Indian Union etc.

INTRODUCTION

The following sets, we will use frequently

in this session and following sessions:

- N : For the set of natural numbers
- Z or I: For the set of integers
- Z^+ or I^+: For the set of all positive integers
- Q : For the set of all rational numbers
- Q^+ : For the set of all positive rational numbers
- R : For the set of all real numbers
- R^+ : For the set of all positive real numbers
- C : For the set of all complex numbers

REPRESENTATION OF A SET

A set is often represented in the following two ways:

(I) Roster method (Tabular form)

In this method a set is described by listing elements separated by commas, within braces { }.

For example, the set of even natural numbers can be described as {2, 4, 6, 8, ...}.

(II) Set Builder Method

In this method, a set is described by a characterizing property P(x) of its element x. In such a case the set is described by {x : P(x) holds} or {x / P(x) holds}

The symbol '|' or ':' is read as 'such that'.

In this representation the set of all even natural numbers can be written as : {x / x = 2n for $n \in N$ }

or $\{x / x = 2n,\ n \in N\}$.

TYPES OF SETS

Empty sets: A set having no element is called an empty set. It is also known as null set or void set. It is denoted by ϕ.

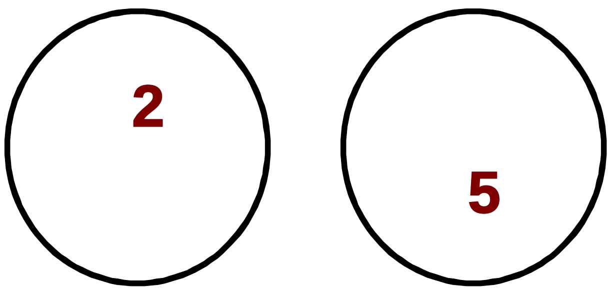
For example:

(a) $A = \{x \in R \,/\, x^2 = -10\} = \phi$

(b) $B = \text{set of immortal man} = \phi$

Singleton set: A set having single element is called singleton set.

For example, {2}, {0}, {5} are singleton set.



Types of Sets

Finite set: A set is called a finite set if it is called either void set or its elements can be counted by natural numbers and process of listing terminates at a certain natural numbers.

For example, {1, 2, 4, 6} is a finite set because it has four elements.

Infinite set: A set which is not a finite set, i.e. the counting up of whose elements is impossible, is called an infinite set.

For example:

(i) The set of all straight line in a given plane.
(ii) The set of all natural numbers.
(iii) The set of real numbers between ‘1’ and ‘2’.

CARDINAL NUMBER OR ORDER OF A FINITE SET

The total number of elements in a finite set is called cardinal number or order of a finite set. It is denoted by n(A).
For example, if

$$A = \{1, 2, 3, 4, 5\} \Rightarrow n(A) = 5 \text{ or } o(A) = 5.$$

Set of sets: A set S having all its elements as sets is called set of sets.

For example:

S = { {1, 2}, {2, 4}, {3, 5, 7} }

But S = { {1, 2}, 4, {3, 5, 7} } is not a set of sets as
is not a set.

$3 \in S$

Equivalent and Equal Sets

Equivalent sets: Two finite sets A and B are equivalent if their cardinal number is same, i.e. $n(A) = n(B)$.

Equal sets: Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

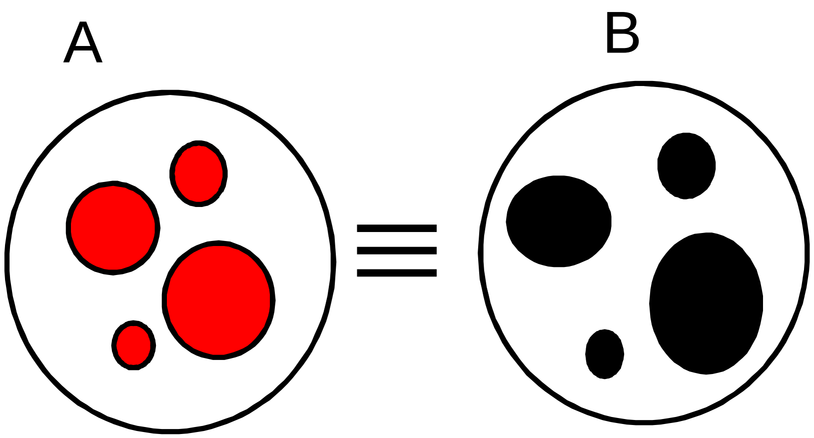
For example:

$A = \{4, 5, 6\}$ and
$B = \{a, b, c\}$ are equivalent but

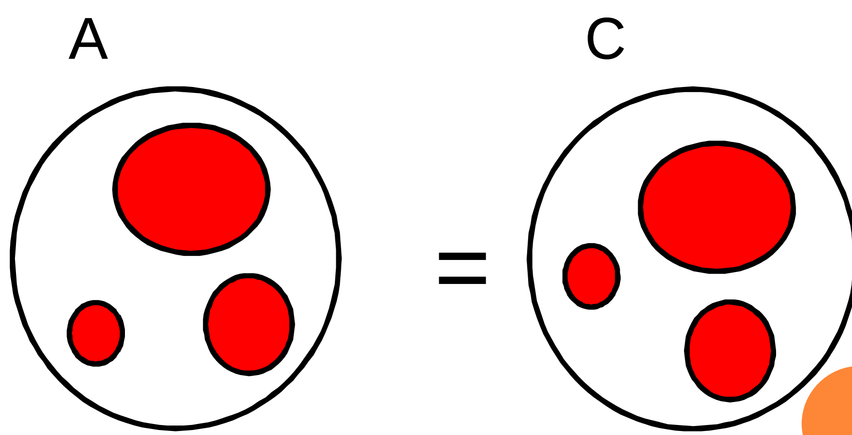
$A = \{4, 5, 6\}$ and
$C = \{6, 5, 4\}$ are equal, i.e. $A = C$.

Equivalent and Equal Sets

$A \equiv B$ if $n(A) = n(B)$



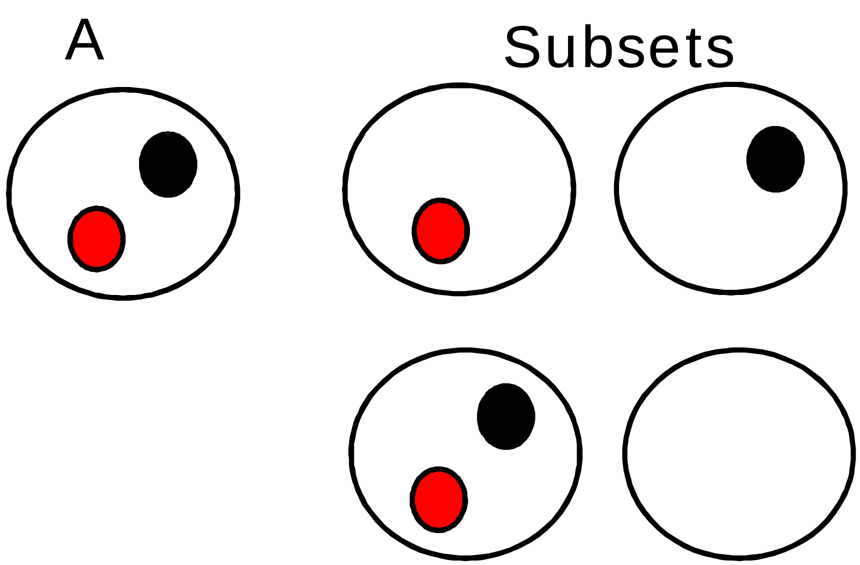
A = C if each element of each set is equal to each other



Subsets and Proper Subsets

Subsets: A set A is said to be a subset of a set B if each element of A is also an element of B.

$$A \subseteq B, \text{ if } x \in A \Rightarrow x \in B$$



For example:

Let A = {2, 4, 6, 8},

B = {2, 4, 6, 8, 10, 12},
then $A \subseteq B$.

SUBSETS AND PROPER SUBSETS

Proper subset: A set A is said to be a proper subset of a set B if every element of A is an element of B and B has at least one element which is not an element of A.

It can be written as $A \subset B$

For example:

Let A = {1, 2, 3}, B = {2, 3, 4, 1, 5}, then $A \subset B$.

Thus if A is a proper subset of B, then there exists an element such that $x \in B$ $x \notin A$.

For example, $\{1\} \subset \{1, 2, 3\}$ but $\{1, 4\} \not\subset \{1, 2, 3\}$.

Procedure for Proving Equality of Sets

As we have discussed earlier that two sets A and B are said to be equal if every element of set A is an element of set B and every element of B is an element of A.

It is clear that $$A = B \Leftrightarrow A \subseteq B \text{ and } B \subseteq A.$$

i.e. $$A = B \Leftrightarrow [x \in A \Leftrightarrow x \in B].$$

Some Results on Subsets

(i) Every set is a subset of itself.

(ii) The empty set ϕ is a subset of every set.

(iii) The total number of subsets of a finite set containing n elements is 2^n.

Proof : We know that ${}^nC_r$ denotes the number of ways for choosing r things from n different things. Therefore each selection of r things gives a subset of the set A containing r elements.

$\therefore$ The number of subsets of A having no element ${}^nC_0$

The number of subsets of A having one element ${}^nC_1$

SOME RESULTS ON SUBSETS

The number of subsets of A having two elements = ${}^nC_2$

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- -

- -

The number of subsets of A having n elements = ${}^nC_n$.

Hence, the total number of subsets of A

$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

[$\because$ We know that $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$.

Putting x = 1, we get $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$.]

Power Set

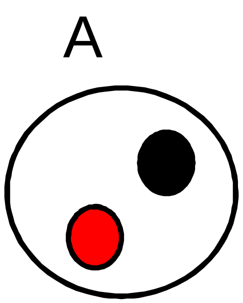
The set of all the subsets of a given set A is said to be the power set of A and is denoted by P(A).

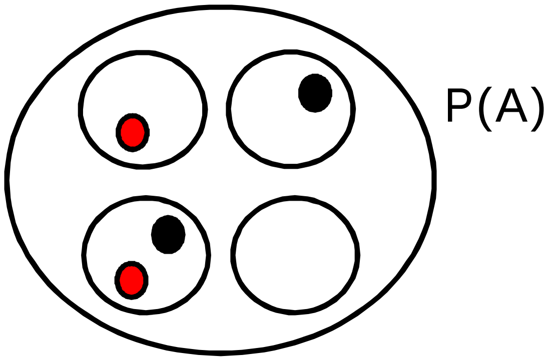
i.e. $P(A) = \{S \mid S \subseteq A\} \Rightarrow S \in P(A) \Leftrightarrow S \subseteq A$

Also, $\phi \in P(A)$ and $A \in P(A)$ for all sets A.

For example, if A = {a, b, c}, then

$$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}.$$





Universal Set

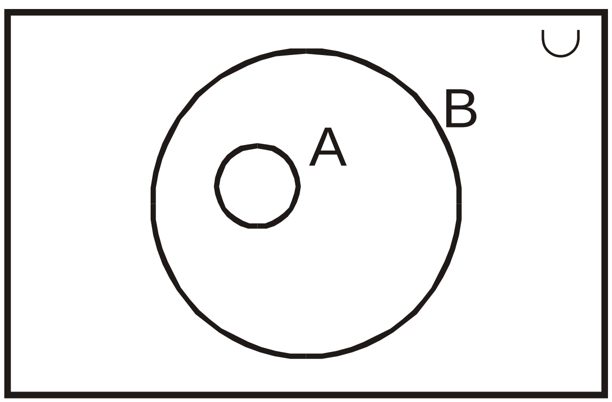
Any set which is super set of all the sets under consideration is called the universal set and is denoted by

Ω or $\cup$.

For example:

(i) When we are using sets containing natural numbers then N is the universal set.

Euler-Venn Diagram



$A \subset B$

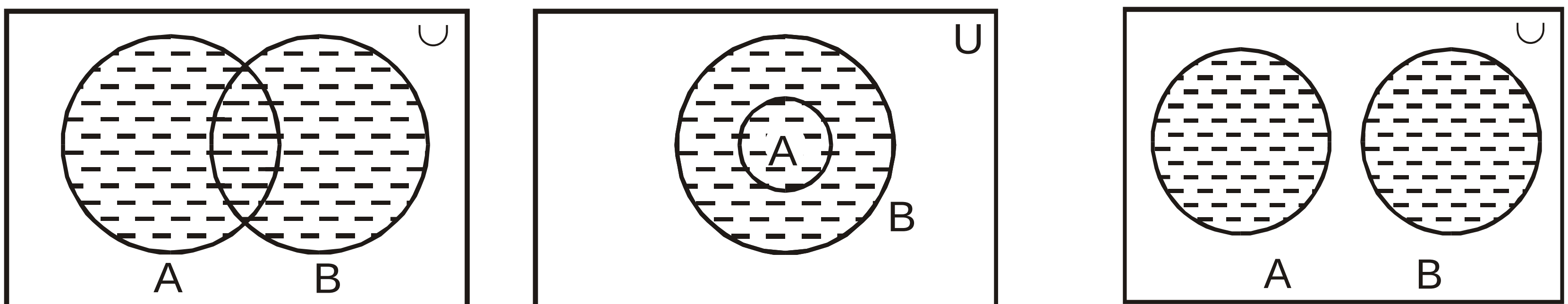
ALGEBRA OF SETS

Union of sets:

$$\therefore \; A \bigcup B = \{x : x \in A \text{ or } x \in B\}.$$

Clearly, if $x \in A \bigcup B \Leftrightarrow x \in A$ or $x \in B$

and $x \notin A \bigcup B \Leftrightarrow x \notin A$ and $x \notin B$



$A \cup B$ (a)

$A \cup B$ (is minimum) (b)

$A \cup B$ (is maximum) (c)

Generalised definition

If $A_1, A_2, \ldots A_n$ is a finite family of sets, then their union is

$A_1 \bigcup A_2 \bigcup A_3 \bigcup \ldots \bigcup A_n$ or $\bigcup\limits_{i=1}^{n} A_n$.

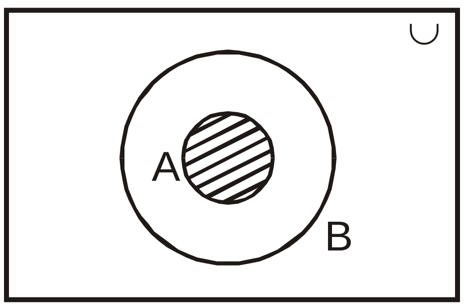
Algebra of Sets

Intersection of sets

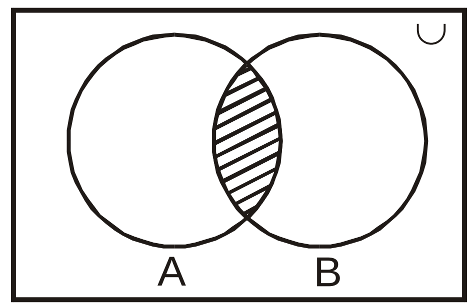
$$A \bigcap B = \{x : \ x \in A \ \text{ and } \ x \in B\}$$

Clearly, if $\quad x \in A \bigcap B \Rightarrow \ x \in A \ \text{ and } \ x \in B$

and $\quad x \notin A \bigcap B \ \Leftrightarrow \ x \notin A \ \text{ or } \ x \notin B.$



$A \cap B$



$A \cap B$

Generalised definition

The intersection of sets $A_1, A_2, \ldots A_n$ is the set of all the elements

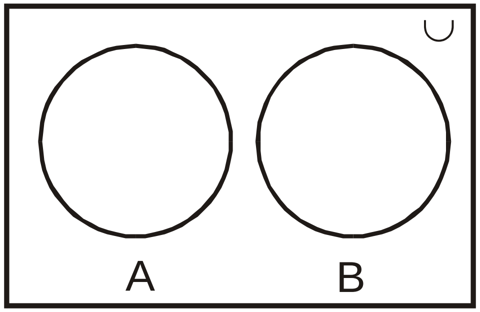
which are common to all the sets $A_1, \ A_2, \ldots \ A_n$. It is denoted by

$$A_1 \bigcap A_2 \bigcap A_3 \ldots \bigcap \ A_n \quad \text{or} \quad \bigcap_{i=1}^{n} A_i$$

Algebra of Sets

Disjoint sets

$$A \bigcap B = \phi$$



$A \cap B$

Difference of sets

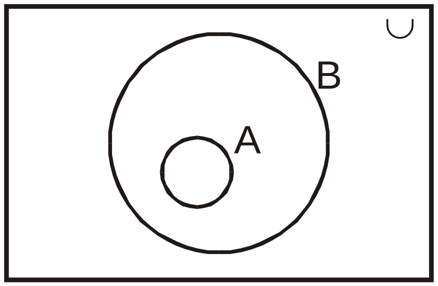
$$A - B = \{x : x \in A \ \text{ and } \ x \notin B\}$$

$$x \in A - B \Leftrightarrow x \in A \ \text{ and } \ x \notin B$$

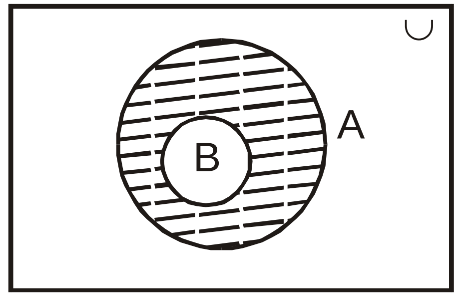
$$x \in B - A = \{x : x \in B \ \text{ and } \ x \notin A\}$$

ALGEBRA OF SETS

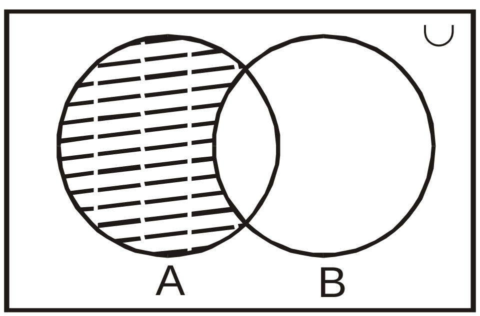
Difference of sets



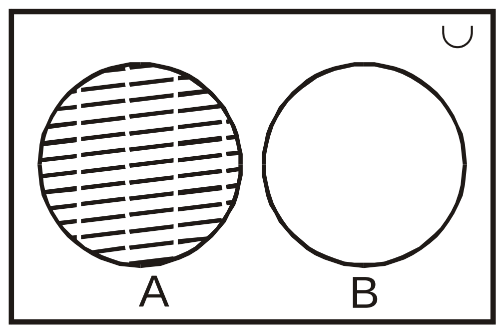
When $A \subset B$
$A - B = \phi$
(a)



When $B \subset A$
(b)



When neither $A \subset B$
nor $B \subset A$
(c)



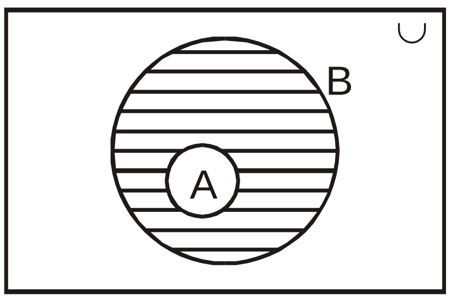
When A and B are
disjoint set
(d)

Algebra of Sets

Symmetric Differences of sets

$$A \,\Delta\, B = (A - B) \cup (B - A)$$

$$x \in A \,\Delta\, B \Rightarrow x \notin A \cap B$$



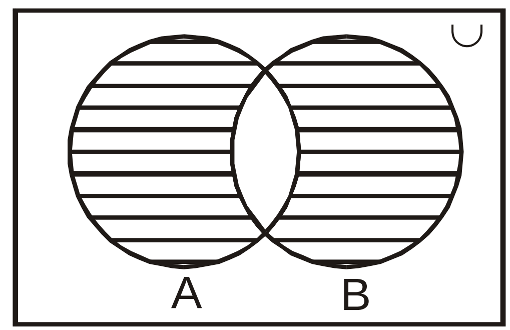
When $A \subset B$

$A \,\Delta\, B = (A - B) \cup (B - A)$

$= \phi \cup (B - A)$

$= B - A$

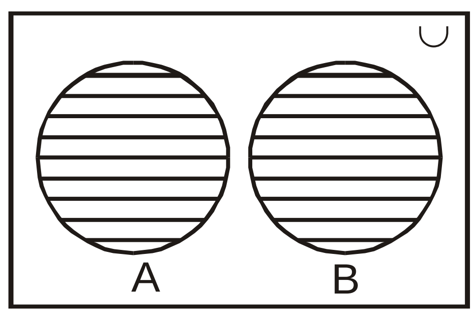
(a)



When neither $B \subset A$

nor $B \subset A$

(b)



When A and B are

disjoint set

(c)

ALGEBRA OF SETS

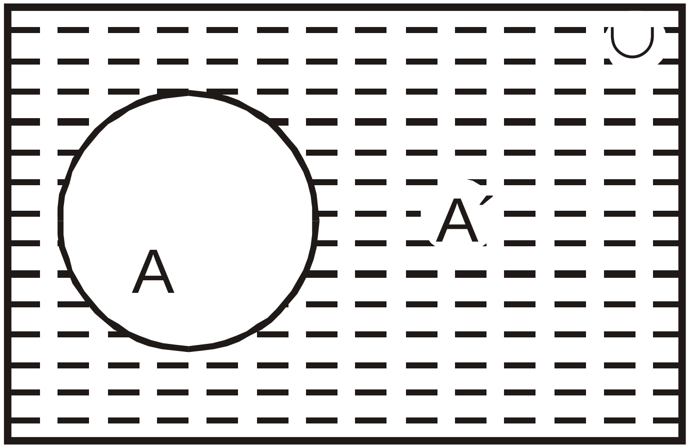
Complement of set

A^c or A'

Clearly, A^c or $A' = U - A$

$\therefore\ A^c$ or $A' = \{x : x \in U \text{ and } x \notin A\}$

$x \in A' \Leftrightarrow x \notin A$



Do You Know

(i) $U' = \{x : x \in U \text{ and } x \notin U\} = \phi.$

(ii) $\phi' = \{x : x \in U \text{ and } x \notin \phi\} = U.$

(iii) $(A')' = \{x : x \in U \text{ and } x \notin A'\}$

$= \{x : x \in U \text{ and } x \in A\} = A$

(iv) $A \cup A' = \{x : x \in U \text{ and } x \in A\} \cup \{x : x \in U \text{ and } x \notin A\} = U$

(v) $A \cap A' = \{x : x \in U \text{ and } x \in A\} \cap \{x : x \in U \text{ and } x \notin A\} = \phi$

LAW OF ALGEBRA OF SETS

Indempotent laws

For any set A, we have

(i) $A \cup A = A$

(ii) $A \cap A = A$

Identity laws

For any set A, we have

(i) $A \cup \phi = A$

(ii) $A \cap U = A$

i.e. ϕ and U are identity elements for union and intersection respectively.

Law of Algebra of Sets

Commutative laws

For any two sets A and B, we have

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

Proof: As we know that two sets X and Y are equal if

$X \subseteq Y$ and $Y \subseteq X$.

(i) Let x be any arbitrary element of $A \cup B$.

$$\Rightarrow x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \in B \text{ or } x \in A$$

Law of Algebra of Sets

$\Rightarrow \ x \in B \cup A$

$\therefore \ A \cup B \subseteq B \cup A$ ...(i)

Similarly, let y be any arbitrary element of . $B \cup A$

$\Rightarrow \ y \in B \cup A \ \Rightarrow \ y \in B \ \text{ or } \ y \in A$

$\Rightarrow \ y \in A \ \text{ or } \ y \in B \qquad \Rightarrow \ y \in A \cup B$

$\therefore \ B \cup A \subseteq A \cup B$ ...(i)

From (i) and (ii)

$A \cup B = B \cup A$

Law of Algebra of Sets

Associative laws

If A, B and C are any three sets, then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

Proof:

(i) Let x be any arbitrary element of $(A \cup B) \cup C$.

$\therefore \; x \in (A \cup B) \cup C \quad \Rightarrow \; x \in (A \cup B) \text{ or } x \in C$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C \quad \Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$

$\Rightarrow \; x \in A \text{ or } (x \in B \cup C) \quad \Rightarrow x \in A \cup (B \cup C)$

Law of Algebra of Sets

$\therefore \ (A \cup B) \cup C \subseteq A \cup (B \cup C)$ ...(i)

Similarly, let y be any arbitrary element of $A \cup (B \cup C)$

$\therefore \ y \in A \cup (B \cup C)$ $\Rightarrow y \in A$ or $y \in (B \cup C)$

$\Rightarrow y \in A$ or $(y \in B$ or $y \in C)$ $\Rightarrow (y \in A$ or $y \in B)$ or $y \in C$

$\Rightarrow (y \in A \cup B)$ or $y \in C$ $\Rightarrow y \in (A \cup B) \cup C$

$\therefore \ A \cup (B \cup C) \subseteq (A \cup B) \cup C$ ...(ii)

From (i) and (ii)

$(A \cup B) \cup C = A \cup (B \cup C)$ Proved.

Law of Algebra of Sets

Distributive laws

If A, B and C are any three sets, then

(i) $$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(ii) $$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

In other words, union and intersection are distributive over intersection and union respectively.

Proof:

(i) Let x be any arbitrary element of $A \cup (B \cap C)$.

$$\therefore\ x \in A \cup (B \cap C)$$

$$\Rightarrow\ x \in A \quad \text{or} \quad x \in (B \cap C)$$

$$\Rightarrow\ x \in A \ \text{ or } \ (x \in B \quad \text{and} \quad x \in C)$$

Law of Algebra of Sets

$$\Rightarrow (x \in A \quad or \quad x \in B) \quad and \quad (x \in A \quad or \quad x \in C)$$

[$\because$ 'or' is distributive over 'and']

$$\Rightarrow x \in (A \cup B) \quad and \quad x \in (A \cup C)$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C)$$

$$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \qquad ...(i)$$

Similarly, let y be any arbitrary element of $(A \cup B) \cap (A \cup C)$.

$$\therefore y \in (A \cup B) \cap (A \cup C) \qquad \Rightarrow y \in (A \cup B) \quad and \quad y \in (A \cup C)$$

$$\Rightarrow (y \in A \quad or \quad y \in B) \ and \ (y \in A \quad or \quad y \in C)$$

$$\Rightarrow y \in A \quad or \quad (y \in B \quad and \quad y \in C)$$

Law of Algebra of Sets

$$\Rightarrow \quad y \in A \quad \text{or} \quad (y \in B \cap C)$$

$$\Rightarrow \quad y \in A \cup (B \cap C)$$

$$\therefore \ (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \qquad \ldots\text{(ii)}$$

From (i) and (ii),

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \qquad \text{Proved.}$$

De Morgan's Law

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof:

(i) Let x be an arbitrary element of the set $(A \cup B)'$.

$\therefore\ x \in (A \cup B)'$ $\Rightarrow\ x \notin (A \cup B)$

$\Rightarrow\ x \notin A \text{ and } x \notin B$ $\Rightarrow\ x \in A' \text{ and } x \in B'$

$\Rightarrow\ x \in A' \cap B'$ $\therefore\ (A \cup B)' \subseteq A' \cap B'$ ...(i)

Again let y be an arbitrary element of $A' \cap B'$.

$\therefore\ y \in A' \cap B'$ $\Rightarrow\ y \in A' \text{ and } y \in B'$

De Morgan's Law

$$\Rightarrow \ y \notin A \ \text{ and } \ y \notin B \qquad \Rightarrow \ y \notin A \cup B$$

$$\Rightarrow \ y \in (A \cup B)'$$

$$\therefore \ A' \cap B' \subseteq (A \cup B)' \qquad \text{...(ii)}$$

From (i) and (ii),

$$(A \cup B)' = A' \cap B'$$

Some Important Results on Cardinal Number of Union, Intersection, Difference, and Symmetric Difference of Sets

If A, B and C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A$ and B are disjoint non-void sets.

(iii) n (A – B) = n(A) – $n(A \cap B)$

i.e., $n(A) = n(A - B) + n(A \cap B)$

(iv) $n(A \Delta B) =$ Number of elements which belong to exactly one of A or B.

$$= n(A) + n(B) - 2n(A \cap B)$$

Some Important Results on Cardinal Number of Union, Intersection, Difference, and Symmetric Difference of Sets

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B and C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(vii) Number of elements in exactly one of the sets A, B and C

$$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$$

Class Test

Class Exercise - 1

Let A = {a, b, c, d}, B = {a, b, c} and C = {b, d}. Find all sets X such that

(i) $X \subset B \text{ and } X \subset C$ (ii) $X \subset A \text{ and } X \not\subset B$

SOLUTION

(i)

$$P(B) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

$$P(C) = \{\phi, \{b\}, \{d\}, \{b, d\}\}$$

$$\because\ X \subset B \text{ and } X \subset C$$

$$\Rightarrow\ X \in P(B) \text{ and } X \in P(C) \Rightarrow X \in P(B) \cap P(C)$$

$$\Rightarrow\ X = \phi,\ \{b\}$$

(ii) Now, $X \subset A$ and $X \not\subset B$

$\Longrightarrow$ X is subset of A but X is not subset of B.

$$\Rightarrow\ X \in P(A) \text{ but } x \notin P(B) \Rightarrow X \in P(A) - P(B)$$

SOLUTION CONTD..

$\therefore \quad X =$ {d}, {a, d} {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}

Here note that to obtain X we have added each element of P(B) with 'd' which is in A not in B.

Class Exercise - 2

For any two sets A and B, prove that

$$A \cup B = A \cap B \Leftrightarrow A = B.$$

SOLUTION

First let A = B. Then

$$A \cup B = A \quad \text{and} \quad A \cap B = A$$

$$\Rightarrow \; A \cup B = A \cap B$$

$$\therefore \; A = B \;\Rightarrow\; A \cup B = A \cap B \qquad \text{...(i)}$$

Conversely, let $A \cup B = A \cap B$.

$$\therefore \; x \in A \quad \Rightarrow \; x \in A \cup B$$

$$\Rightarrow \; x \in A \cap B$$

$$\Rightarrow \; x \in A \;\text{ and }\; x \in B$$

$$\Rightarrow \; x \in B$$

Solution contd..

$$\therefore \quad A \subseteq B \qquad \text{...(ii)}$$

Now let

$$y \in B \quad \Rightarrow \quad y \in A \cup B$$

$$\Rightarrow \quad y \in A \cap B$$

$$\Rightarrow \quad y \in A \quad \text{and} \quad y \in B$$

$$\Rightarrow \quad y \in A$$

$$\therefore \quad B \subseteq A \qquad \text{...(iii)}$$

From (ii) and (iii), we get A = B

$$\text{Thus, } A \cup B = A \cap B \quad \Rightarrow \quad A = B \qquad \text{...(iv)}$$

$$\text{From (i) and (iv), } A \cup B = A \cap B \quad \Leftrightarrow \quad A = B$$

Class Exercise - 3

If $a \in N$ such that $aN = \{ax : x \in N\}$, describe the set $3N \cap 7N$.

SOLUTION

We have $aN = \{ax : x \in N\}$

$\therefore\ 3N = \{3x : x \in N\} = \{3, 6, 9, 12, 15, \ldots\}$

$7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

Hence, $3N \cap 7N = \{21, 42, 63, \ldots\} = \{21x : x \in N\} = 21N$

Note that $aN \cap bN = cN$ where c = LCM of a, b.

CLASS EXERCISE - 4

If A, B and C are any three sets, then prove that $A-(B\cap C)=(A-B)\cup(A-C)$.

SOLUTION

Let x be any element of $\cdot A-(B\cap C)$

$$\therefore\ x\in A-(B\cap C)\Rightarrow x\in A\ \text{ and }\ x\notin(B\cap C)$$

$$\Rightarrow\ x\in A\ \text{ and }\ (x\notin B\ \text{ or }\ x\notin C)$$

$$\Rightarrow\ (x\in A\ \text{ and }\ x\notin B)\ \text{ or }\ (x\in A\ \text{ and }\ x\notin C)$$

$$\Rightarrow\ x\in(A-B)\ \text{ or }\ x\in(A-C)$$

$$\Rightarrow\ x\in(A-B)\cup(A-C)$$

$$\therefore\ A-(B\cap C)\subseteq(A-B)\cup(A-C)\qquad\text{...(i)}$$

SOLUTION CONTD..

Again y be any element of $(A-B)\cup(A-C)$

$$\therefore\ y\in(A-B)\cup(A-C)\Rightarrow y\in(A-B) \text{ or } y\in(A-C)$$

$$\Rightarrow (y\in A \text{ and } y\notin B) \text{ or } (y\in A \text{ and } y\notin C)$$

$$\Rightarrow y\in A \text{ and } (y\notin B \text{ or } y\notin C)$$

$$\Rightarrow y\in A \text{ and } (y\notin(B\cap C)) \qquad \Rightarrow y\in A-(B\cap C)$$

$$\therefore\ (A-B)\cup(A-C)\subseteq A-(B\cap C) \qquad \text{...(ii)}$$

From (i) and (ii),

$$A-(B\cap C)=(A-B)\cup(A-C) \qquad \text{Proved.}$$

CLASS EXERCISE - 5

Let A, B and C be three sets such that $A \cup B = C$ and $A \cap B = \phi$, then prove that $A = C - B$

SOLUTION

$\because$ We have $A \cup B = C$.

$\therefore$ $C - B = (A \cup B) - B$

$= (A \cup B) \cap B'$ $\quad [\because \; X - Y = X \cap Y']$

$= (A \cap B') \cup (B \cap B')$ $\quad$ [By distributive law]

$= (A \cap B') \cup \phi$

$= A \cap B'$

$= A - B$

$= A$ $\quad [\because \; A \cap B = \phi]$ $\quad$ Proved.

Class Exercise - 6

If A, B and C are the sets such that $A \subset B$, then prove that $C - B \subset C - A$.

SOLUTION

Let x be any arbitrary element of C – B.

$$\therefore \quad x \in C - B \Rightarrow x \in C \text{ and } x \notin B$$

$$\Rightarrow \quad x \in C \text{ and } x \notin A \qquad [\because \; A \subset B]$$

$$\Rightarrow \quad x \in C - A$$

$$\therefore \quad C - B \subset C - A \qquad \text{Proved.}$$

CLASS EXERCISE - 7

If A, B and C are the three sets and U is the universal set such that n(U) = 700, n(A) = 200, n(B) = 300 and, find

$n\left(A' \cap B'\right).$

SOLUTION

$\because \quad A' \cap B' = (A \cup B)'$ By De Morgan's law

$$\therefore \; n(A' \cap B') = n\left((A \cup B)'\right)$$

$$= n(U) - n(A \cup B)$$

$$= n(U) - \left[n(A) + n(B) - n(A \cap B)\right]$$

= 700 – [200 + 300 – 100]

= 300

CLASS EXERCISE - 8

In a class of 35 students, 17 have taken mathematics, 10 have taken mathematics but not physics. Find the number of students who have taken both mathematics and physics and the number of students who have taken physics but not mathematics, if it is given that each student has taken either mathematics or physics or both.

SOLUTION

Method I:

Let M denote the set of students who have taken mathematics and P be the set of students who have taken physics.

Given that

$$n(M \cup P) = 35,\ n(M) = 17,\ n(M - P) = 10$$

We know that $n(M - P) = n(M) - n(M \cap P)$

$$\Rightarrow\ 10 = 17 - n(M \cap P)$$

$\Rightarrow\ n(M \cap P) = 17 - 10 = 7 \Rightarrow$ 7 students have taken both mathematics and physics.

Solution contd..

Now we want to find n(P – M).

$$\therefore\ n(M \cup P) = n(M) + n(P) - n(M \cap P)$$

$\Rightarrow$ 35 = 17 + n(P) – 7

$\Rightarrow$ n(P) = 35 – 10 = 25

$$\therefore\ n(P - M) = n(P) - P(M \cap P)$$

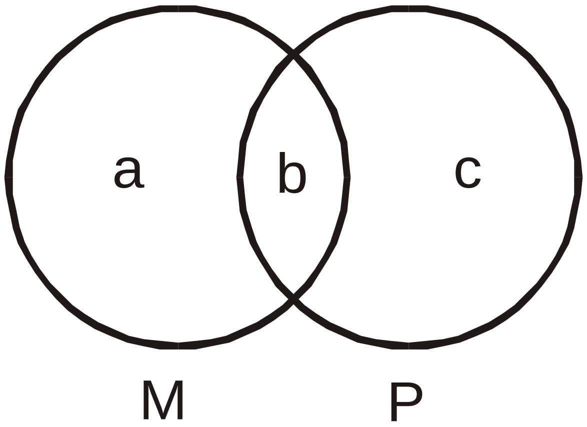
= 25 – 7
= 18

$\Rightarrow$ 18 students have taken physics but not mathematics.

Solution contd..

Method II:

Venn diagram method:



Given that $n(M \cup P) = a + b + c = 35$ ...(i)

$n(M) = a + b = 17$ ...(ii)

$n(M - P) = a = 10$ ...(iii)

Solution contd..

We want to find b and c

From (ii) and (iii),
$b = 17 - 10 = 7 \Rightarrow$ 7 students have taken both physics and mathematics.

From (i), $10 + 7 + c = 35$

$c = 35 - 17 = 18$

$\Rightarrow$ 18 students have taken physics but not mathematics.

CLASS EXERCISE - 9

If A and B be the two sets containing 3 and 6 elements respectively, what can be the minimum and maximum number of elements in $A \cup B$?

SOLUTION

As we know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$\therefore$ $n(A \cup B)$ is minimum or maximum accordingly as

$n(A \cap B)$ is maximum or minimum respectively.

Case I: When $n(A \cap B)$ is minimum, i.e. $n(A \cap B) = 0$

This is possible only when . $A \cap B = \phi$

$$\therefore\ n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

= 3 + 6 – 0

= 9

$\therefore$ Maximum number of elements in $A \cup B$ = 9

SOLUTION CONTD..

Case II: When $n\left(A \cap B\right)$ is maximum

This is possible only when $A \subseteq B$ In this case

$A \cup B = B$

$$\therefore\ n\left(A \cup B\right) = n\left(B\right) = 6$$

$\therefore$ Minimum number of elements in $A \cup B$ is 6.

CLASS EXERCISE - 10

Out of 880 boys in a school, 224 play cricket, 240 play hockey, and 336 play basketball. Of the total, 64 play both basketball and hockey; 80 play cricket and basketball and 40 play cricket and hockey; 24 play all the three games. Find the number of boys who did not play any game.

Solution

Method I:

Let C, B and H denote the set of boys playing cricket, basketball and hockey respectively.

Here given that

$n(C) = 224$, $n(H) = 240$, $n(B) = 336$

$$n(B \cap H) = 64,\ \ n(C \cap B) = 80,\ n(C \cap H) = 40$$

$$n(C \cap B \cap H) = 24$$

$\because$ We know that

$$n(C \cup B \cup H) = n(C) + n(B) + n(H) - n(C \cap B) - n(B \cap H) - n(C \cap H) + n(C \cap B \cap H)$$

$= 224 + 336 + 240 – 80 – 64 – 40 + 24$

$= 640$

SOLUTION CONTD..

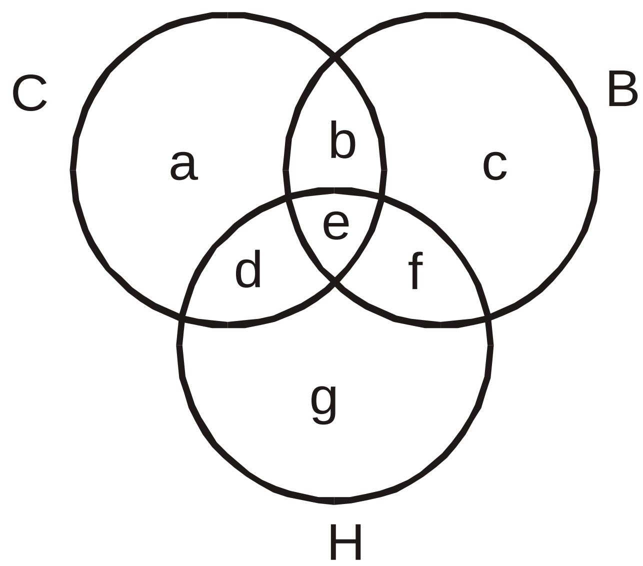
$\therefore$ Number of boys not playing any game is

Total number of students – $n\left(C \cup B \cup H\right)$

= 880 – 640 = 240

Method II:

Venn diagram method:



It is given that

$n(C) = a + b + d + e = 224$ ...(i)

$n(H) = d + e + f + g = 240$ ...(ii)

$n(B) = b + c + e + f = 336$ ...(iii)

Solution contd..

$n(B \cap H) = e + f = 64$ ...(iv)

$n(C \cap B) = b + e = 80$ ...(v)

$n(C \cap H) = d + e = 40$ ...(vi)

$n(C \cap B \cap H) = e = 24$ ...(vii)

$\because\ d + e = 40 \Rightarrow d = 40 - 24 = 16$

$b + e = 80 \Rightarrow b = 80 - 24 = 56$

$e + f = 64 \Rightarrow f = 64 - 24 = 40$

$\because\ b + c + e + f = 336 \Rightarrow c = 336 - 56 - 24 - 40 = 216$

Again $d + e + f + g = 240 \Rightarrow g = 240 - 16 - 24 - 40$

$= 240 - 80$

$= 160$

Solution contd..

and $a + b + d + e = 224$

$\Rightarrow\ a = 224 - 56 - 16 - 24$

$= 224 - 96$

$= 128$

$\therefore$ Required number of students not playing any game

$= 880 - (a + b + c + d + e + f + g)$

$= 880 - (128 + 56 + 216 + 16 + 24 + 40 + 160)$

$= 880 - 640$

$= 240$