LECTURE 14

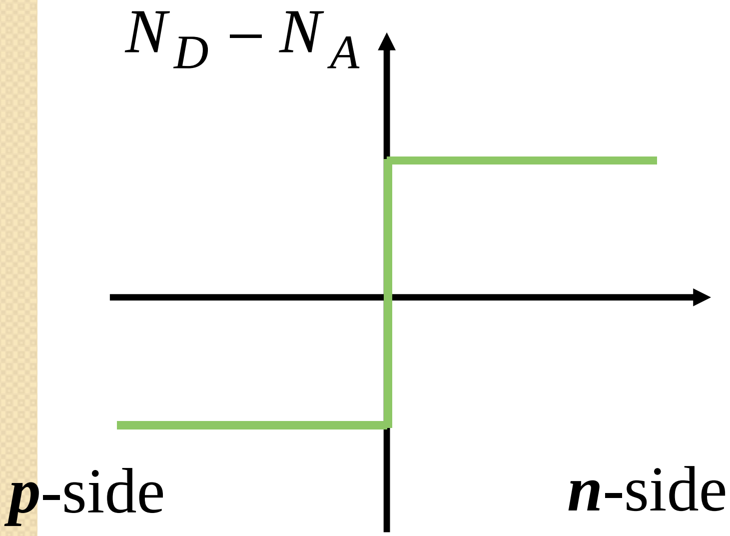
-Breakdown Mechanism

Topics to be covered

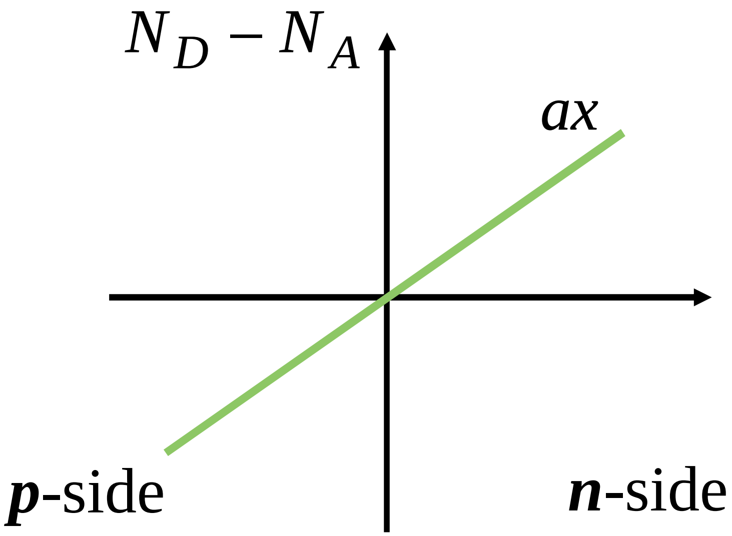
- Breakdown Mechanism

1. PN-junctions - General Consideration:

- PN-junction is a **two terminal** device.
- Based on the **doping profile**, PN-junctions can be separated into two major categories:
    - step junctions
    - linearly-graded junctions

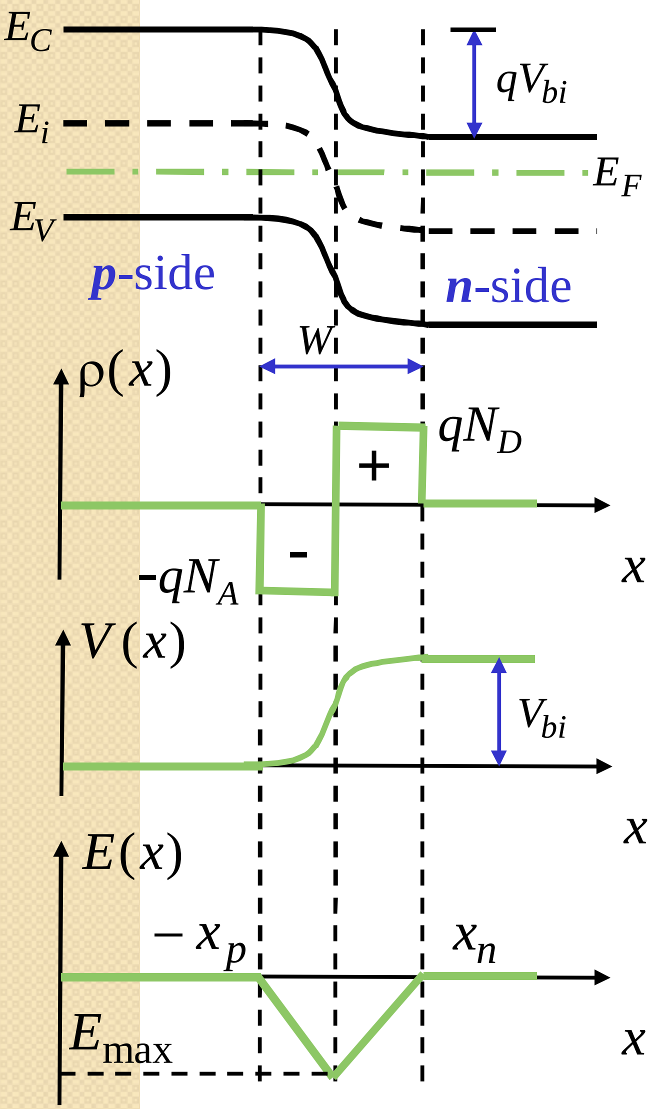


Step junction



Linearly-graded junction

(A) Equilibrium analysis of step junctions



(a) Built-in voltage V_{bi}:

$$qV_{bi} = (E_i - E_F)_p + (E_F - E_i)_n$$

$$n_{n0} = n_i \exp[(E_F - E_i)/k_B T]$$

$$p_{p0} = n_i \exp[E_i - E_F / k_B T]$$

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{p_{p0} n_{n0}}{n_i^2}\right) \approx V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

(b) Majority- minority carrier relationship:

$$p_{n0} = p_{p0} \exp[-V_{bi} / V_T]$$

$$n_{p0} = n_{n0} \exp[-V_{bi} / V_T]$$

(c) Depletion region width:

➔ Solve 1D Poisson equation using depletion charge approximation, subject to the following boundary conditions: $V(-x_p) = 0, \quad V(x_n) = V_{bi}, \quad E(-x_n) = E(x_p) = 0$

$$\textbf{p}\text{-side:}\; V_p(x) = \frac{qN_A}{2k_s\varepsilon_0}\left(x + x_p\right)^2$$

$$\textbf{n}\text{-side:}\; V_n(x) = -\frac{qN_D}{2k_s\varepsilon_0}(x_n - x)^2 + V_{bi}$$

➔ Use the continuity of the two solutions at x=0, and charge neutrality, to obtain the expression for the depletion region width W:

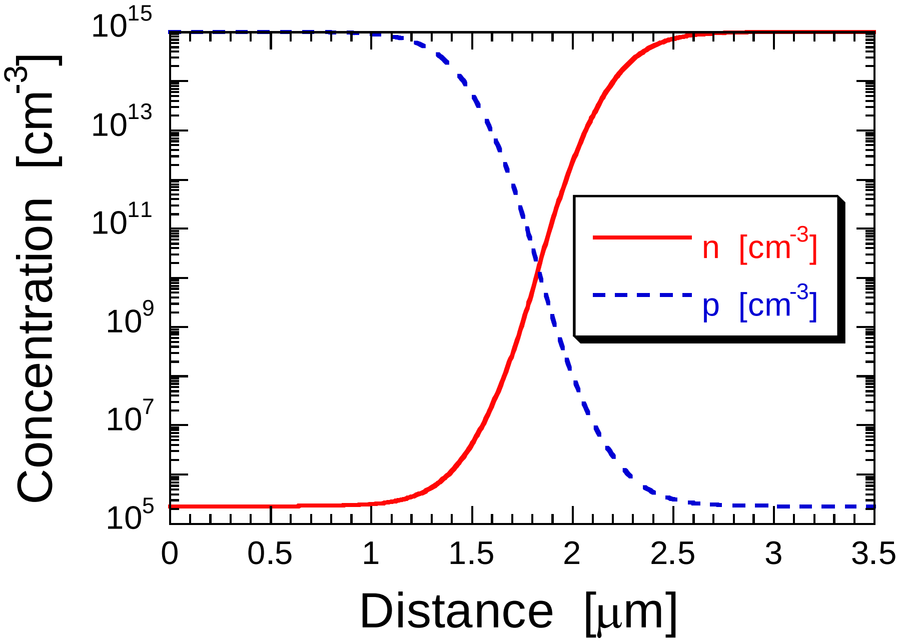
$$\left.\begin{array}{c} x_n + x_p = W \\ V_p(0) = V_n(0) \\ N_A x_p = N_D x_n \end{array}\right\} \rightarrow W = \sqrt{\frac{2k_s\varepsilon_0(N_A + N_D)V_{bi}}{qN_A N_D}}$$

(d) Maximum electric field:

The maximum electric field, which occurs at the metallurgical junction, is given by:

$$E_{\max} = -\left.\frac{dV}{dx}\right|_{x=0} = -\frac{qN_A N_D W}{k_s \varepsilon_0 (N_A + N_D)}$$

(e) Carrier concentration variation:



$$N_A = N_D = 10^{15}\,cm^{-3}$$

$$W_{calc} = 1.23\,\mu m$$

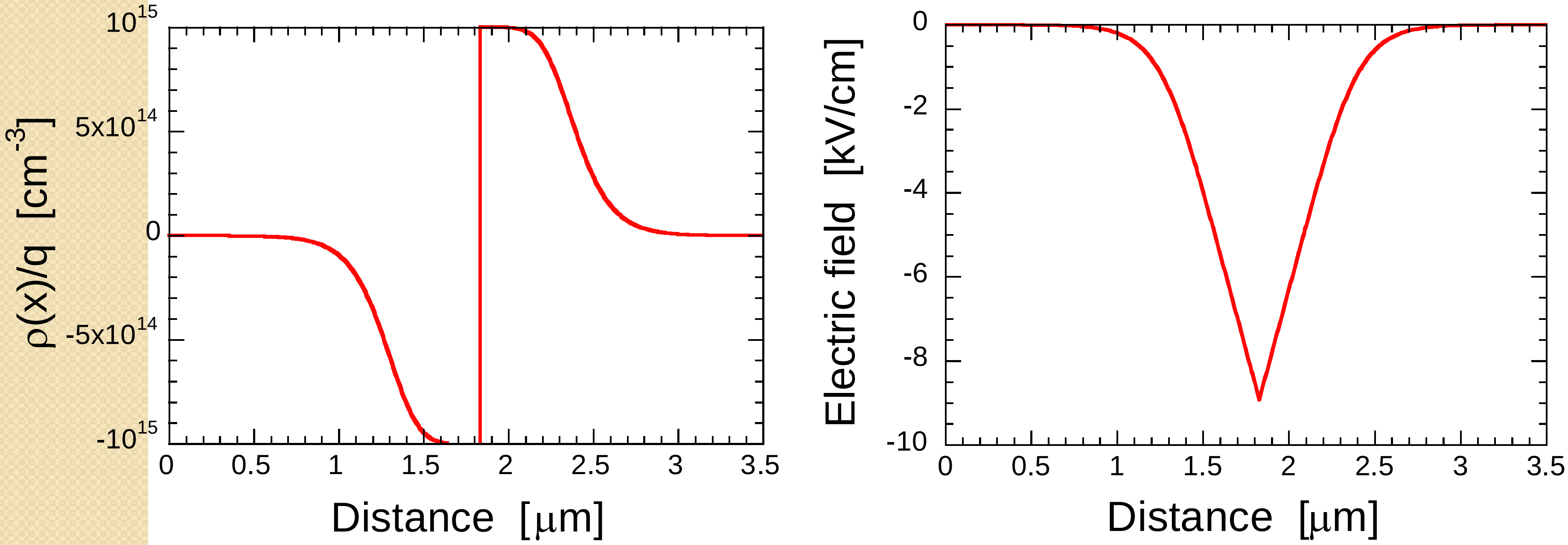
$$E_{\max(DC)} = 9.36\,kV/cm$$

$$E_{\max(sim)} = 8.93\,kV/cm$$

(f) Analytical vs. numerical data

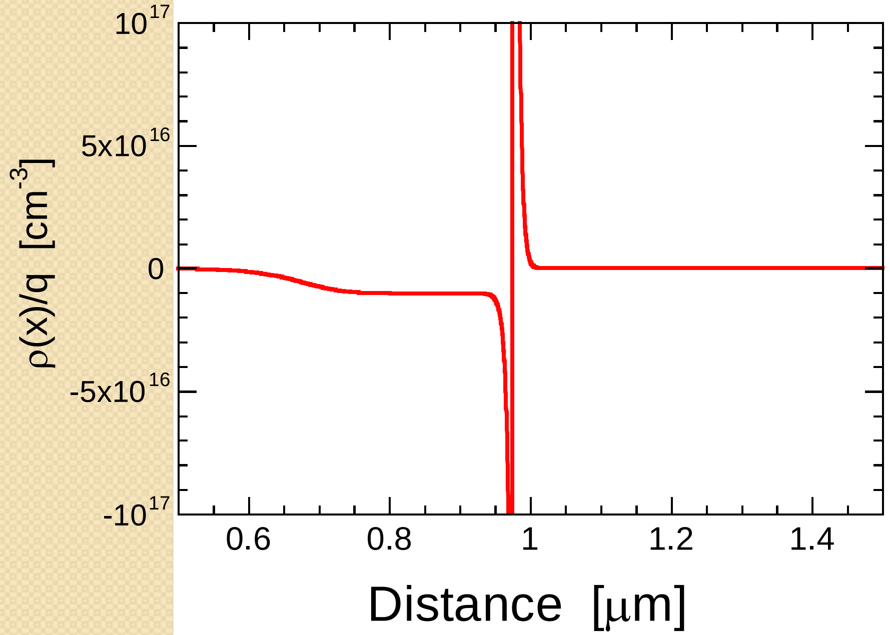
$$N_A = N_D = 10^{15}\,cm^{-3}$$

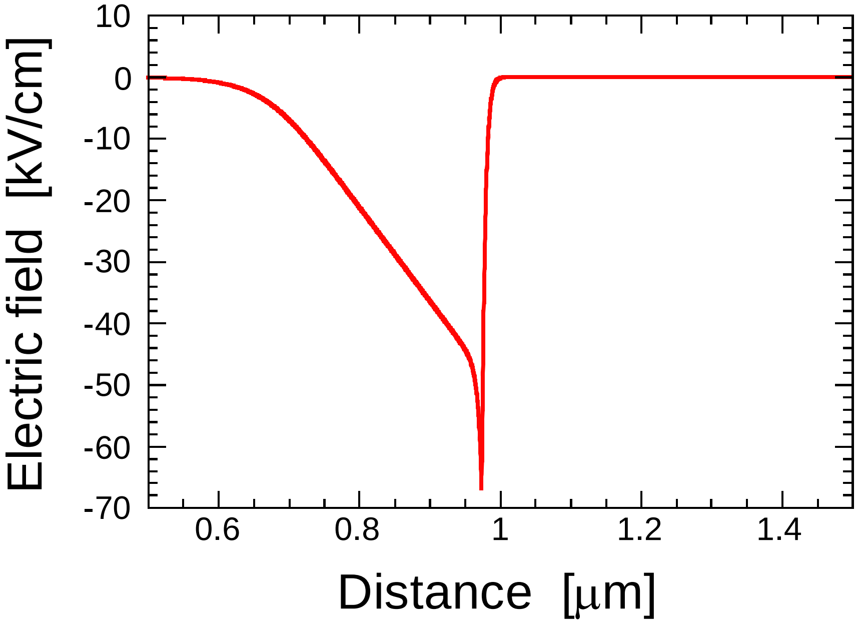
$$W_{calc} = 1.23\,\mu m,\;\; E_{\max(DC)} = 9.36\,kV/cm,\;\; E_{\max(sim)} = 8.93\,kV/cm$$



$$N_A = 10^{16}\,cm^{-3},\ N_D = 10^{18}\,cm^{-3}$$

$$W_{calc} = 0.328\ \mu m,\ E_{\max(DC)} = 49.53\ kV/cm,\ E_{\max(sim)} = 67\ kV/cm$$





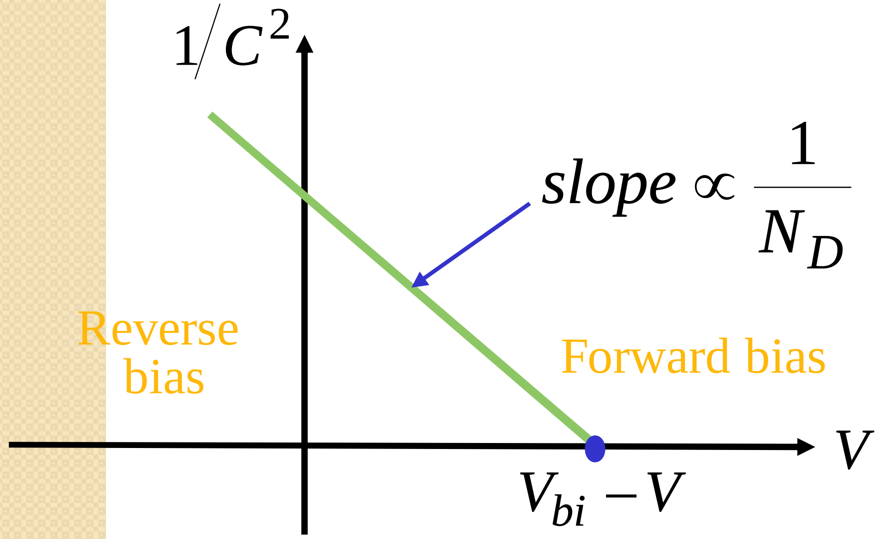
(g) Depletion layer capacitance:

➔ Consider a $p^{+}n$, or one-sided junction, for which:

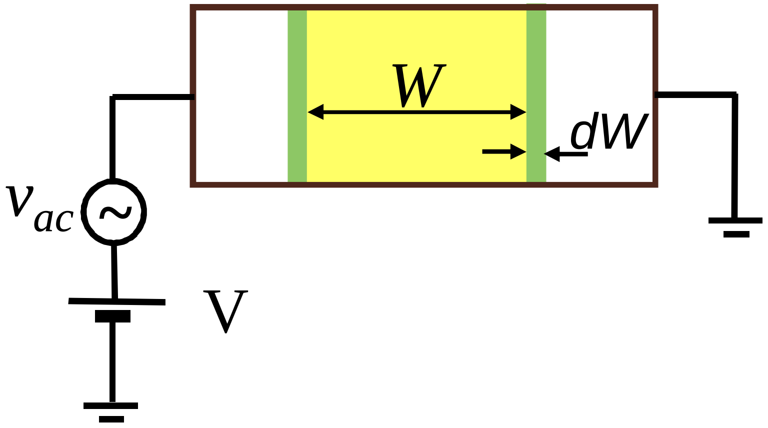
$$W = \sqrt{\frac{2k_s\varepsilon_0\left(V_{bi} \mp V\right)}{qN_D}}$$

➔ The depletion layer capacitance is calculated using:

$$C = \frac{dQ_c}{dV} = \frac{qN_D dW}{dV} = \sqrt{\frac{qN_D k_s\varepsilon_0}{2(V_{bi} \mp V)}} \rightarrow \frac{1}{C^2} = \frac{2(V_{bi} \mp V)}{qN_D k_s\varepsilon_0}$$



Measurement setup:



(B) Equilibrium analysis of linearly-graded junction:

(a) Depletion layer width:

$$W = \left[\frac{12 k_s \varepsilon_0 \left(V_{bi} \mp V\right)}{qa}\right]^{1/3}$$

(c) Maximum electric field:

$$E_{\max} = -\frac{qaW^2}{8 k_s \varepsilon_0}$$

(d) Depletion layer capacitance:

$$C = \left[\frac{qa k_s^2 \varepsilon_0^2}{12\left(V_{bi} \mp V\right)}\right]^{1/3}$$

Based on accurate numerical simulations, the depletion layer capacitance can be more accurately calculated if V_{bi} is replaced by the gradient voltage V_g:

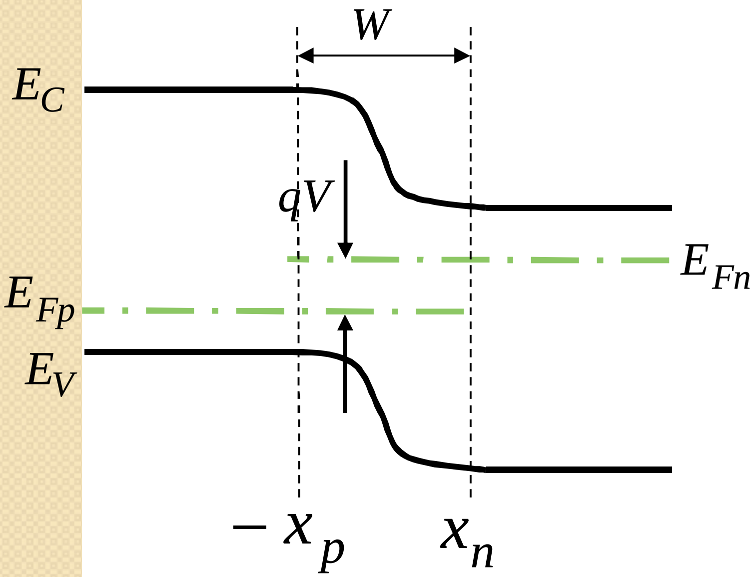
$$V_g = \frac{2}{3} V_T \ln\left[\frac{a^2 k_s \varepsilon_0 V_T}{8 q n_i^3}\right]$$

(2) Ideal Current-Voltage Characteristics:

Assumptions:

- Abrupt depletion layer approximation
- Low-level injection ➔ injected minority carrier density much smaller than the majority carrier density
- No generation-recombination within the space-charge region (SCR)

(a) Depletion layer:



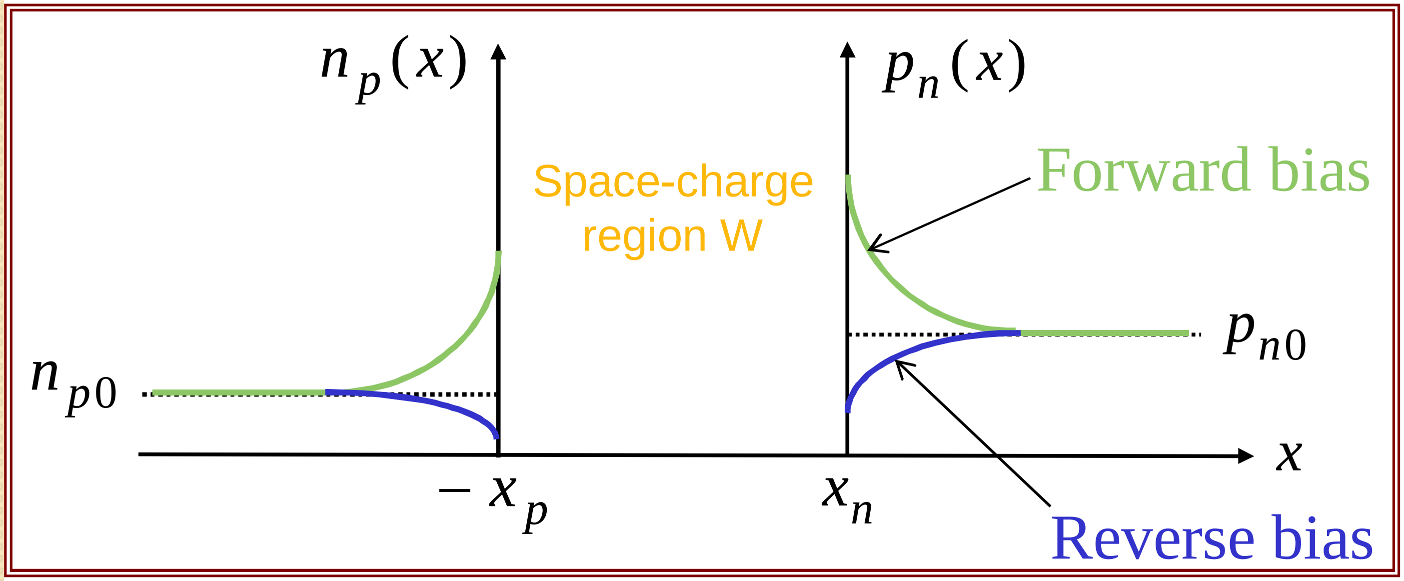
$$np = n_i^2 \exp(V/V_T)$$
$$n_p(-x_p) = n_{p0} \exp(V/V_T)$$
$$p_n(x_n) = p_{n0} \exp(V/V_T)$$

(b) Quasi-neutral regions:

- Using minority carrier continuity equations, one arrives at the following expressions for the excess hole and electron densities in the quasi-neutral regions:

$$\Delta p_n(x) = p_{n0}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$

$$\Delta n_p(x) = n_{p0}(e^{V/V_T} - 1)e^{(x+x_p)/L_n}$$

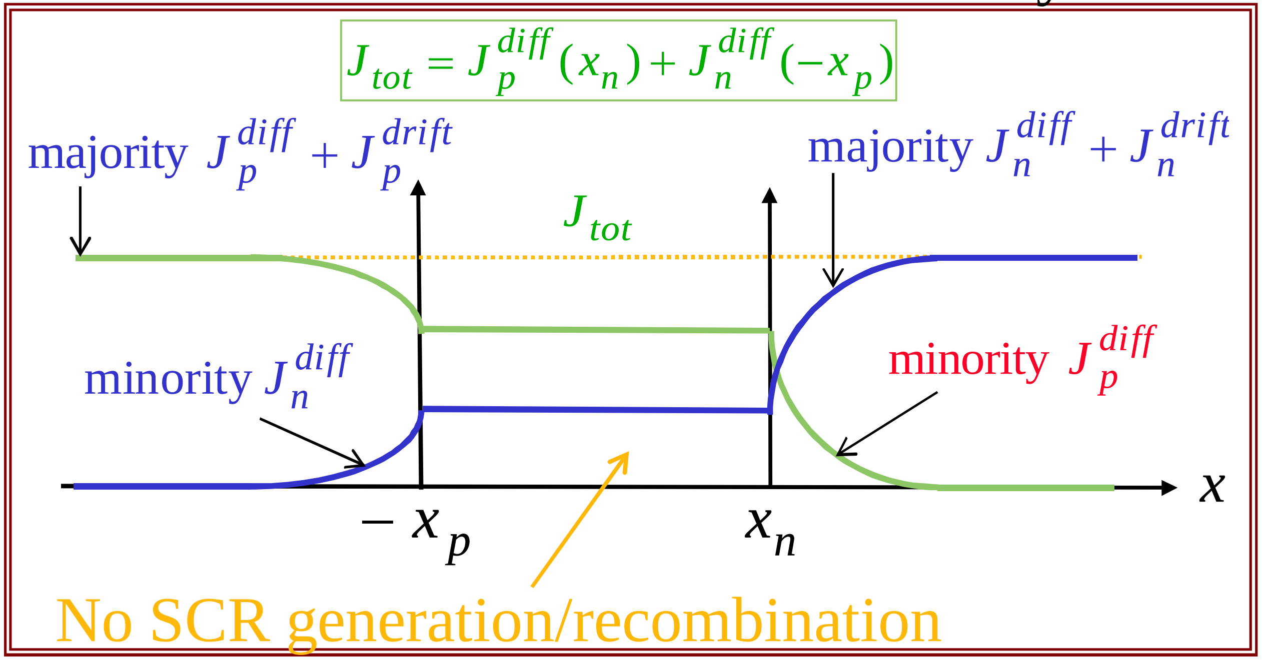


- Corresponding minority-carriers **diffusion** current densities are:

$$J_p^{diff}(x) = \frac{qD_p p_{n0}}{L_p}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$

$$J_n^{diff}(x) = \frac{qD_n n_{p0}}{L_n}(e^{V/V_T} - 1)e^{(x+x_p)/L_n}$$

Shockley model



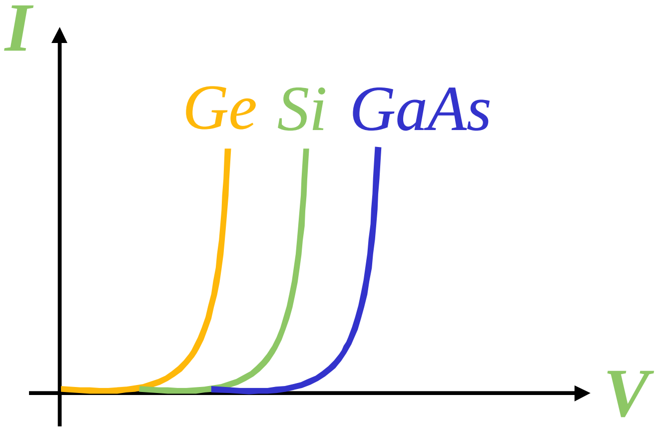
$$\boxed{J_{tot} = J_p^{diff}(x_n) + J_n^{diff}(-x_p)}$$

No SCR generation/recombination

(c) Total current density:

- Total current equals the sum of the minority carrier diffusion currents defined at the edges of the SCR:

$$I_{tot} = I_p^{diff}(x_n) + I_n^{diff}(-x_p)$$
$$= qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right)\left(e^{V/V_T} - 1\right)$$

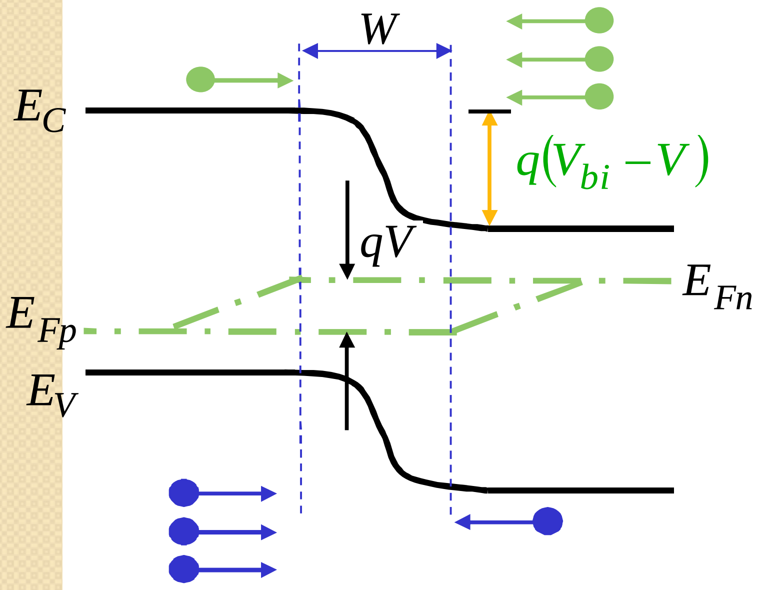


- Reverse saturation current I_S:

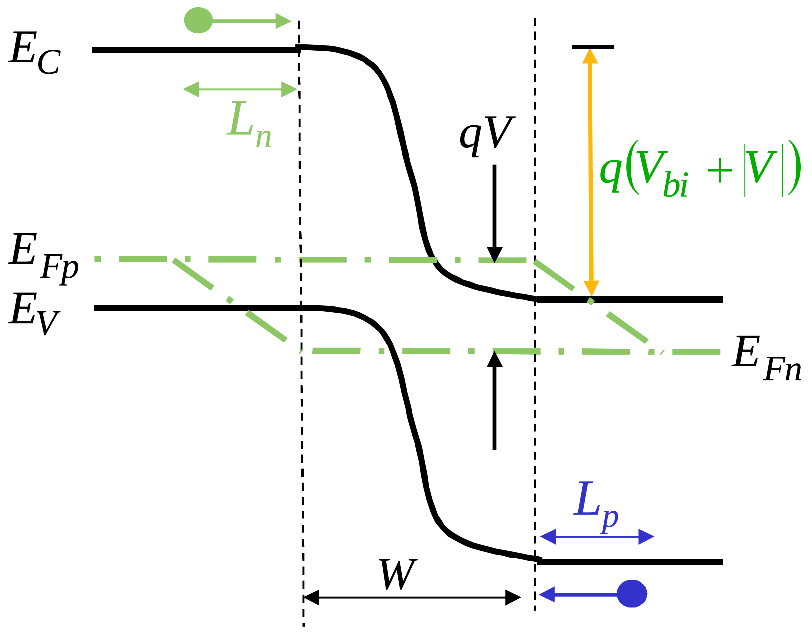
$$I_s = qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right) = qAn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

(d) Origin of the current flow:

Forward bias:



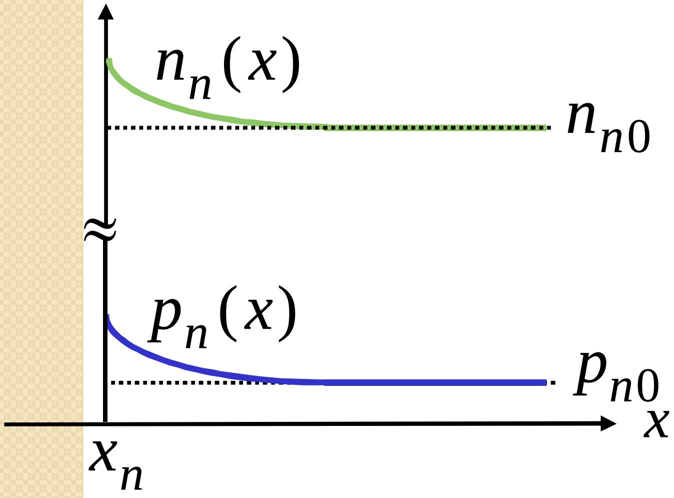
Reverse bias:



Reverse saturation current is due to minority carriers being collected over a distance on the order of the diffusion length.

(e) Majority carriers current:

- Consider a forward-biased diode under low-level injection conditions:



Quasi-neutrality requires:

$$\Delta n_n(x) \approx \Delta p_n(x)$$

This leads to:

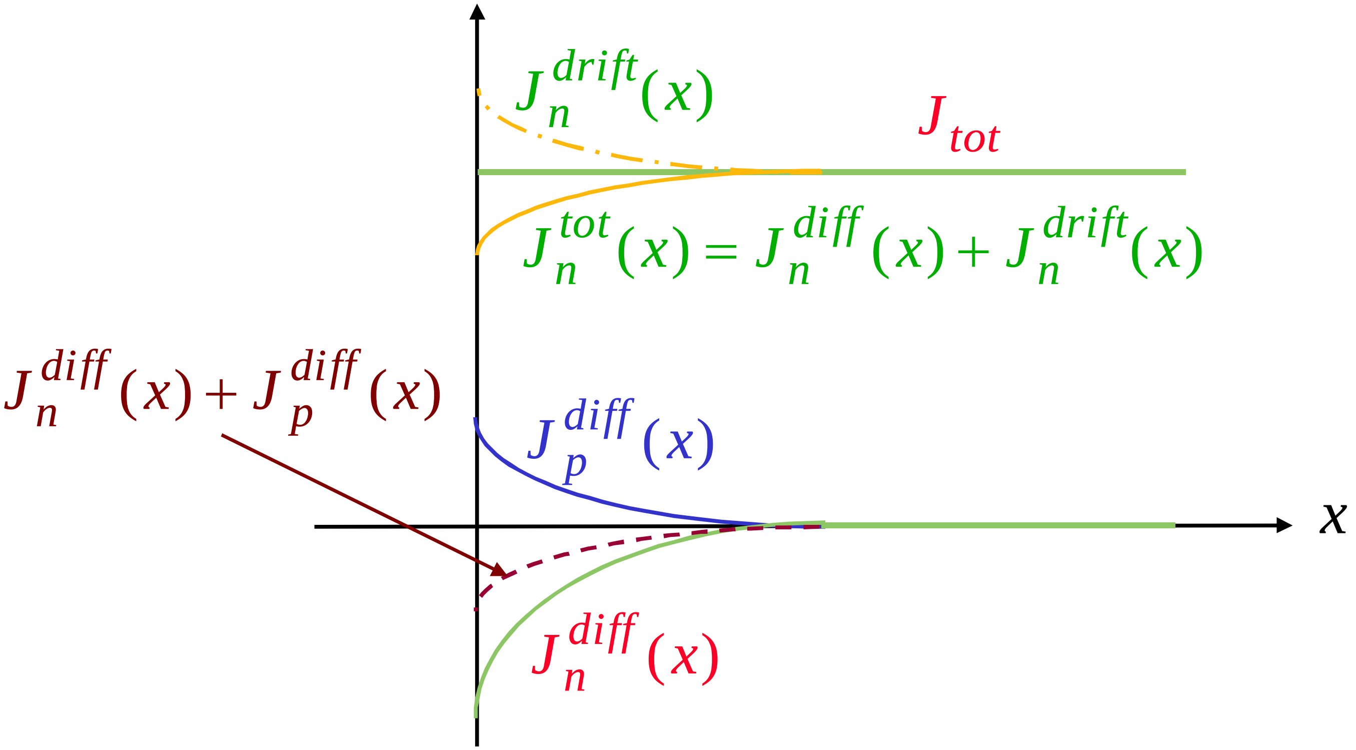
$$J_n^{diff}(x) = -\frac{D_n}{D_p} J_p^{diff}(x)$$

- Total hole current in the quasi-neutral regions:

$$J_p^{tot}(x) = J_p^{diff}(x) + J_p^{drift}(x) \approx J_p^{diff}(x)$$

- Electron drift current in the quasi-neutral region:

$$J_n^{diff}(x) = J_{tot} + \left(\frac{D_n}{D_p} - 1\right) J_p^{diff}(x), \quad E(x) \approx \frac{1}{qn(x)\mu_n} J_n^{diff}(x)$$



(f) Limitations of the Shockley model:

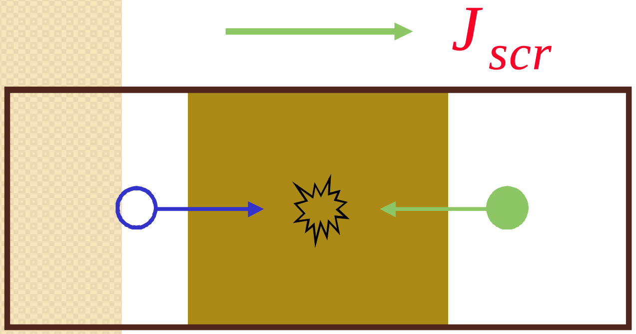
- The simplified Shockley model accurately describes *IV*-characteristics of Ge diodes at low current densities.
- For Si and Ge diodes, one needs to take into account several important non-ideal effects, such as:

➔ Generation and recombination of carriers within the depletion region.

➔ Series resistance effects due to voltage drop in the quasi-neutral regions.

➔ Junction breakdown at large reverse biases due to tunneling and impact ionization effects.

(3) Generation and Recombination Currents



- Continuity equation for holes:

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p$$

- Steady-state and no light generation process: $\partial p / \partial t = 0, G_p = 0$

- Space-charge region recombination current:

$$\int_{-x_p}^{x_n} dJ_p(x) = J_p(x_n) - J_p(-x_p) = -q \int_{-x_p}^{x_n} R_p dx$$

$$J_{scr} = q \int_{-x_p}^{x_n} R_p dx$$

Reverse-bias conditions:

- Concentrations *n* and *p* are negligible in the depletion region:

$$R \approx \frac{-n_i^2}{\tau_p n_1 + \tau_n p_1} = -\frac{n_i}{\tau_g}, \quad \tau_g = \tau_p \exp\left(\frac{E_t - E_i}{k_B T}\right) + \tau_n \exp\left(\frac{E_i - E_t}{k_B T}\right)$$

Generation lifetime

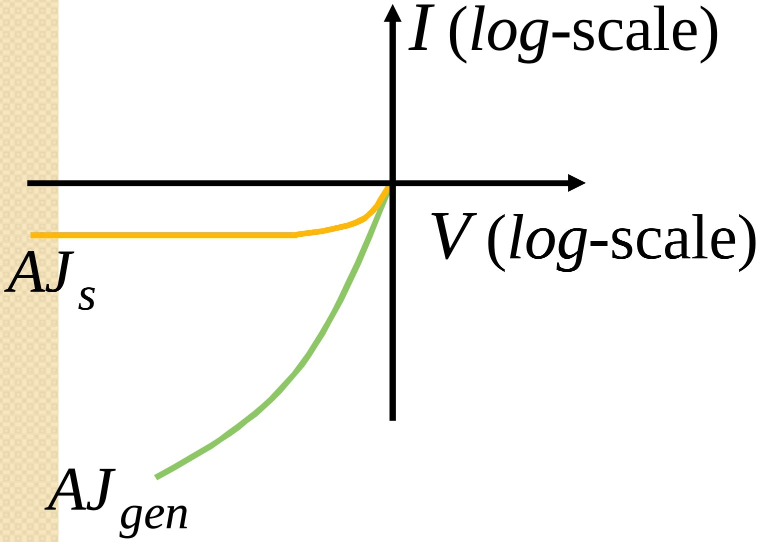
- Space-charge region current is actually generation current:

$$J_{scr} = -J_{gen} = -\frac{q n_i W}{\tau_g} \rightarrow J_{gen} = \frac{q n_i W}{\tau_g} \propto \sqrt{V_{bi} - V}$$

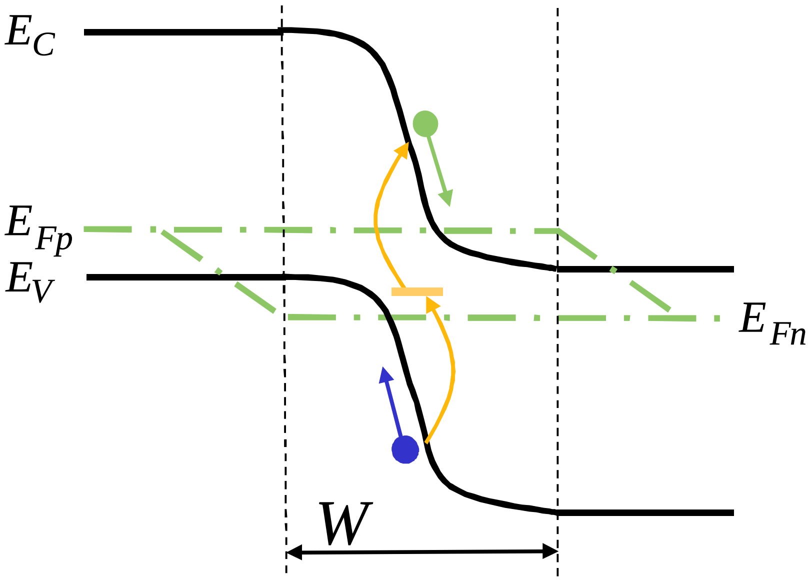
- Total reverse-saturation current:

$$J = J_s\left(e^{V/V_T} - 1\right) + J_{scr} \xrightarrow[|V|>V_T]{} -\left(J_s + J_{gen}\right)$$

- Generation current dominates when n_i is small, which is always the case for *Si* and *GaAs* diodes.



IV-characteristics
under reverse bias conditions



Generated carriers are swept away from the depletion region.

Forward-bias conditions:

- Concentrations *n* and *p* are large in the depletion region:

$$np = n_i^2 e^{V/V_T} \rightarrow R = \frac{n_i^2\left(e^{V/V_T}-1\right)}{\tau_p\left(n+n_1\right)+\tau_n\left(p+p_1\right)}$$

- Condition for maximum recombination rate:

$$n = p = n_i e^{V/2V_T}$$

Recombination lifetime

$$R_{\max} \approx \frac{n_i^2 e^{V/V_T}}{n\tau_p + p\tau_n} = \frac{n_i}{\tau_{rec}} e^{V/2V_T}, \quad \tau_{rec} = \tau_p + \tau_n$$

- Estimate of the recombination current:

$$J_{scr}^{\max} = \frac{qn_iW}{\tau_{rec}} e^{V/2V_T}$$

- Exact expression for the recombination current:

$$J_{scr} = \frac{qn_i\Phi}{\tau_{rec}} e^{V/2V_T}, \quad \Phi = \sqrt{\frac{\pi}{2}} V_T \frac{1}{E_{np}}, \quad E_{np} = \sqrt{\frac{qN_D(2V_{bin} - V)}{k_s\varepsilon_0}}$$

- Corrections to the model:

$$J_{scr} = \frac{qn_i\Phi}{\tau_{rec}} e^{V/m_r V_T}$$

- Total forward current:

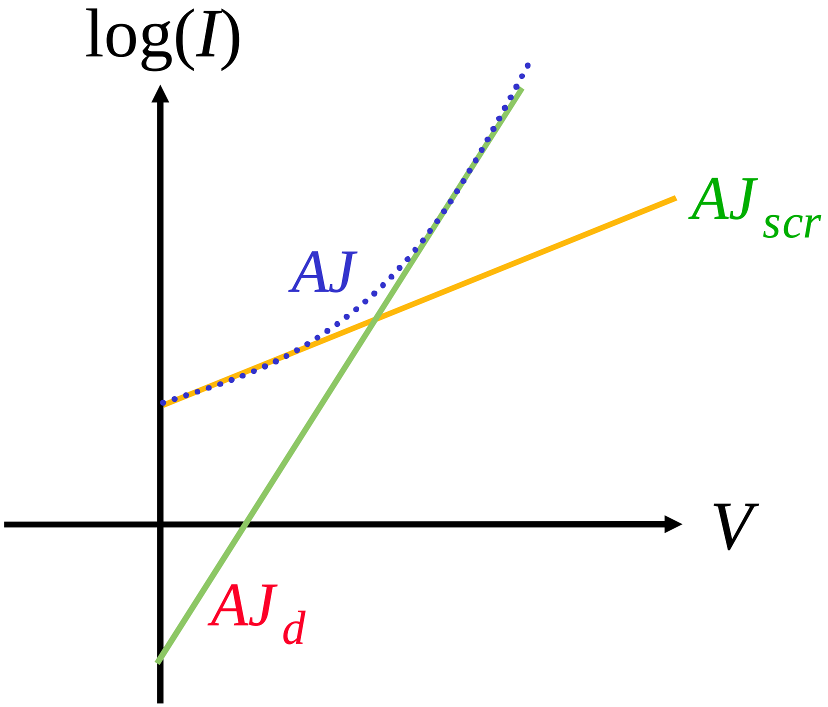
$$J = J_s\left(e^{V/V_T} - 1\right) + \frac{qn_i\Phi}{\tau_{rec}} e^{V/m_r V_T} = J_{s,eff}\left(e^{V/\eta V_T} - 1\right)$$

η → ideality factor. Deviations of η from unity represent an important measure for the recombination current.

- Importance of recombination effects:

  Low voltages, small n_i → recombination current dominates

  Large voltages → diffusion current dominates



(4) Breakdown Mechanisms

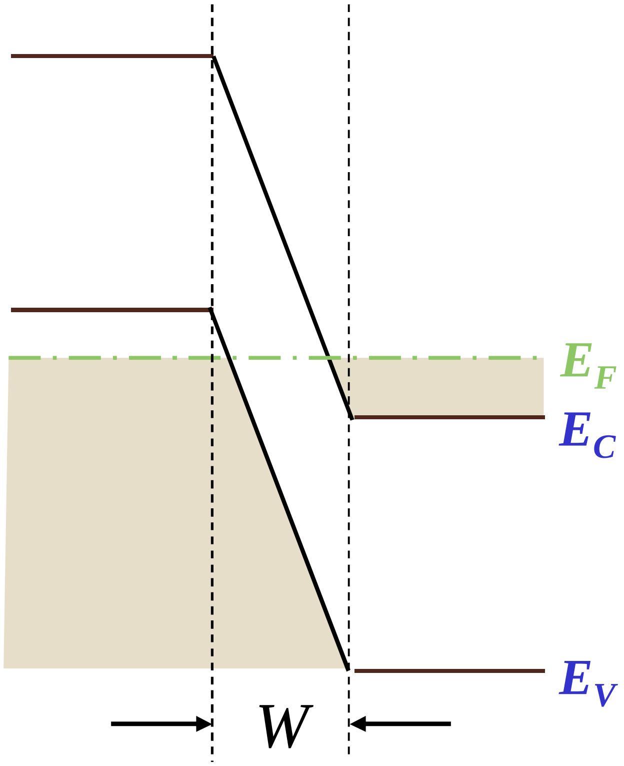
- Junction breakdown can be due to:
  - tunneling breakdown
  - avalanche breakdown

- One can determine which mechanism is responsible for the breakdown based on the value of the breakdown voltage V_{BD} :
  - $V_{BD} < 4E_g/q$ → tunneling breakdown
  - $V_{BD} > 6E_g/q$ → avalanche breakdown
  - $4E_g/q < V_{BD} < 6E_g/q$ → both tunneling and avalanche mechanisms are responsible

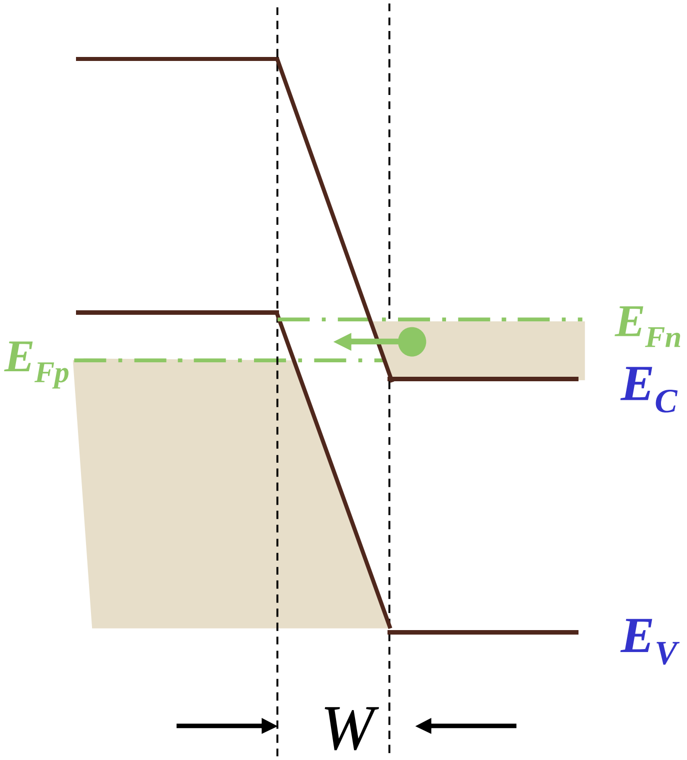
Tunneling breakdown:

- Tunneling breakdown occurs in heavily-doped *pn*-junctions in which the depletion region width *W* is about 10 nm.

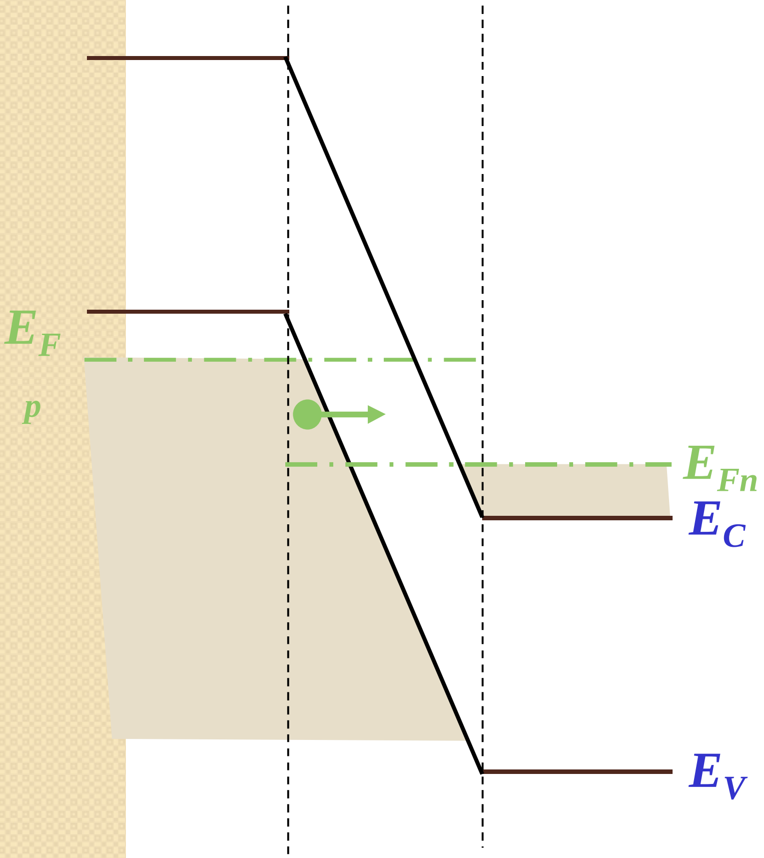
Zero-bias band diagram:



Forward-bias band diagram:



Reverse-bias band diagram:



- Tunneling current (obtained by using WKB approximation):

$$I_t = \frac{\sqrt{2m^*}\, q^3 F_{cr} V A}{4\pi^2 \hbar^2 E_g^{1/2}} \exp\left(- \frac{4\sqrt{2m^*}\, E_g^{3/2}}{3\hbar q F_{cr}} \right)$$

F_{cr} ➔ average electric field in the junction

- The critical voltage for tunneling breakdown, V_{BR}, is estimated from:

$$I_t(V_{BR}) \propto 10 I_S$$

- With $T\uparrow$, $E_g\downarrow$ and $I_t\uparrow$.

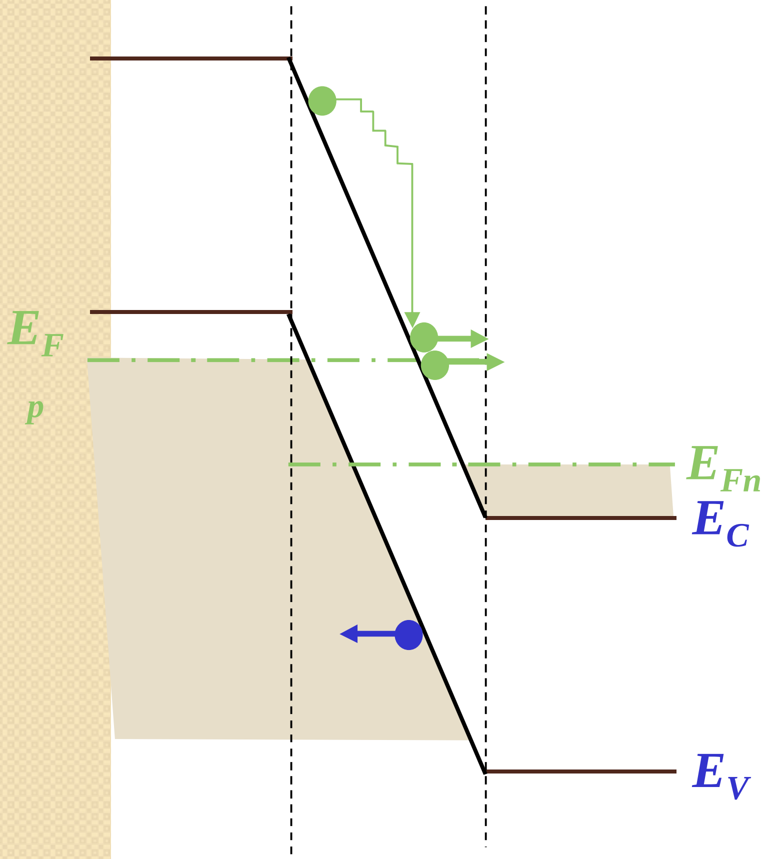
Avalanche breakdown:

- Most important mechanism in junction breakdown, i.e. it imposes an upper limit on the reverse bias for most diodes.
- Impact ionization is characterized by ionization rates α_n and α_p, defined as probabilities for impact ionization per unit length, i.e. how many electron-hole pairs have been generated per particle per unit length:

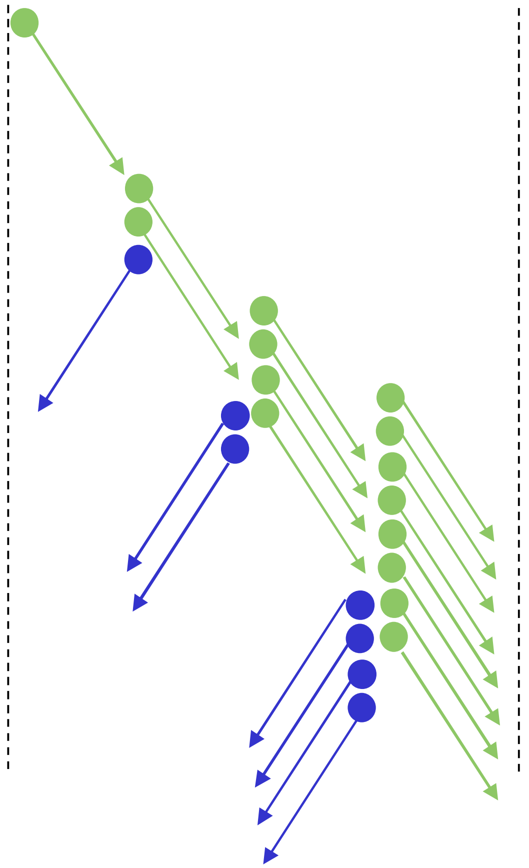
$$\alpha_i \propto \exp\left(-\frac{E_i}{q\lambda F_{cr}}\right)$$

- E_i ➔ critical energy for impact ionization to occur
- F_{cr} ➔ critical electric field
- λ ➔ mean-free path for carriers

Avalanche mechanism:

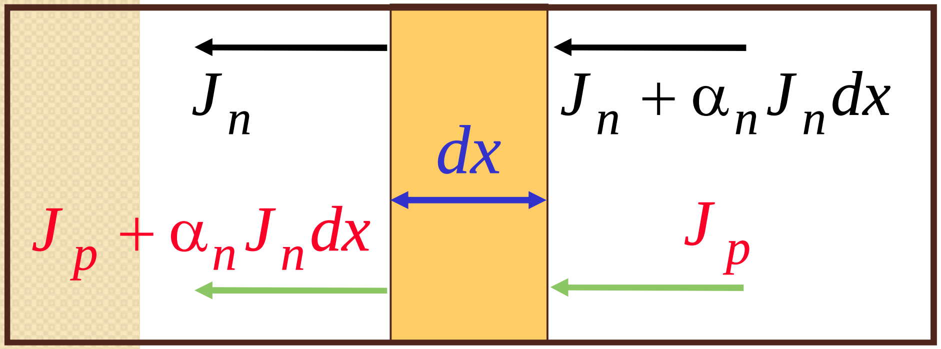


Generation of the excess electron-hole pairs is due to impact ionization.



Expanded view of the depletion region

- Description of the avalanche process:



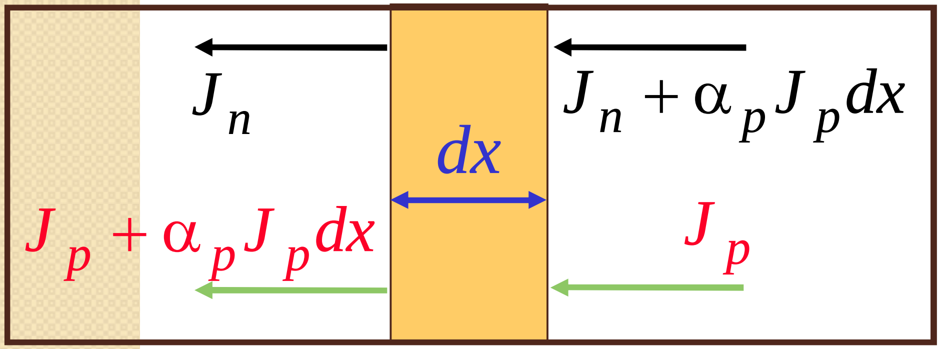
Impact ionization initiated by electrons.

$$\frac{dJ_n}{dx} > 0, \frac{dJ_p}{dx} < 0$$

$$\frac{dJ_n}{dx} = -\frac{dJ_p}{dx}$$

$$\Downarrow$$

$$J = J_n + J_p = const.$$



Impact ionization initiated by holes.

Multiplication factors for electrons and holes:

$$M_n = \frac{J_n(W)}{J_n(0)}, \quad M_p = \frac{J_p(0)}{J_p(W)}$$

- Breakdown voltage ➔ voltage for which the multiplication rates M_n and M_p become infinite. For this purpose, one needs to express M_n and M_p in terms of α_n and α_p:

$$\begin{cases} \dfrac{dJ_n}{dx} = \alpha_n J_n + \alpha_p J_p \\ \dfrac{dJ_p}{dx} = -\alpha_n J_n - \alpha_p J_p \end{cases} \Rightarrow \begin{cases} 1 - \dfrac{1}{M_n} = \displaystyle\int_0^W \alpha_n e^{-\int_0^x (\alpha_n - \alpha_p) dx'} dx \\ 1 - \dfrac{1}{M_p} = \displaystyle\int_0^W \alpha_p e^{-\int_0^x (\alpha_n - \alpha_p) dx'} dx \end{cases}$$

The breakdown condition does not depend on which type of carrier initiated the process.

- Limiting cases:

(a) $\alpha_n=\alpha_p$ (semiconductor with equal ionization rates):

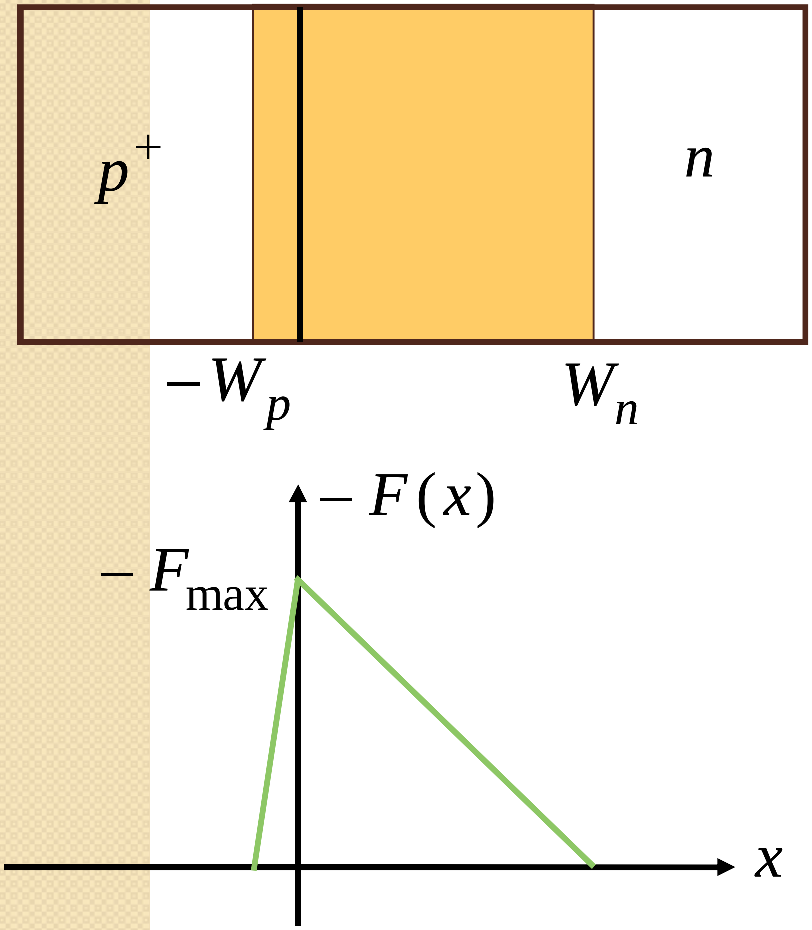
$$\begin{cases} 1-\dfrac{1}{M_n}=\displaystyle\int_0^W \alpha_n dx \rightarrow M_n=\dfrac{1}{1-\displaystyle\int_0^W \alpha_n dx} \\ 1-\dfrac{1}{M_p}=\displaystyle\int_0^W \alpha_p dx \rightarrow M_p=\dfrac{1}{1-\displaystyle\int_0^W \alpha_p dx} \end{cases}$$

(b) $\alpha_n >> \alpha_p$ (impact ionization dominated by one carrier):

$$M_n=e^{\int_0^W \alpha_n dx} \approx 1+\int_0^W \alpha_n dx$$

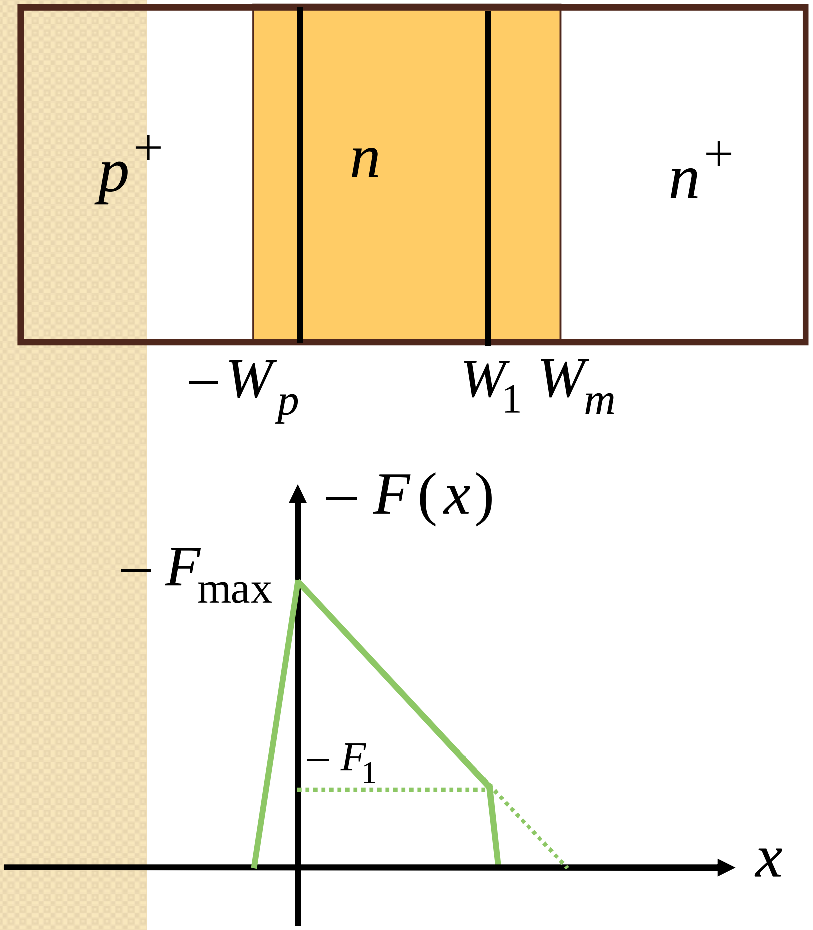
Breakdown voltages:

(a) Step *p*+*n*-junction



- For one sided junction we can make the following approximation: $W = W_n + W_p \approx W_n$
- Voltage drop across the depletion region on the *n*-side: $V_n = \frac{1}{2} F_{\max} W_n \approx V_{BD} \approx \frac{1}{2} F_{\max} W$
- Maximum electric field: $F_{\max} = \frac{qN_D W}{k_s \varepsilon_0} \Rightarrow V_{BD} = \frac{k_s \varepsilon_0}{2qN_D} F_{\max}^2$
- Empirical expression for the breakdown voltage V_{BD}: $V_{BD} \approx 60 \left(\frac{E_g}{1.1}\right)^{3/2} \left(\frac{N_D}{10^{16}}\right)^{-3/4} \left[\frac{kV}{cm}\right]$

(b) Step *p*+-*n*-*n*+ junction



- Extension of the *n*-layer large:

$$V_{BD} = \frac{1}{2} F_{\max} W_m$$

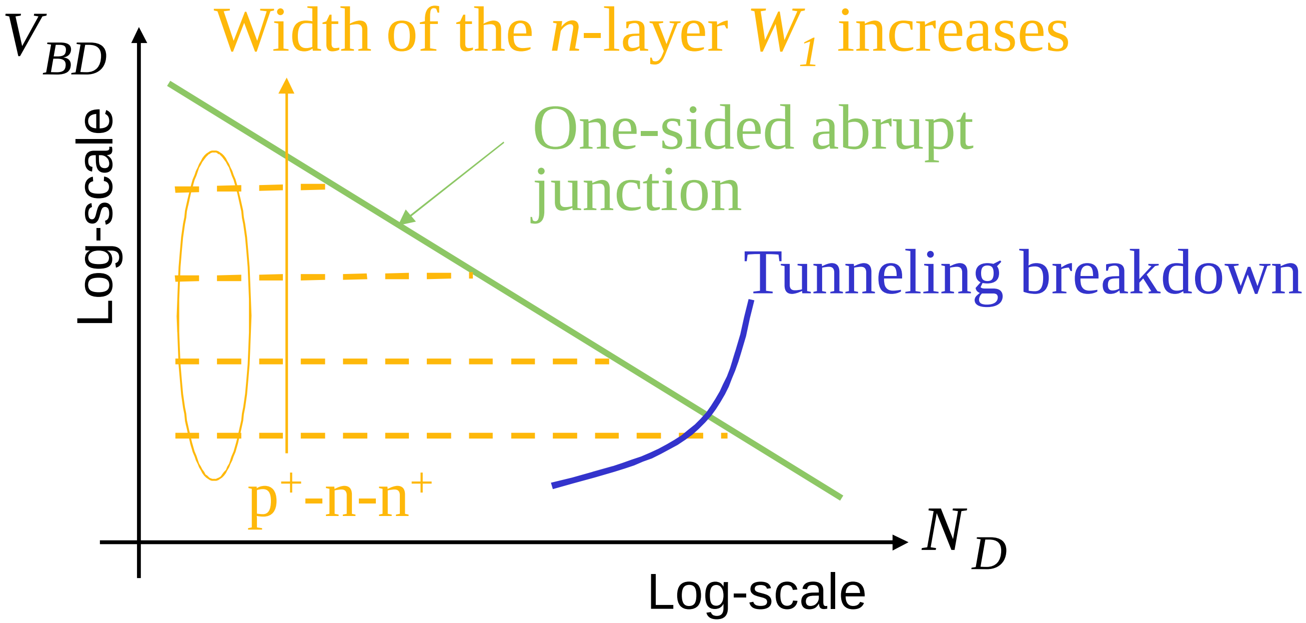
- Extension of the *n*-layer small:

$$V_P = \frac{1}{2} F_{\max} W_m - \frac{1}{2} F_1 \left(W_m - W_1 \right)$$

- Final expression for the punch-through voltage V_P:

$$V_P = V_{BD} \frac{W_1}{W_m} \left(2 - \frac{W_1}{W_m} \right)$$

- Doping-dependence of the breakdown voltage V_{BD}:

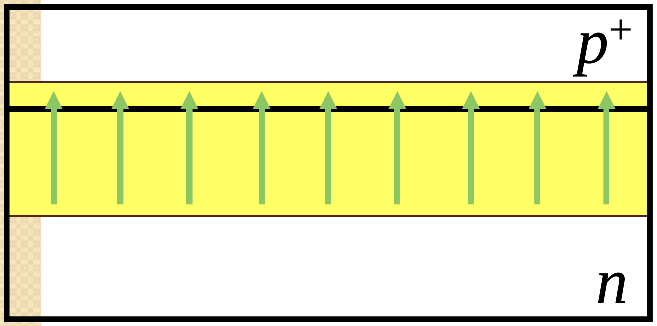


- Temperature dependence:

  As temperature increases, lattice scattering increases which makes impact ionization less probable. As a result of this, the breakdown voltage increases.

(c) Plane vs. planar or cylindrical junction

- Plane junction:

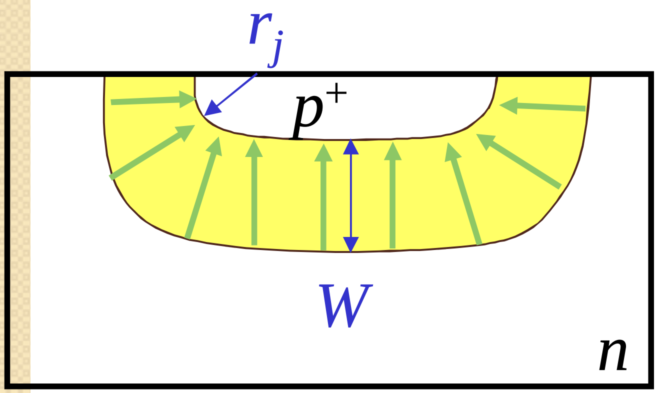


Maximum electric field:

$$F_{\max} = \frac{Q}{k_s \varepsilon_0} = \frac{qN_D W}{k_s \varepsilon_0}$$

Except for surface effects, this is an **ideal junction**.

- Planar junction:



Maximum electric field:

$$F_{\max} = \frac{qN_D W}{k_s \varepsilon_0}\left(1 + \frac{W}{2r_j}\right)$$

The smaller the radius r_j, the larger the electric field **crowding**.

(5) AC-Analysis and Diode Switching

(a) Diffusion capacitance and small-signal equivalent circuit

- This is capacitance related to the change of the minority carriers. It is important (even becomes dominant) under forward bias conditions.
- The diffusion capacitance is obtained from the device impedance, and using the continuity equation for minority carriers:

$$\frac{d\Delta p_n}{dt} = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p}$$

- Applied voltages, currents and solution for Δp_n:

$$V(t) = V_0 + V_1 e^{i\omega t},\ V_1 << V_0$$
$$J(t) = J_0 + J_1 e^{i\omega t},\ J_1 << J_0$$

$$\longrightarrow \quad p_n(x,t) = p_{ns}(x) + p_{n1}(x) e^{i\omega t}$$

- Equation for $p_{n1}(x)$:

$$\frac{d^2 p_{n1}}{dx^2} - \frac{1 + i\omega\tau_p}{D_p \tau_p} p_{n1}(x) = 0 \rightarrow \frac{d^2 p_{n1}}{dx^2} - \frac{p_{n1}(x)}{L_{p'}^2} = 0$$

- Boundary conditions:

$$p_n(\infty, t) = p_{n0} \rightarrow p_{n1}(\infty) = 0$$

$$p_n(0,t) = p_{n0} \exp\left(\frac{V_0 + V_1 e^{i\omega t}}{V_T}\right) \rightarrow p_{n1}(0) = \frac{p_{n0} V_1}{V_T} \exp\left(\frac{V_0}{V_T}\right)$$

- Final expression for $p_{n1}(x)$:

$$p_{n1}(x,t) = \frac{p_{n0} V_1}{V_T} \exp\left(\frac{V_0}{V_T}\right) \exp\left(-\frac{x}{L_{p'}}\right)$$

- Small-signal hole current:

$$I_1 = -AqD_p \left.\frac{dp_{n1}}{dx}\right|_{x=0} = \frac{AqD_p p_{n0} V_1}{L_p V_T} \sqrt{1 + i\omega\tau_p}\, \exp\left(\frac{V_0}{V_T}\right) = YV_1$$

- Low-frequency limit for the admittance *Y*:

$$Y = \frac{AqD_p p_{n0}}{L_p V_T} \exp\left(\frac{V_0}{V_T}\right)\left(1 + \frac{1}{2} i\omega\tau_p\right) = G_d + i\omega C_{dif}$$

$$G_d = \frac{AqD_p p_{n0}}{L_p V_T} \exp\left(\frac{V_0}{V_T}\right) = \frac{I_s e^{V_0/V_T}}{V_T} = \frac{I}{V_T} = \frac{dI}{dV}, \quad I \rightarrow \text{Forward current}$$

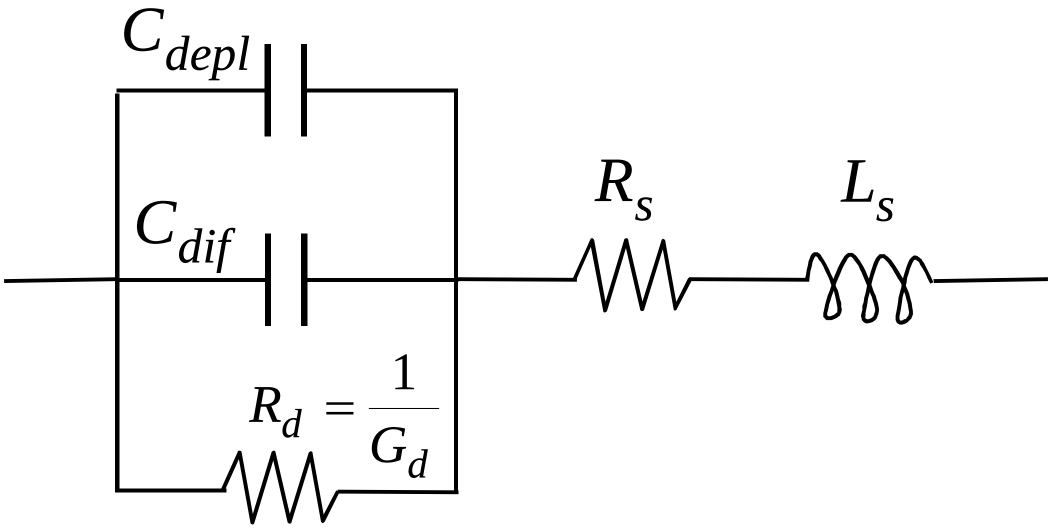
$$C_{dif} = \frac{1}{2} \frac{AqD_p p_{n0}}{L_p V_T} \tau_p \exp\left(\frac{V_0}{V_T}\right) = \frac{1}{2} \frac{I}{V_T} \tau_p$$

- *RC*-constant:

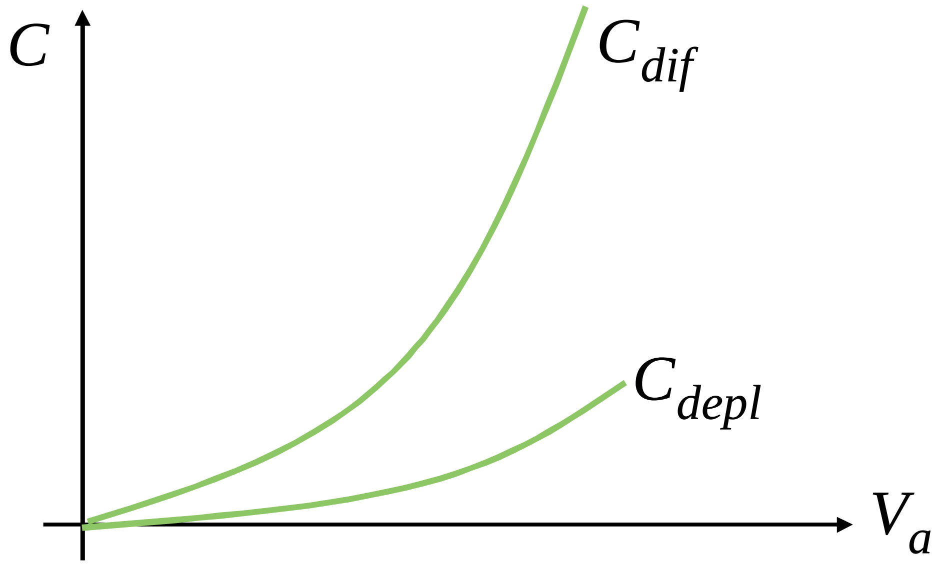
$$R_d C_{dif} = \frac{\tau_p}{2}$$

The characteristic time constant is on
the order of the minority carriers lifetime.

- Equivalent circuit model for forward bias:



- Bias dependence:



(b) Diode switching

- For switching applications, the transition from forward bias to reverse bias must be nearly abrupt and the transit time short.
- Diode turn-on and turn-off characteristics can be obtained from the solution of the continuity equations:

$$\frac{d\Delta p_n}{dt} = -\frac{1}{q}\nabla\cdot J_p - R_p \xrightarrow{1D} -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p_n}{\tau_p}$$

$$\downarrow$$

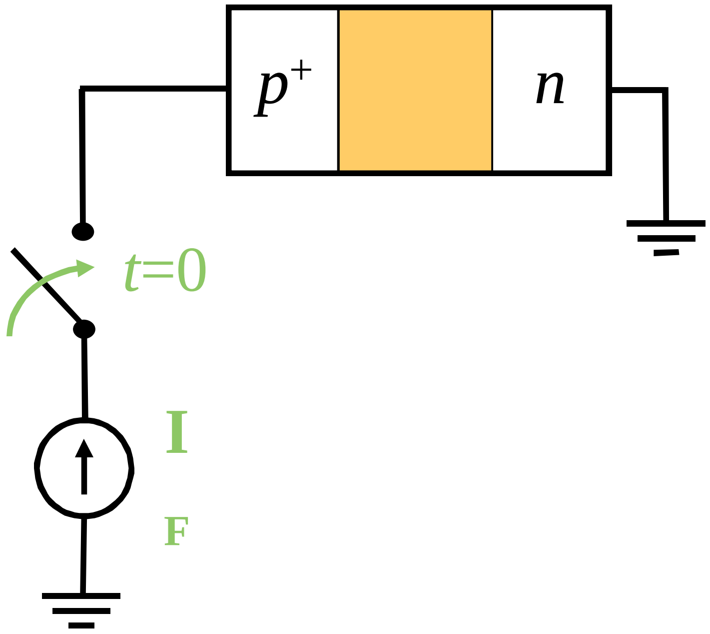
$$\frac{dQ_p}{dt} = I_p(t) - \frac{Q_p}{\tau_p} \rightarrow \boxed{I(t) \approx I_p(t) = \frac{dQ_p}{dt} + \frac{Q_p}{\tau_p}}$$

$Q_p(t)$ = excess hole charge

$\uparrow$

Valid for p^+n diode

Diode turn-on:



- For *t*<0, the switch is open, and the excess hole charge is:

$$Q_p(t<0) = Q_p(0^-) = 0$$

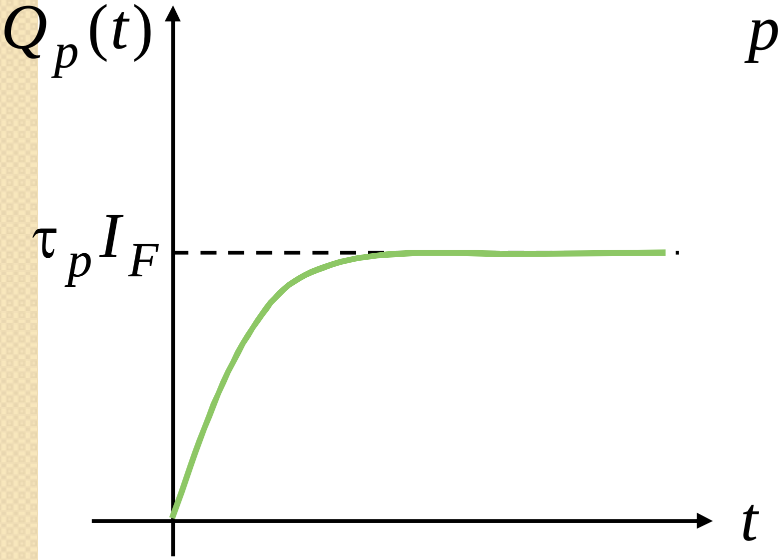
- At *t*=0, the switch closes, and we have the following boundary condition:

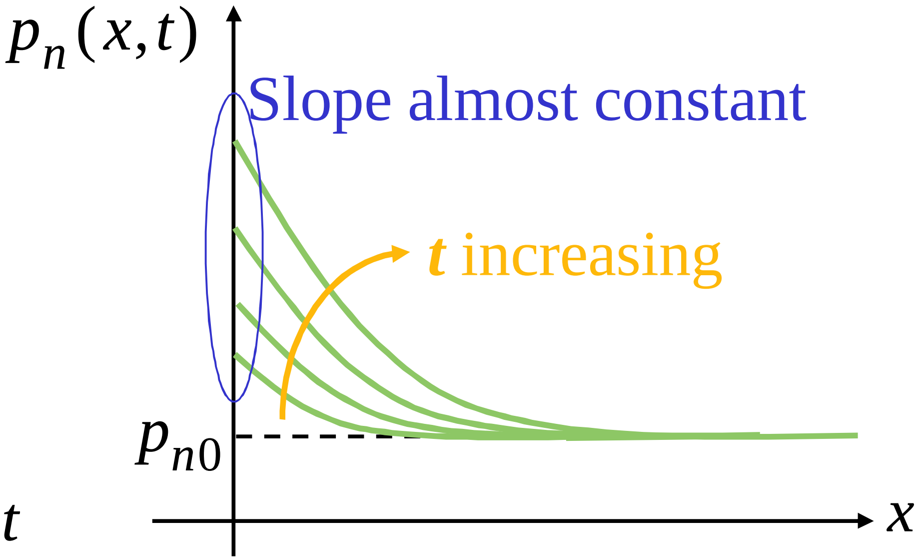
$$Q_p(0^-) = Q_p(0^+) = 0$$

- Final expression for the excess hole charge:

$$Q_p(t) = A + Be^{-t/\tau_p} = \tau_p I_F\left[1 - e^{-t/\tau_p}\right]$$

- Graphical representation:





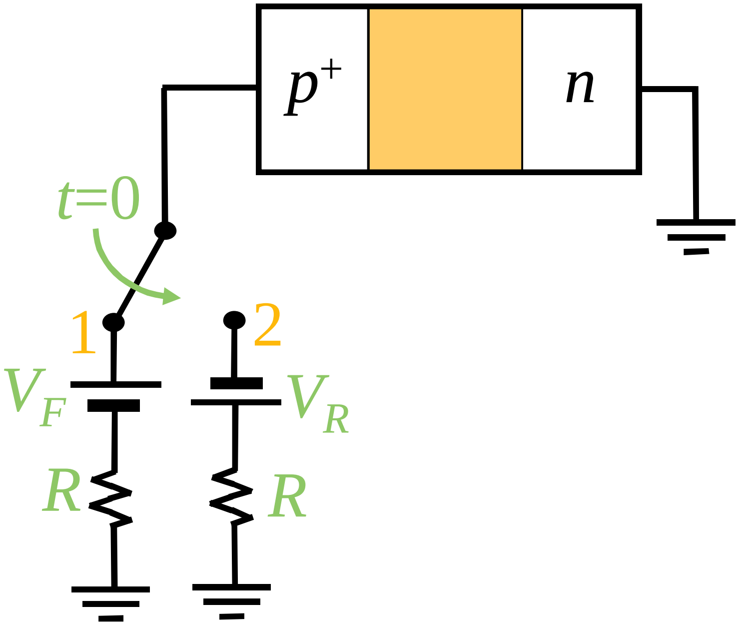
- Steady state value for the bias across the diode:

$$\Delta p_n(x) = p_{n0}\left(e^{V_a/V_T}-1\right)e^{-x/L_p} \rightarrow Q_p = Aqp_{n0}L_p\left(e^{V_a/V_T}-1\right)$$

$$\downarrow$$

$$\boxed{V_a = V_T \ln\left(1+\frac{I_F}{I_S}\right)}$$

Diode turn-off:



- For $t<0$, the switch is in position 1, and a steady-state situation is established:

$$I_F \approx \frac{V_F}{R}$$

- At t=0, the switch is moved to position 2, and up until time $t=t_1$ we have:

$$p_n(0,t) \geq p_{n0} \rightarrow V_a \geq 0$$

- The current through the diode until time t_1 is:

$$I_R \approx -\frac{V_R}{R}$$

- To solve exactly this problem and find diode switching time, is a rather difficult task. To simplify the problem, we make the crucial assumption that I_R remains constant even beyond t_1.
- The differential equation to be solved and the initial condition are, thus, of the form:

$$-I_R = \frac{dQ_p}{dt} + \frac{Q_p}{\tau_p}, \quad Q_p(0^-) = Q_p(0^+) = \tau_p I_F$$

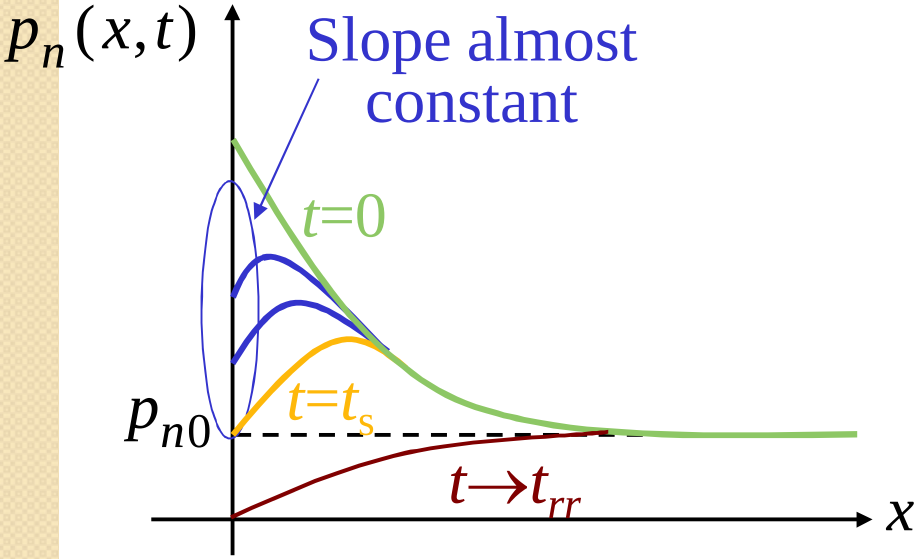
- This gives the following final solution:

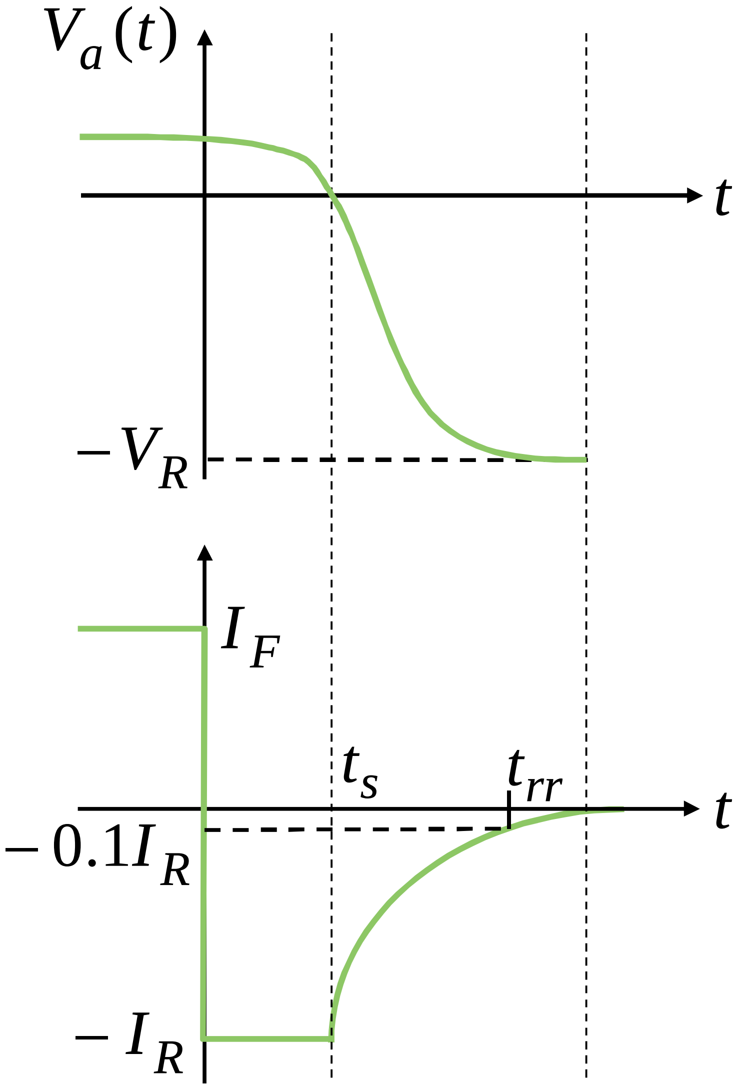
$$Q_p(t) = -\tau_p I_R + \tau_p (I_F + I_R) e^{-t/\tau_p}$$

- Diode switching time:

$$Q_p(t_{rr}) = 0 \rightarrow \boxed{t_{rr} = \tau_p \ln\left(1 + \frac{I_F}{I_R}\right)}$$

- Graphical representation:





$t_s \to$ switching time

$t_{rr} \to$ reverse recovery time