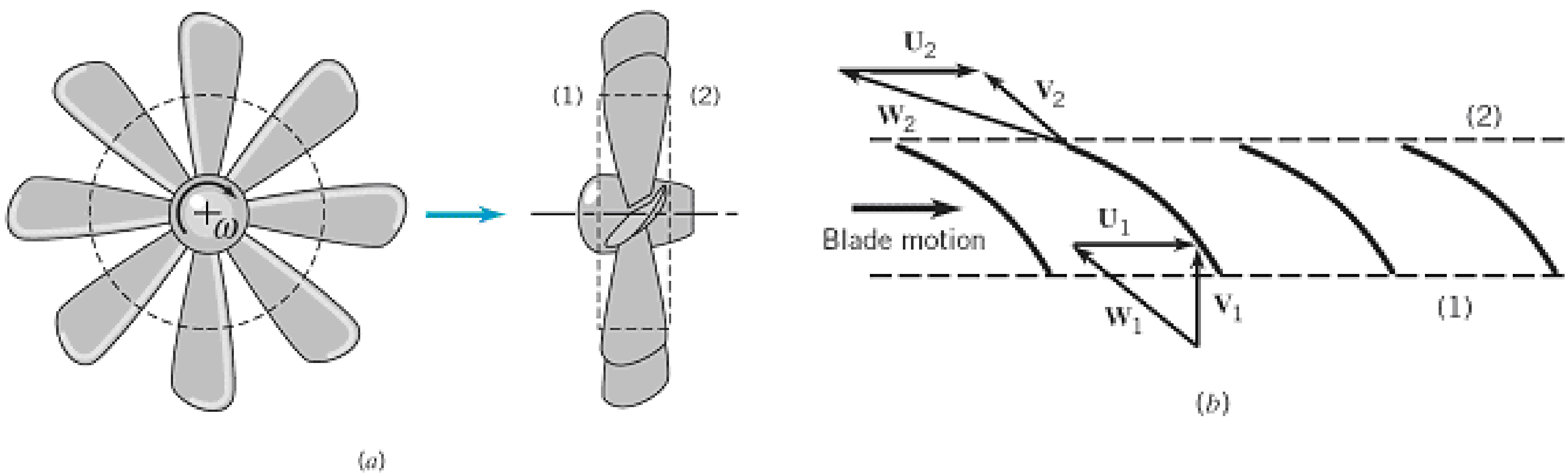
DRONACHARYA
College of Engineering

Fluid Mechanics

Hydraulic Turbines

Windmill

❖ Consider the windmill. Rather than the rotor being driven by a motor, it is rotated in the opposite direction by the wind blowing through the rotor.



Idealized flow through a windmill: (a) windmill blade geometry; (b) absolute velocity, V; relative velocity, W, and blade velocity, U at the inlet and exit of the windmill blade section.

Basic Angular Momentum Considerations

Angular Momentum Considerations 1/6

- ❖ Work transferred from a fluid flowing through a turbine occurs by interaction between moving rotor blades and the fluid.
  - ⇨ Turbine: The energy transfer is from the fluid to the rotor. (The torque exerted by the shaft on the rotor is opposite to the direction of rotation)

Angular Momentum Considerations 2/6

- All of the turbomachines involve the rotation of an impeller or a rotor about a central axis, it is appropriate to discuss their performance in terms of **torque and angular momentum**.

Angular Momentum Considerations 3/6

- In a turbomachine a series of particles (a continuum) passes through the rotor.
- For steady flow, the moment of momentum equation applied to a control volume

$$\sum(\vec{r}\times\vec{F}) = \int_{CS}(\vec{r}\times\vec{V})\rho\vec{V}\cdot\vec{n}\,dA$$

Sum of the external torques

Net rate of flow of moment-of-momentum (angular momentum) through the control volume

Angular Momentum Considerations 4/6

Applied to the one-dimensional simplification of flow through a turbomachine rotor, the axial component

$$T_{shaft} = -\dot{m}_1(r_1 V_{\theta 1}) + \dot{m}_2(r_2 V_{\theta 2}) \qquad (2)$$

Shaft work applied to the contents of the control volume

"+" : in the same direction as rotation

"-" : in the opposite direction as rotation

Euler turbomachine equation

Euler turbomachine equation : the shaft torque is directly proportional to the mass flowrate. The torque also depends on the tangential component of the absolute velocity, V_θ.

Angular Momentum Considerations 5/6

(2) ⟹

$$\dot{W}_{shaft} = T_{shaft}\,\omega \qquad (3)$$

$$\dot{W}_{shaft} = -\dot{m}_1 (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \qquad (4)$$

$$w_{shaft} = \frac{\dot{W}_{shaft}}{\dot{m}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) \qquad (5)$$

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

(3) (4) (5) :**The basic governing equations for pumps or turbines whether the machines are radial-, mixed, or axial-flow devices and for compressible and incompressible flows.**

Turbines

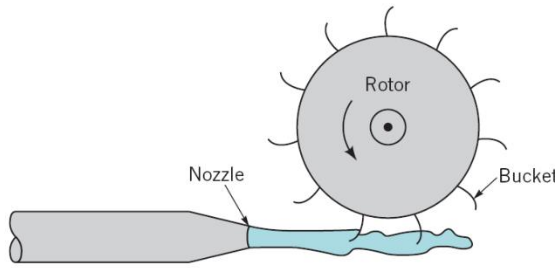
Turbines 1/6

- ❖ Turbines are devices that extract energy from a flowing fluid.
- ❖ The geometry of turbines is such that the fluid exerts a torque on the rotor in the direction of its rotation.
- ❖ The shaft power generated is available to derive generators or other devices.
- ❖ The two basic types of hydraulic turbines are **impulse and reaction turbines.**

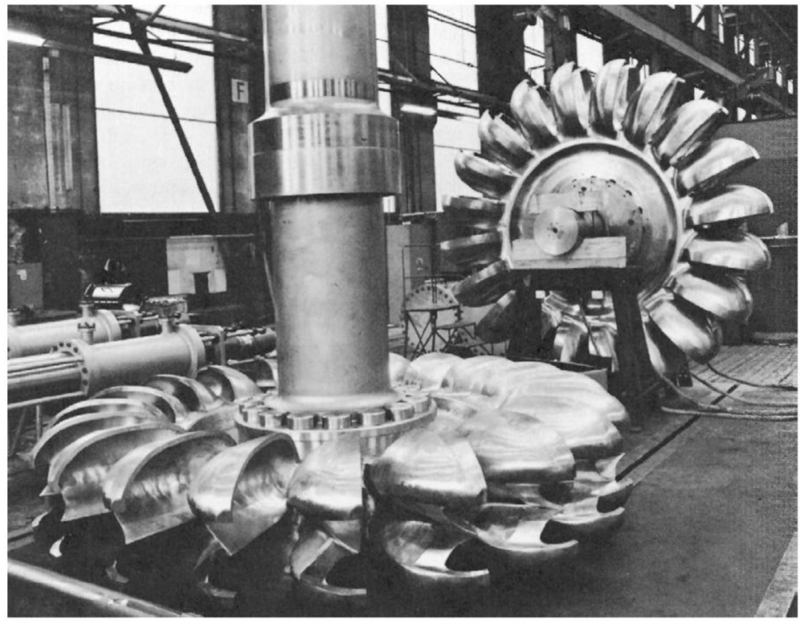
Turbines 2/6

❖ For hydraulic impulse turbines, the pressure drop across the rotor is zero; all of the pressure drop across the turbine stages occurs in the nozzle row.

❖ The Pelton wheel is a classical example of an impulse turbines.

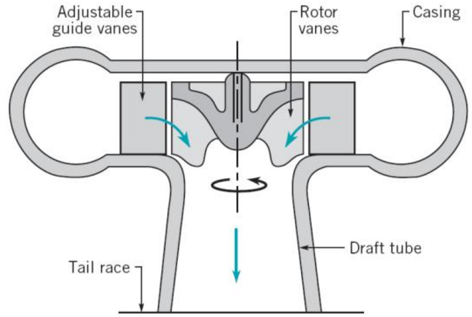


(a)

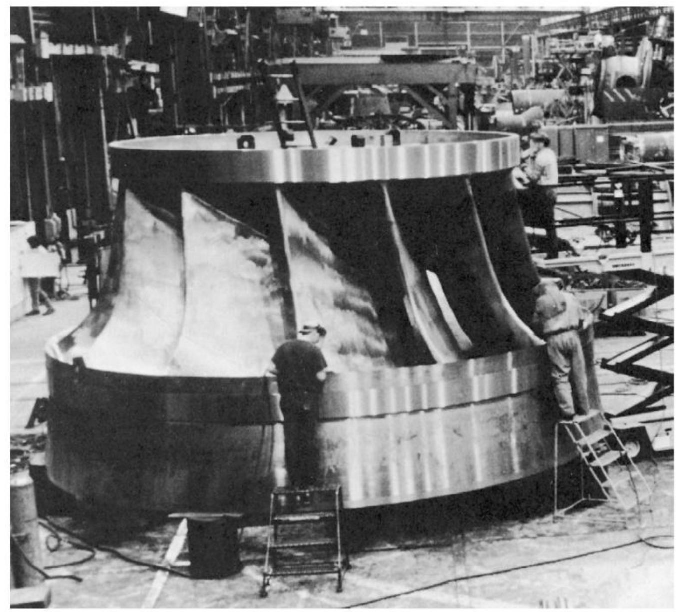

(b)

Turbines 3/6

❖ Diagram shows a reaction turbine.



(a)



(b)

Turbines 4/6

- For impulse turbines
  - The total head of the incoming fluid is converted into a large velocity head at the exit of the supply nozzle.
  - Both the pressure drop across the bucket (blade) and the change in relative speed of the fluid across the bucket are negligible.
  - The space surrounding the rotor is not completely filled with fluid.
  - The individual jets of fluid striking the buckets that generates the torque.

Turbines 5/6

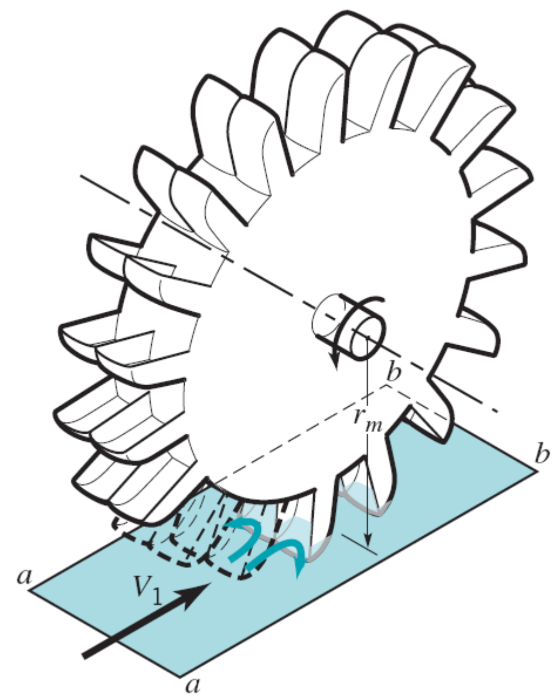
- For reaction turbines
  - There is both a pressure drop and a fluid relative speed change across the rotor.
  - Guide vanes act as nozzle to accelerate the flow and turn it in the appropriate direction as the fluid enters the rotor.
  - Part of the pressure drop occurs across the guide vanes and part occurs across the rotor,

Turbines 6/6

- **Summary**
  - **Impulse turbines: High-head, low flowrate devices.**
  - **Reaction turbines: Low-head, high-flowrate devices.**

Impulse Turbines 1/6

- The easiest type of impulse turbines design is the Pelton wheel.
- Lester Pelton (1829~1908), an American mining engineer during the California gold-mining days, is responsible for many of still-used features of this type of turbine.

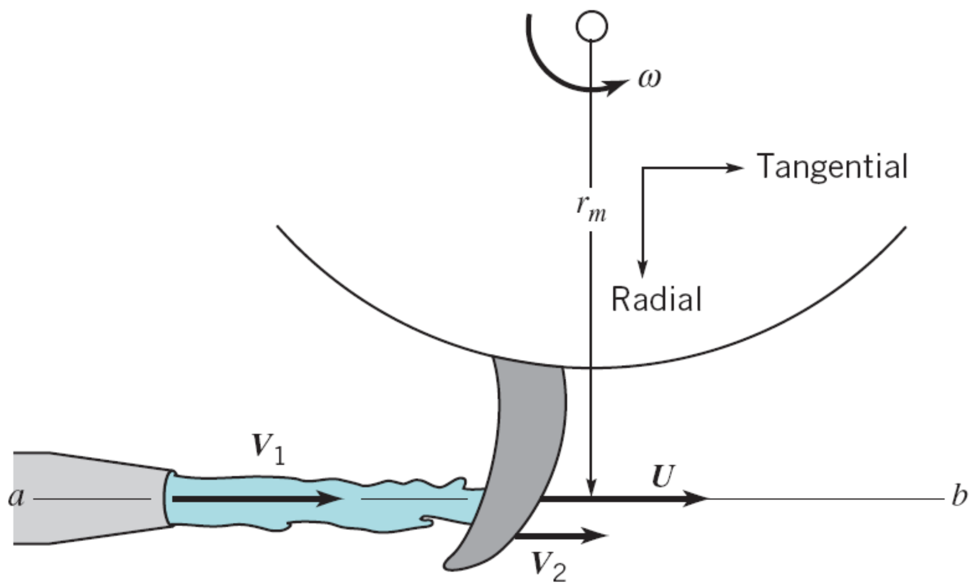


Impulse Turbines 2/6

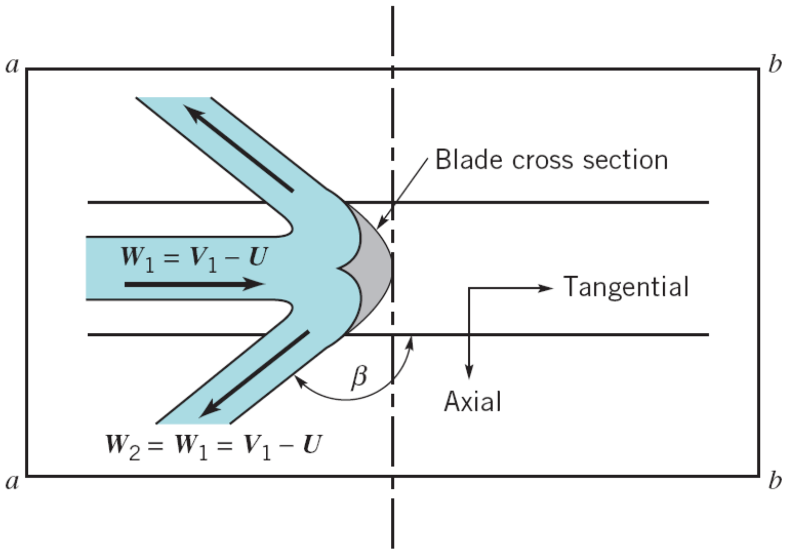
- A high-speed jet of water strikes the Pelton wheel buckets and is deflected.
- The water enters and leaves the control volume surrounding the wheel as free jet.
- A person riding on the bucket would note that the speed of the water doest not change as it slides across the buckets. That is, the magnitude of the relative velocity does not change, but its direction does.

Impulse Turbines 3/6

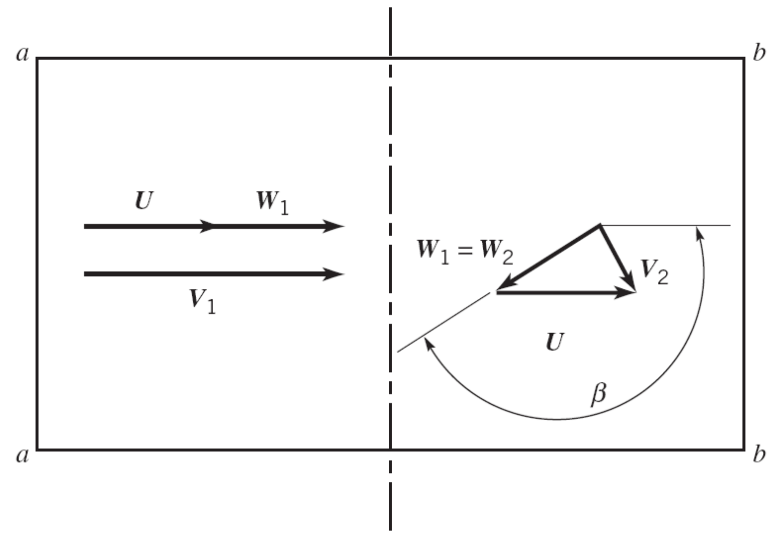
- Ideally, the fluid enters and leaves the control volume with no radial component of velocity.
- The buckets would ideally turn the relative velocity through a 180º turn, but physical constraints dictate that β, the angle of the exit edge of the blade, is less than 180 º



Impulse Turbines 4/6



Flow as viewed by an observer riding on the Pelton wheel – relative velocities



Inlet and exit velocity triangles for a Pelton wheel turbine.

$$V_{\theta 1} = V_1 = W_1 + U \quad (48)$$

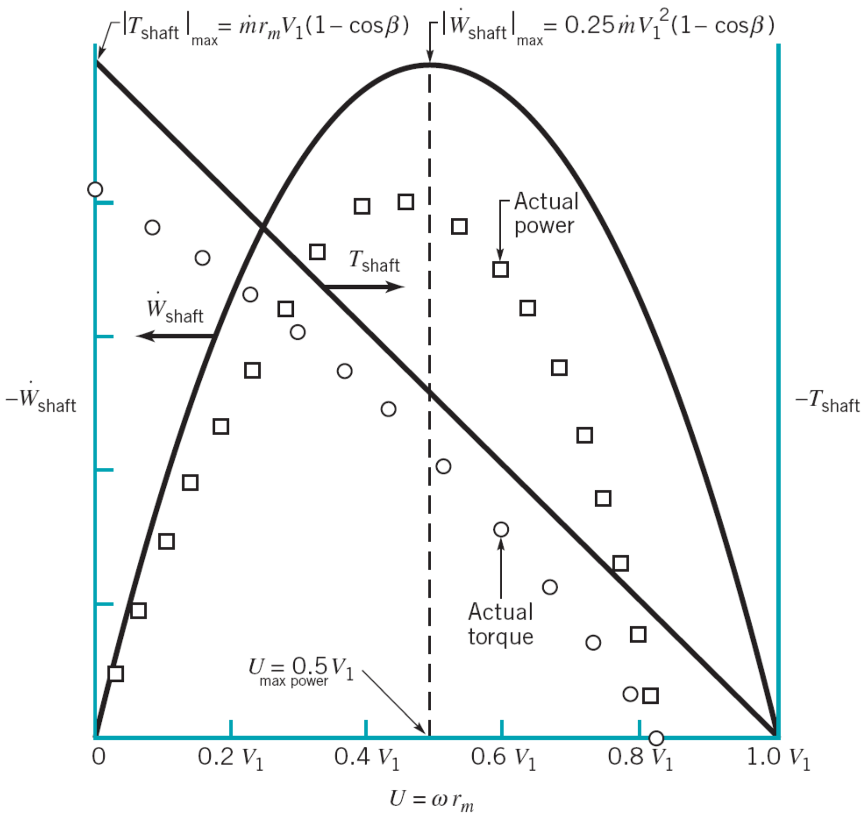
$$V_{\theta 2} = W_2 \cos\beta + U \quad (49)$$

With $W_1 = W_2$ (48)+(49) ⇒

$$V_{\theta 2} - V_{\theta 1} = (U - V_1)(1 - \cos\beta) \quad (50)$$

Impulse Turbines 5/6

(50)+(2)+(4) ⇒ $$T_{shaft} = \dot{m} r_m (U - V_1)(1 - \cos\beta)$$



$$\dot{W}_{shaft} = T_{shaft}\,\omega$$
$$= \dot{m} U (U - V_1)(1 - \cos\beta) \quad (51)$$

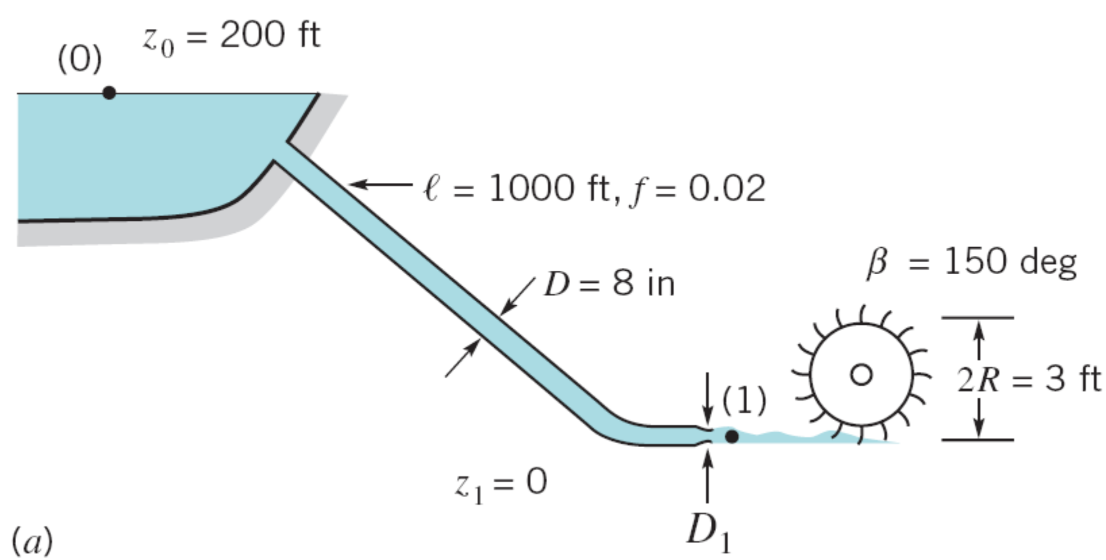
Typical theoretical and experimental power and torque for a Pelton wheel turbine as a function of bucket speed.

Impulse Turbines 6/6

- From above results:
  - The power is a function of β. A typical value of $\beta=165^o$ results in a relatively small reduction in power since $1-\cos165^o=1.966$.
  - Although torque is maximum when the wheel is stopped ($U=0$), there is no power under this condition – to extract power one needs force and motion.
  - The power output is a maximum when $U=V/2$. (52)
  - The maximum speed occurs when $T_{shaft}=0$.

Example 12.6 Pelton Wheel Turbine Characteristics

- Water to drive a Pelton wheel is supplied through a pipe from a lake as indicated in Fig. E12.6a. Determine the nozzle diameter, D_1, that will give the maximum power output. Include the head loss due to friction in the pipe, but neglect minor losses. Also determine this maximum power and the angular velocity of the rotor at this condition.



Example 12.6 Solution[1/3]

(51) $$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$$

The nozzle exit speed, V_1, can be obtained by applying the energy equation between a point on the lake surface (where V_0=p_0=0) and the nozzle outlet (where z_1=p_1=0) to give

$$z_0 = \frac{V_1^2}{2g} + h_L \qquad h_L = f\frac{\ell}{D}\frac{V^2}{2g}$$

$$z_0 = \left[1 + f\frac{\ell}{D}\left(\frac{D_1}{D}\right)^4\right]\frac{V_1^2}{2g} = \frac{113.5}{\sqrt{1 + 152 D_1^4}}$$

$$Q = \pi D_1^2 V_1 / 4$$

Example 12.6 Solution$^{2/3}$

$$\dot{W}_{shaft} = \frac{323UD_1^2}{\sqrt{1+152D_1^4}}\left[U - \frac{113.5}{\sqrt{1+152D_1^4}}\right]$$

The maximum power occurs when $U=V_1/2$

$$\dot{W}_{shaft} = \frac{1.04\times10^6 D_1^2}{(1+152D_1^4)^{3/2}}$$

The maximum power possible occurs when $d\dot{W}_{shaft}/dD_1 = 0$

D_1=0.239ft

$$\dot{W}_{shaft} = \frac{1.04\times10^6 D_1^2}{(1+152D_1^4)^{3/2}} = -3.25\times10^4 \,ft\cdot lb/s = -59.0hp$$

Example 12.6 Solution[3/3]

The rotor speed at the maximum power condition can be obtained from

$$U = \omega R = \frac{V_1}{2} \qquad \omega = \frac{V_1}{2R} = 295\,\text{rpm}$$

Example 12.7 Maximum Power Output for a Pelton Wheel Turbine

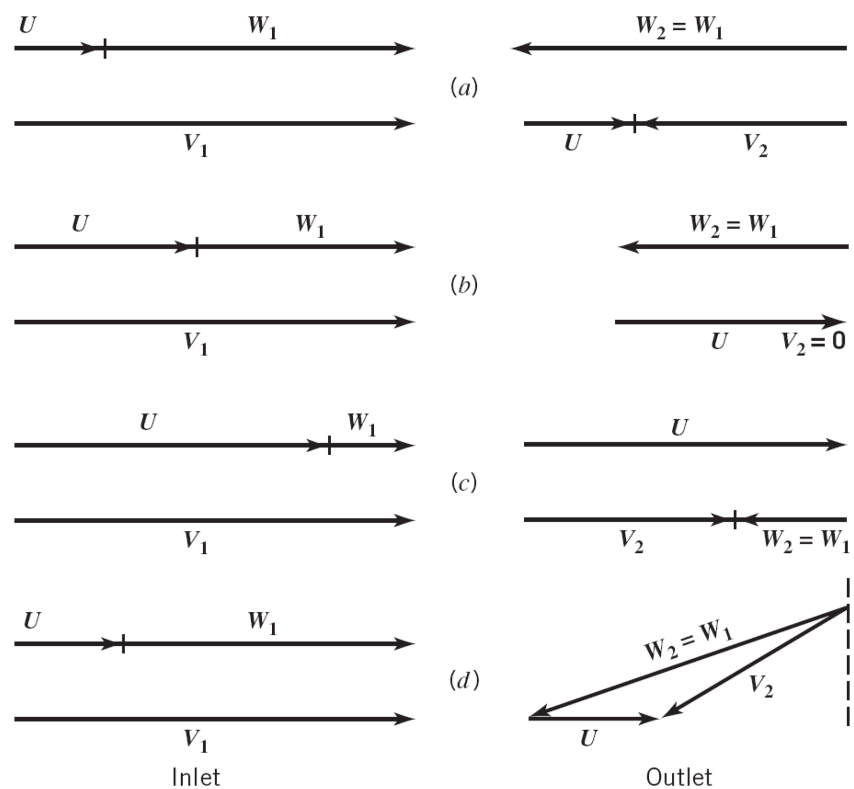
- Water flows through the Pelton wheel turbine shown in Fig. 12.24. For simplicity we assume that the water is turned 180° by the blade. Show, based on the energy equation, that the maximum power output occurs when the absolute velocity of the fluid exiting the turbine is zero.

Example 12.7 Solution[1/2]

(51) $$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta) = 2\rho Q (U^2 - U V_1)$$

For this impulse turbine with $\beta = 180^o$, the velocity triangles simplify into the diagram types shown in Fig. E12.7. Three possibilities are indicated:

(a) The exit absolute velocity, V_2, is directed back toward the nozzle.



Example 12.7 Solution$^{2/2}$

(b) The absolute velocity at the exit is zero, or
(c) The exiting stream flows in the direction of the incoming stream.

The maximum power occurs when $U=V_1/2$.
If viscous effects are negligible, when $W_1=W_2$ and we have $U=W_2$, which gives $V_2=0$

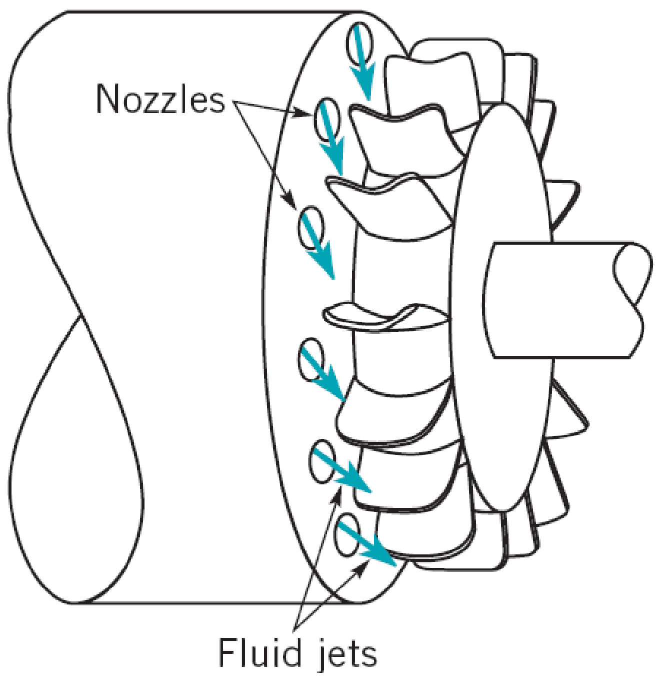
Consider the energy equation for flow across the rotor we have

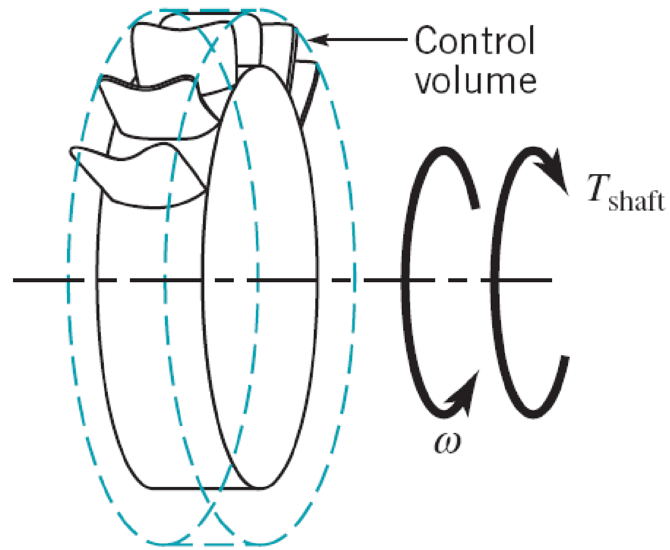
$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_T+h_L$$

$$\Rightarrow h_T=\frac{V_1^2-V_2^2}{2g}-h_L$$

⇒ $V_2=0$

Second Type of Impulse Turbines 1/3

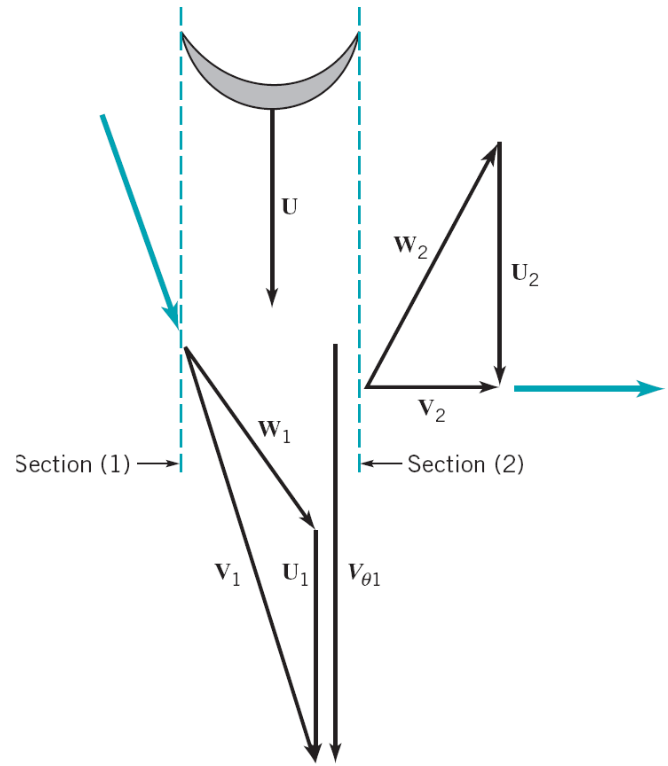




A multinozzle, non-Pelton wheel impulse turbine commonly used with air as the working fluid.

Second Type of Impulse Turbines 2/3

- ❖ A circumferential series of fluid jets strikes the rotating blades which, as with the Pelton wheel, alter both the direction and magnitude of the absolute velocity.
- ❖ The inlet and exit pressure are equal.
- ❖ The magnitude of the relative velocity is unchanged as the fluid slides across the blades.



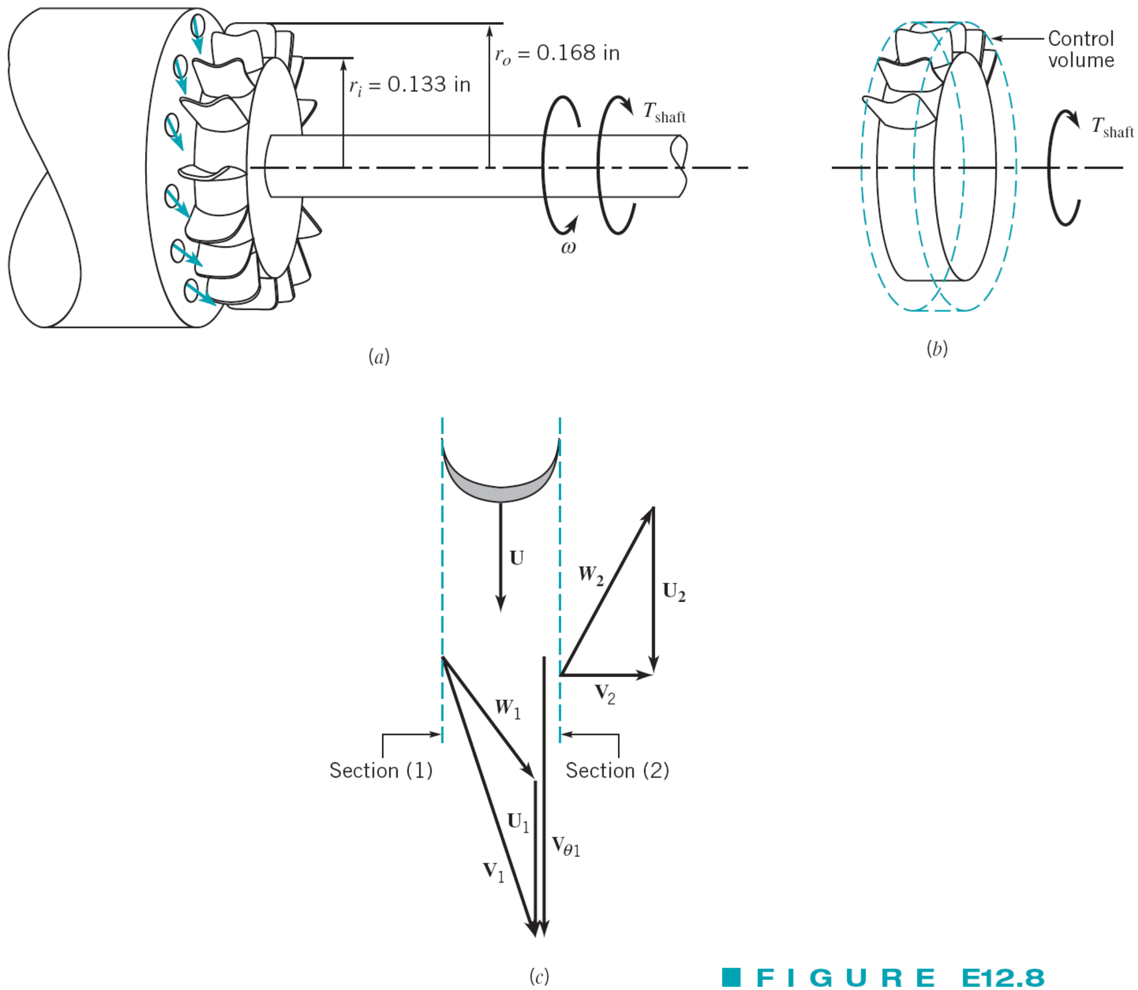
Second Type of Impulse Turbines 3/3

- ❖ In order for the absolute velocity of the fluid to be changed as indicated during its passage across the blade, the blade must push on the fluid in the direction opposite of the blade motion.
  - ⇨ The fluid pushes on the blade in the direction f the blades motion – the fluid does work on the blade.

Example 12.8 Non-Pelton Wheel Impulse Turbine 1/2

- An air turbine used to drive the high-speed drill used by your dentist is shown in Fig. E12.8a. Air exiting from the upstream nozzle holes force the turbine blades to move in the direction shown. Estimate the shaft energy per unit mass of air flowing through the turbine under the following conditions. The turbine rotor speed is 300,000 rpm, the tangential component of velocity out of the nozzle is twice the blade speed, and the tangential component of the absolute velocity out of the rotor is zero.

Example 12.8 Non-Pelton Wheel Impulse Turbine 2/2



■ FIGURE E12.8

Example 12.8 Solution

For simplicity we analyze this problem using an arithmetic mean radius

$$r_m = \frac{1}{2}(r_o + r_i)$$

(5)

$$w_{shaft} = -U_1 V_{\theta 1} + U_2 V_{\theta 2}$$

$$V_{\theta 1} = 2U \qquad V_{\theta 2} = 0$$

$$U = \omega r_m = \ldots = 394 ft/s$$

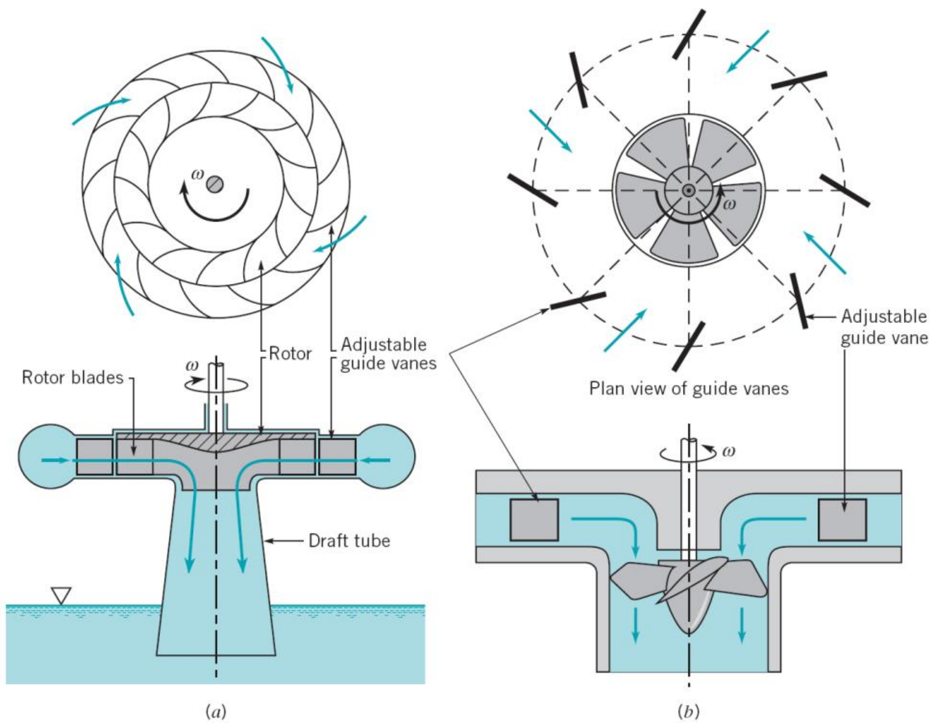
$$w_{shaft} = \ldots = -9640 ft \cdot lb/lbm$$

Reaction Turbines 1/2

- Best suited for higher flowrate and lower head situations such as are often encountered in hydroelectric power plants associated with a dammed river.
- The working fluid completely fills the passageways through which it flows.
- The angular momentum, pressure, and the velocity of the fluid decrease as it flows through the turbine rotor – the turbine rotor extracts energy from the fluid.

Reaction Turbines 2/2

❖ The variety of configurations: radial-flow, mixed flow, and axial-flow.



(*a*) Typical radial-flow Francis turbine. (*b*) typical axial-flow Kaplan turbine.

Dimensionless Parameters for Turbines 1/2

- As with pumps, incompressible flow turbine performance is often specified in terms of appropriate dimensionless parameters

The flow coefficient $C_Q = \dfrac{Q}{\omega D^3}$

Head rise coefficient $C_H = \dfrac{g h_T}{\omega^2 D^2}$

Power coefficient $C_p = \dfrac{\dot{W}_{shaft}}{\rho \omega^3 D^5}$

Dimensionless Parameters for Turbines 2/2

- On the other head, turbine efficiency is the inverse of pump efficiency

$$\eta = \frac{\dot{W}_{shaft}}{\rho g Q h_T}$$

Similarity Laws for Turbines

- For geometrically similar turbines and for negligible Reynolds number and surface roughness difference effects, the relationship between the dimensionless parameters are given

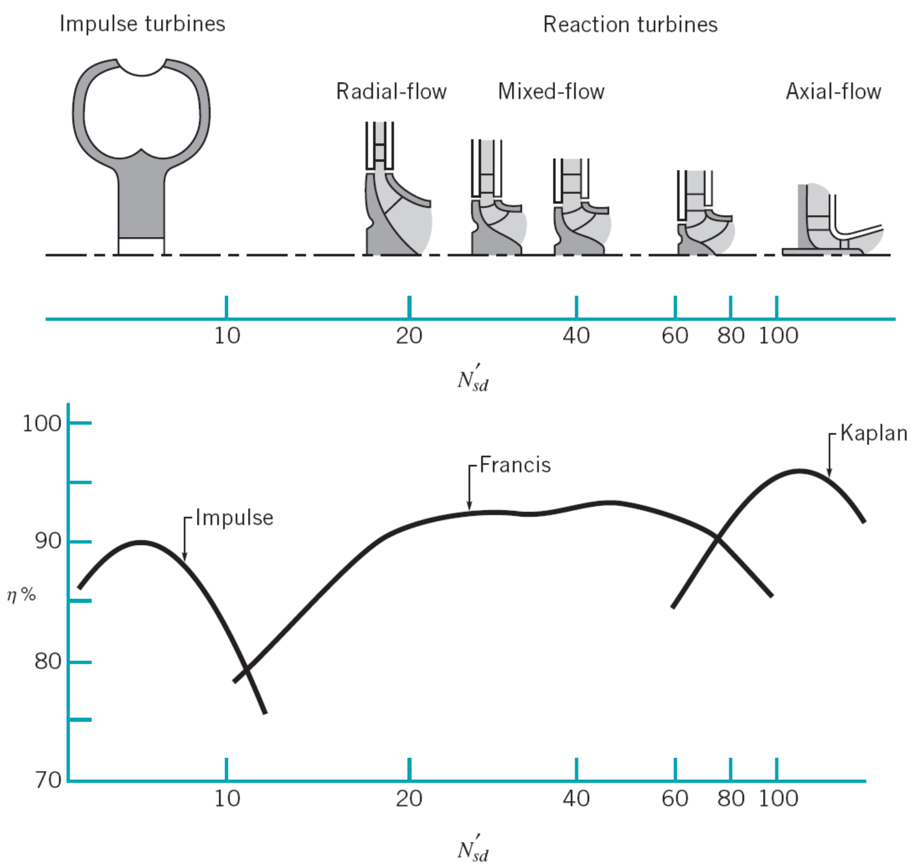
$$C_H = \phi_1(C_Q) \qquad C_p = \phi_2(C_Q) \qquad \eta = \phi_3(C_Q)$$

Power Specific Speed 1/2

- The design engineer has a variety of turbine types available for any given application.
- It is necessary to determine which type of turbine would best fit the job before detailed design work is attempted.
- As with pump, the use of a specific speed parameter can help provide this information

$$N'_s = \frac{\omega\sqrt{\dot{W}_{shaft}/\rho}}{(gh_T)^{5/4}} \Rightarrow N'_{sd} = \frac{\omega(rpm)\sqrt{\dot{W}_{shaft}(bhp)}}{[h_T(ft)]^{5/4}} \quad (53)$$

Power Specific Speed 2/2



Typical turbine cross sections and maximum efficiencies as a function of specific speed.

- Provide a guide for turbine-type selection.
- The actual turbine efficiency for a given turbine depends very strongly on the detailed design of the turbine.

Example 12.9 Use of Specific Speed to Select Turbine Type

- A hydraulic turbine is to operate at an angular velocity of 6 rev/s, a flowrate of 10 ft^3/s (0.2832 m3/s), and a head of 20 ft (6.1 m). What type of turbine should be selected? Explain.

Example 12.9 Solution

$$\omega = 6 rev / s = 360 rpm$$

Assumed efficiency $\eta = 94\%$

$$\dot{W}_{shaft} = \gamma Q z \eta = (62.4 lb/ft^3)(10 ft^3/s)\left[\frac{20 ft (0.94)}{550 ft \cdot lb/s \cdot hp}\right] = 21.3 hp$$

$$N'_{sd} = \frac{\omega \sqrt{\dot{W}_{shaft}}}{(h_T)^{5/4}} = 39.3$$

(Fig. 12.32) → **A mixed-flow Francis turbine would probably give the highest efficiency and an assumed efficiency of 0.94 is appropriate.**